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Neural Control for Power Ascent of a TRIGA Reactor

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Abstract—**A basic control problem in a nuclear research reactor consists of increasing or decreasing the neutron power from a certain level R_0 to a new desired level R_1 and maintain the reactor stable at the new power level. For security reasons, this task must be performed in such a way that, during the power ascent, the instantaneous period of the reactor must always be greater than or equal to a lower limit value. To solve this problem, avoiding the difficulties associated with the physical modeling of the nuclear process, in this paper, we propose to use an indirect adaptive control scheme in which a single layer second order differential neural network achieves the on-line identification based only on three variables: the external reactivity, the fuel temperature, and the neutron power. The mathematical model provided by this identification process is employed to accomplish the control action in two stages. During the transient stage, the controller objective is to maintain the plant on a constant period. Once the desired power is reached, the control action is switched to a regulation stage. This identifier-controller is tested by simulation. Instead of the real plant, an eighth order physical model of a TRIGA reactor considered as a black box is used. The results show a good performance of the suggested approach.**

I. Introduction

During almost five decades, TRIGA reactors have constituted an important factor for the development of peaceful applications of nuclear energy. Nowadays, as a result of their prompt negative temperature coefficient of reactivity which guarantees an intrinsically safe operation, they are the most widely used research reactors in the world with an installed base of 65 reactors in 24 countries [1]. In Mexico, the National Institute of Nuclear Research (ININ) has a 1-MW TRIGA Mark III reactor which is mainly used for the study of radiation effects in several substances (neutron activation analysis, aging analysis, etc.). Likewise, it is used for personnel training and for production of radioisotopes which are employed in medical, industrial and agricultural applications. During all these activities, the reactor power is increased or decreased from a certain level R_0 to a new desired level R_1 and then the reactor is maