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Online Vehicle Mass Estimation Using Recursive Least Squares and Supervisory Data Extraction

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Abstract—**This paper examines the online estimation of onroad vehicles' mass. It classifies existing estimators based on the dynamics they use for estimation and whether they are *event-seeking* or *averaging*. It then proposes an algorithm comparable to this literature in accuracy and speed, but unique in its minimal instrumentation needs and ability to provide conservative mass error estimates, in the 3σ sense. The algorithm builds on the simple idea, inspired by *perturbation theory*, that inertial dynamics dominate vehicle motion over certain types of maneuvers. A *supervisory algorithm* searches for those maneuvers, and feeds the resulting filtered data into a recursive least squares-based mass estimator and conservative mass error estimator. Both simulation and field data demonstrate the viability of the resulting approach.**

Keywords: mass estimation, singular perturbation theory, recursive least squares, supervisory data extraction

I. INTRODUCTION

GROUND vehicle accidents annually kill at least 40,000 Americans and injure 2.8 million more [1]. They are the leading killer for 2- to 33-year old civilians [1], and a recognized fatal threat to troops and peacekeepers as well [2]. They cost America $230.6 billion in the year 2000 alone.

The past few decades have seen concerted efforts to improve vehicle safety. These efforts have revolutionized automotive safety technologies, but progress in road safety has not been commensurate [3]. Factors explaining this discrepancy include increased traffic, increased road speeds, and a certain degree of *risk compensation*, perhaps to the extent of *homeostasis*, via *behavioral feedback* [3]. This implies that further technology leaps will be needed for making roads safer as they become more congested, vehicles become larger and more powerful, and drivers possibly become more aggressive.

Active safety technologies appear slated to spawn significant improvements in road safety in the coming years. One particularly promising active safety technology is electronic stability control. It has the potential to increase drivers' likelihood to maintain control of their vehicles during aggressive maneuvers by as much as 34% [4]. This may reduce sedan and SUV rollover likelihoods in single-vehicle accidents by up to 71% and 84%, respectively [5]. The resulting safety gains may be enormous, since rollovers account for 62% of all deaths in SUV accidents [6]. For these reasons, new American legislation mandates electronic stability control on all vehicles heavier than 10,000lb by 2009, and on all other vehicles by 2012. Similar mandates have been legislated in Europe and other parts of the world [7].

The viability of an active safety controller depends on the accuracy of the vehicle model used in designing it. This makes active safety system calibration challenging for vehicles with highly variable loads, such as trucks and SUVs. Calibration to nominal loading may compromise safety at high loads, while calibration to maximal loading may visibly penalize handling at lower loads. This necessitates *adapting* active safety systems to variations in loading. Such adaptation requires the online estimation of inertial parameters, especially mass and c.g. height. This paper focuses on mass estimation. A following paper will address c.g. height estimation.

The literature presents many excellent mass estimators, but few vehicles outside the pricier market sectors incorporate them. There are important *practical* needs that a mass estimator should meet to be viable, especially for economy-priced vehicles. It should, for instance, be:

- **Simple** enough to run in real time despite onboard processing limitations;
- **Accurate** enough to estimate mass within, say, 3-5%;
- **Fast** enough to detect changes in a vehicle's loading shortly after it is started and driven onto the road;
- **Reliable** enough to operate successfully despite instrumentation failures;
- **Robust** to road disturbances (e.g., road grade) and variations in vehicle dynamics (e.g., drag);
- Capable of estimating not only vehicle mass, but also **error bounds** on this mass, in the 3σ sense; and,
- **Inexpensive** enough to penetrate the economy-priced vehicle market. This often translates into a **minimal instrumentation** requirement.

With these requirements in mind, this paper begins by surveying the many vehicle mass estimation algorithms described by the literature (Section II). These algorithms are broadly classified based on whether they are *event-seeking* or *averaging*, and also based on the vehicle dynamics they utilize for mass estimation. None of these algorithms are found to satisfy all of the above requirements simultaneously. Furthermore, a closer examination of two particular averaging algorithms that utilize longitudinal

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dynamics for mass estimation reveals that the absence of road grade measurements can significantly penalize estimation speeds (Section III). To address this difficulty, we use simple ideas from *perturbation theory* to reveal the existence of a range of vehicle states and excitation frequencies over which inertial effects dominate longitudinal vehicle dynamics (Section IV). Using this key idea, we construct a novel mass estimator that combines supervisory data extraction with recursive least squares to estimate both vehicle mass and mass error, in the 3σ sense (Section V). The viability of this estimator is demonstrated both in simulation and using field test data (Section VI). Finally, the paper presents a discussion of these results plus some conclusions (Section VII).

II. SURVEY OF MASS ESTIMATION LITERATURE

The literature presents many algorithms for online vehicle mass estimation. We classify these algorithms based on whether they are *event-seeking* or *averaging*. An *averaging* algorithm is one that continuously updates its mass estimate based on measurements of certain vehicle inputs and outputs. *Event-seeking* algorithms, in contrast, continuously monitor vehicle motion for events – such as very sharp accelerations – that conduce to mass estimation, and only estimate mass during such events. We also classify the published mass estimation solutions based on the dynamics they use for such estimation. In particular, the literature presents algorithms for mass estimation based on *suspension dynamics*, *lateral/yaw dynamics*, *powertrain dynamics*, and *longitudinal dynamics*. Each of these categories is reviewed briefly below.

- **Suspension dynamics:** Since vehicle mass directly affects vertical suspension deflections, the availability of suspension sensors such as LVDTs provides an excellent opportunity for mass estimation. Rajamani and Hedrick capitalize on this fact in developing adaptive observers for estimating both suspension states and parameters, including vehicle mass [8]. Furthermore, Kim and Ro demonstrate the viability of quarter-car suspension models as bases for such estimation [9]. Both papers adopt an averaging approach to mass estimation.
- **Lateral/yaw dynamics:** The mass of a vehicle directly affects the relationship between its lateral accelerations and the forces (e.g., due to banking, steering, etc.) causing them. Both Best and Gordon [10] and Wenzel *et al.* [11] capitalize on this fact in developing extended Kalman filters that estimate both vehicle states (e.g., yaw rate) and parameters (including mass) from lateral motion measurements. This is an averaging approach.
- **Powertrain dynamics:** The first natural frequency – also known as the *shuffle frequency* – of a vehicle's cardan shaft depends directly on the vehicle's mass. Fremd [12] creatively capitalizes on this fact in proposing an averaging mass estimator whose input is a direct measurement of this shuffle frequency.
- **Longitudinal dynamics – averaging methods:** The mass of a vehicle directly affects the relationship between the net longitudinal force on the vehicle and its longitudinal acceleration. Many researchers capitalize on this simple fact in proposing vehicle mass estimators. Bae *et al.* , for instance, propose an averaging recursive least squares estimator that utilizes longitudinal force, acceleration, and GPS-based road grade measurements to determine vehicle mass and aerodynamic drag [13]. Grieser proposes a similar averaging estimator in which aerodynamic drag forces are simulated online and subtracted from force measurements, rather than estimated [14]. Vahidi *et al.* propose a similar averaging estimator that does not require road grade measurements and estimates vehicle mass, drag, and road grade simultaneously using minimal instruments [15-17]. The algorithm accommodates the time-varying nature of aerodynamic drag and road grade through *multi-rate forgetting* [15-17]. Finally, Winstead and Kolmanovsky propose an extended Kalman filter that estimates both longitudinal vehicle states and parameters (including mass) for adaptive cruise control [18].
- **Longitudinal dynamics – event-seeking methods:** Sharp longitudinal accelerations and decelerations excite a vehicle's mass significantly, thereby making this mass easier to estimate. With this in mind, Breen proposes using sharp controlled accelerations and decelerations as part of an event-seeking mass estimation method [19]. Similarly, Klatt proposes to estimate vehicle mass specifically during the sharp accelerations and decelerations introduced by gear shifting [20]. Reiner *et al.* propose a similar mass estimator that explicitly compensates for wheel inertia [21]. Further interesting extensions of this event-seeking approach are proposed by Genise [22], Zhu *et al.* [23-24], and Bellinger *et al.* [25].

Given the wealth and depth of the above literature, one may wonder why many commercial vehicles have yet to be equipped with online mass estimation. A full examination of this question is beyond the scope of this paper, which briefly highlights some key limitations of this literature instead. Many of the above estimation algorithms require sensors that may be too expensive for economy-priced cars. Shuffle frequency sensors, for example, may not be available on all cars due to their cost. Similarly, suspension stroke sensors may also be too expensive for economy-priced cars, especially since every tractor and trailer in an articulated vehicle must be equipped with them in order for total vehicle mass to be estimated. Event-seeking estimators are often susceptible to noise during the events used for mass estimation. They are also often vehicle-specific. A mass estimator that targets gear shift events as a basis for estimation, for instance, may not be applicable in vehicles with continuously variable transmissions. Estimating vehicle mass by correlating longitudinal or lateral forces and accelerations through an averaging algorithm (e.g., recursive least squares or extended Kalman filtering) is promising. However, one must keep in mind that the total longitudinal or lateral force acting on a vehicle is influenced not only by inertial effects, but also by road banking/grade, aerodynamic drag, etc. This leads to an important question: *can vehicle mass be estimated without direct measurement of road grade and aerodynamic drag?* To answer this question, Section III compares two mass estimators that utilize averaging and longitudinal dynamics for estimation, namely, Vahidi *et al.*'s algorithm [15-17] and Bae *et al.*'s algorithm [13]. The former estimates road grade and accommodates for its variation through multi-rate forgetting, while the latter measures road grade directly.

III. Influence of Road Grade Measurements on Mass Estimation

Figure 1 examines the problem of estimating the mass of a generic sport utility vehicle (from CarSim [26]) using the algorithms by Vahidi *et al.* (labeled: RLS-MFF) and Bae *et al.* (labeled: RLS with GPS). It shows the mass estimates based on these two algorithms, plus the vehicle's true mass and ±500 lb bounds on this mass. In implementing these algorithms, we manually tuned the forgetting factors in Vahidi *et al.*'s algorithm to furnish near-optimal results. We also simulated ideal noise-free sensors for all measured quantities. The vehicle follows a standard FTP-72 velocity profile, as shown in Figure 2.a. Very small road grades are imparted on the vehicle through the terrain profile in Figure 2.b.

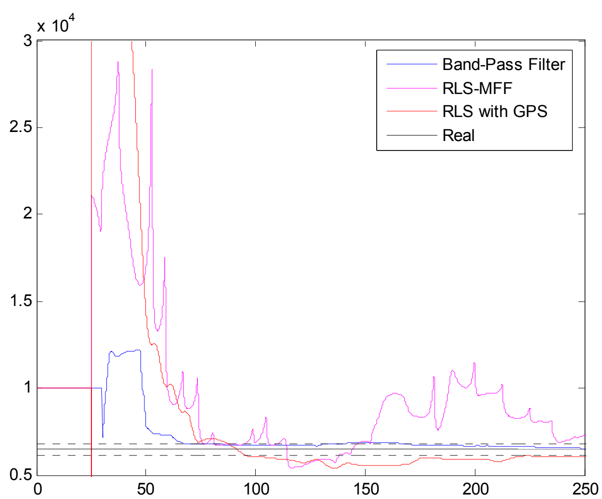

Figure 1: Mass Estimation Comparison

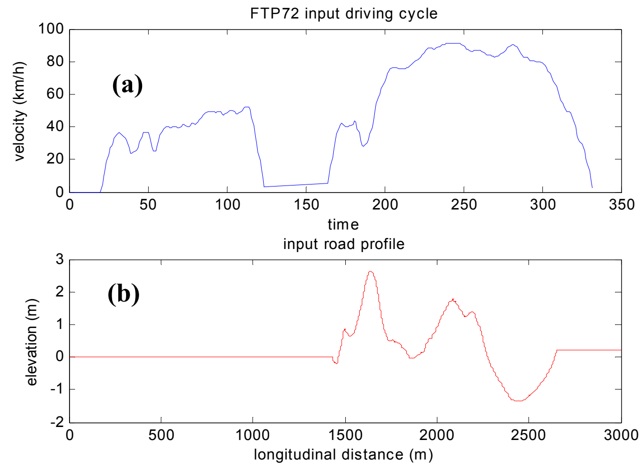

Figure 2: Inputs for Estimator Comparison

As Figure 1 shows, both Vahidi *et al.*'s and Bae *et al.*'s algorithms gradually converge towards the correct vehicle mass range. The convergence is generally faster, however, when direct road grade measurements are available through, say, GPS sensors. In other words, the absence of road grade measurements can significantly affect the speed of vehicle mass estimation. This raises an interesting question, namely, *is it possible to eliminate road grade from the vehicle mass estimation problem?* In other words, is it possible to estimate vehicle mass without measuring or estimating road grade? Section IV addresses this question in depth.

IV. Relative Dominance of Inertial, Drag, and Road Grade Forces

Figure 3 compares the power spectral densities of the various forces acting on the vehicle examined in Section III for the maneuver in Figure 2.

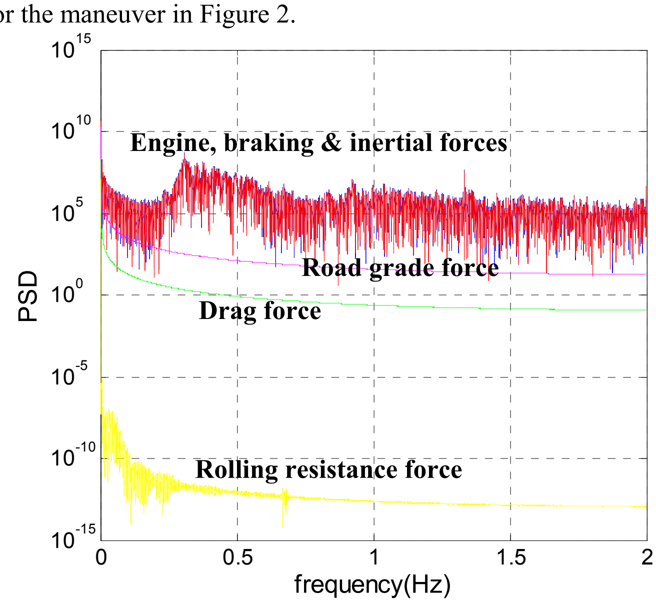

Figure 3: Comparison of Longitudinal Force Spectral Densities

Examination of Figure 3 shows that inertial forces dominate longitudinal vehicle dynamics, and hence almost equal the difference between the engine and brake forces, over almost all frequencies. Drag, road grade-induced, and rolling resistance forces only affect vehicle dynamics at low frequencies. To gain further insight into these observations, consider the following simple longitudinal vehicle dynamics model:

$$m\dot{v}_x = F_e - F_b - \frac{1}{2}\rho A_d C_d v_x^2 - mg\sin\varsigma - mgf_r\cos\varsigma \quad (1)$$

In this equation, m denotes the mass of the vehicle and v_x denotes its longitudinal velocity. The net longitudinal force acting on the vehicle equals the effective engine force at the wheels, F_e, minus the effective braking force at the wheels, F_b, the aerodynamic drag force, the road grade-induced longitudinal force, and the rolling resistance force. Aerodynamic drag is expressed in terms of air density, ρ, effective frontal area, A_d, a drag coefficient, C_d, and longitudinal vehicle velocity squared. Both the rolling resistance force and the road grade-induced force depend on the vehicle's mass, the acceleration of gravity, g, and the road grade angle, ς. Finally, total rolling resistance is assumed to be a constant fraction, f_r, of vehicle weight. Linearizing Equation (1) for very small deviations around a constant velocity and constant road grade furnishes:

$$\begin{aligned} m\delta\dot{v}_x &= \delta F_e - \delta F_b - \rho A_d C_d v_x \delta v_x \\ &\quad - mg\cos\varsigma\delta\varsigma + mgf_r\sin\varsigma\delta\varsigma \end{aligned} \quad (2)$$

where the symbol δ denotes a small deviation in a given quantity. Now suppose that the velocity deviation, δv_x, is a sinusoidal function of time with some frequency ω. Then, using the Bachman-Landau order symbols [27], we find that:

$$\left.\begin{aligned} m\delta\dot{v}_x = O(\omega^2) \\ \rho A_d C_d v_x \delta v_x = O(\omega) \end{aligned}\right\} \Rightarrow m\delta\dot{v}_x >> \rho A_d C_d v_x \delta v_x \quad (3)$$

where the order symbols are defined in the limit as $\omega \uparrow \infty$. In other words, we conclude that inertial effects dominate Equation (2) vis-à-vis aerodynamic drag at infinitely increasing frequencies. This is a very intuitive result, but the Bachman-Landau order symbols and the broader theory of *singular perturbations* provide it with some mathematical grounding [27]. Furthermore, suppose that road grade deviations, $\delta\varsigma$, diminish more rapidly as $\omega \uparrow \infty$ than velocity deviations, δv_x, i.e., $\delta\varsigma << \delta v_x$. Then we conclude that inertial dynamics will dominate the longitudinal behavior of the given vehicle at increasing frequencies. This simple idea is the foundation for the mass estimation algorithm in Section V.

V. Proposed Mass and Mass Error Estimation Algorithms

The proposed mass estimator builds on the simple proposition that when vehicle motion is predominantly longitudinal, the *high-frequency component* of that motion obeys:

$$m\delta\dot{v}_x = \delta F_e - \delta F_b \quad (4)$$

Equation (4) is a *singularly perturbed* version of Equation (2), valid only in the limit as $\omega \uparrow \infty$. The proposed estimator explicitly searches for conditions under which Equation (4) is approximately valid, then uses Equation (4) as a basis for mass estimation. Specifically, mass estimation proceeds in five steps:

Step 1: At every estimation time step, measure/obtain the following quantities:

1. Longitudinal acceleration, dv_x/dt, using an onboard accelerometer;
2. Vehicle yaw rate, obtained from a yaw rate sensor;
3. An estimate of the total longitudinal engine and braking force acting on the vehicle, $F_e - F_b$, available from the vehicle's electronic control units (ECUs);
4. An estimate of vehicle velocity, v_x, obtained from its wheel speed sensors; and,
5. An estimate of tire slip ratios, obtained from the vehicle's electronic stability controller.

Step 2: Use a *fuzzy supervisor* to determine whether the vehicle's motion is significant, and predominantly longitudinal. Specifically, test to make sure that:

1. The vehicle's yaw rate is less than 1 degree/sec;
2. Vehicle acceleration exceeds 1 m/s^2;
3. Vehicle velocity exceeds 10 km/h;
4. Wheel slip ratios do not exceed 0.05; and,
5. The net longitudinal force exerted by the engine and brakes on the vehicle (estimated by the vehicle's ECUs) exceeds 500N.

These conditions do not provide a rigorous definition of "significant, predominantly longitudinal" motion. They do, however, act as a potentially trainable/tunable *supervisor* for the proposed mass estimator. This supervisor can be strict, in the sense of only allowing mass estimation to proceed if the above rules are met exactly. Alternatively, it can be implemented as a *fuzzy* supervisor which gradually switches mass estimation off based on proximity to the above rules.

Step 3: Use a simple lead-lag band-pass filter to extract the high-frequency components of the vehicle acceleration and longitudinal force signals, while attenuating measurement noise. Denote those high-frequency force and acceleration components by $F_e^* - F_b^*$ and a_x^*, respectively.

Step 4: Assuming that $F_e^* - F_b^* = ma_x^*$, use the recursive least squares algorithm [28] to estimate vehicle mass.

Step 5: Estimate the variance of the mass estimate as follows [see 28 for justification]:

$$\sigma_M^2 = \frac{\sigma^2}{\sum_i (a_{x,i}^*)^2} \tag{5}$$

In this equation, σ_M^2 denotes the predicted variance of the mass estimate. Using this variance, one can provide conservative mass error bounds, in the $\pm 3\sigma_M$ sense. The quantity $a_{x,i}^*$ denotes the filtered longitudinal acceleration measurement for some time step i, and the denominator of Equation (5) squares this quantity at every estimation time step and sums the result over time. The numerator, σ^2, of Equation (5) denotes the variance of the effective longitudinal engine and brake force measurement/estimation process. The standard recursive least squares algorithm provides means for computing this quantity [28], but in this work, we assume that an upper bound on this sensor error variance is known *a priori*. We use this upper bound in computing mass estimation error, thereby obtaining a conservative mass error estimate.

This concludes the description of the mass and mass error estimators. Section IV demonstrates these estimators' viability.

VI. Mass Estimation Case Studies

Figure 1 compares the proposed mass estimation algorithm to Bae *et al.'s* and Vahidi *et al.*'s algorithms. The proposed algorithm's mass estimate (without error bounds) is labeled: "Band-Pass Filter". It converges to the ±500lb error range faster than the estimates based on Bae *et al.*'s and Vahidi *et al.*'s algorithms. This is quite encouraging if one considers the fact that Bae *et al.*'s algorithm, in particular, assumes the availability of GPS-based road grade measurements, whereas the proposed algorithm does not. A key strength of the proposed algorithm is that it filters road grade effects out, thereby eliminating the need for measurements or estimates of road grade. To see this, examine Figures 4-6 below.

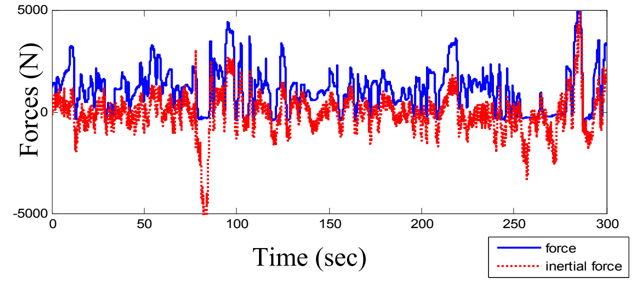

Figure 4: Raw Force and Inertial Force Measurements, Chrysler Proving Grounds

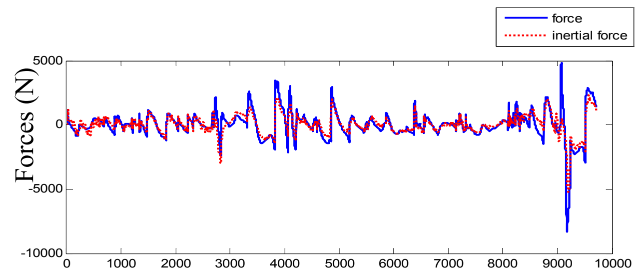

Figure 5: Filtered Force and Inertial Force Measurements, Chrysler Proving Grounds

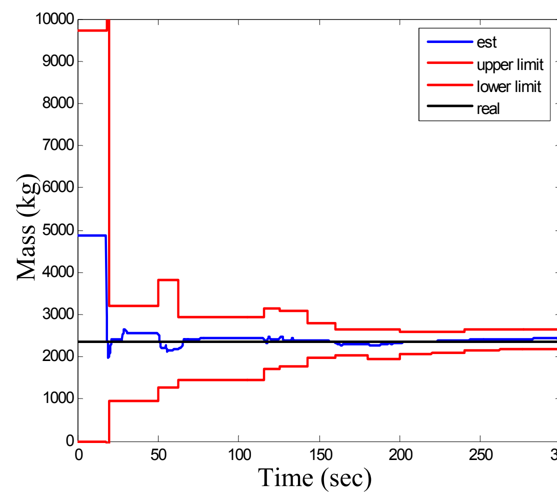

Figure 6: Mass Estimation Results, Chrysler Proving Grounds

Figures 4-6 were generated using experimental data obtained from an instrumented SUV tested in the Chrysler Proving Grounds (Chelsea, MI). Figure 4 plots the vehicle's measured acceleration multiplied by its true mass vis-à-vis its net wheel force. Clearly, these two quantities do not match, because a significant portion of wheel force compensates for aerodynamic drag, road grade loads, rolling resistance, etc. Figure 5, in comparison, plots the vehicle's *filtered* net longitudinal wheel force vis-à-vis the vehicle's true mass multiplied by its *filtered* acceleration. These filtered force and acceleration quantities were obtained using Steps 1-3 in the proposed estimation algorithm. The fact that they clearly match one another demonstrates the viability of the algorithm. Finally, Figure 6 plots the estimated vehicle mass and its upper and lower bounds versus time. The mass estimate converges very quickly to an accurate value, but the upper and lower bounds do not converge nearly as fast. This reflects the fact that the upper and lower bounds are quite *conservative*. Thus, we have succeeded in developing a mass estimator that converges quickly and accurately to vehicle mass, but simultaneously provides very conservative upper and lower error bounds on this mass. Those upper and lower bounds are more likely to be useful in practical implementation than the mass estimate itself.

VII. Discussion and Conclusions

This paper examines the problem of estimating an onroad vehicle's mass online, in real time, using minimal instrumentation. Such estimation is essential for adaptive calibration of active safety control systems. Such adaptive safety controllers can potentially furnish significant improvements in vehicle safety – especially at higher loads – with minimal compromises in maneuverability at low loads. The literature presents many algorithms for vehicle mass estimation. We review them herein, and classify them both based on the dynamics they use for mass estimation and based on whether they are *averaging* or *event-seeking*. An important conclusion from this literature review is that while many excellent solutions exist for online vehicle mass estimation, few are ideally suited for economy-priced vehicles. In particular, mass estimation based on longitudinal dynamics can be challenging in the absence of potentially expensive road grade measurement capabilities. This paper proposes an alternative approach that builds on simple ideas from singular perturbation theory. Specifically, we emphasize the fact that the high-frequency components of vehicle dynamics are dominated by simple inertial effects when vehicle motion is significant and predominantly longitudinal. We translate these conditions into a *fuzzy supervisor* that focuses only on those components of vehicle maneuvers dominated by inertial effects. Using this supervisor in conjunction with recursive least squares results in an effective mass estimator plus a conservative mass error estimator. Both simulation and experimental results demonstrate the viability of the proposed approach.

In addition to its important advantages, the proposed approach does have some interesting caveats. First, like many other mass estimators, its convergence is dependent on persistence of excitation, which in turn typically depends on driver aggressiveness. Benign drivers are less likely than aggressive drivers to excite inertial dynamics to the point of enabling rapid mass estimation. While one may argue that the need for mass estimation is lessened by benign driving, rapid convergence to accurate mass estimates may still be desirable even in benign driving. In such situations, it may be necessary to intentionally disturb or *dither* the given vehicle's dynamics to obtain good mass estimates. Such dithering may provide persistence of excitation guarantees that may not exist otherwise. Secondly, it is interesting to note that focusing mass estimation on those frequencies where vehicle dynamics are dominated by inertia but sensor measurements are still accurate may render the proposed estimator particularly vulnerable to powertrain *shuffle dynamics*. If a vehicle shuffles sufficiently to prevent accurate mass estimation using the proposed approach, it may be necessary to limit estimation to braking conditions, which are not as vulnerable to drivetrain shuffle. This is an interesting implementation caveat that the authors have yet to address. Despite these two caveats, the authors believe that the proposed mass estimator addresses many practical mass estimation needs, and thus promises to be a potentially viable solution for practical applications.

VIII. Acknowledgments

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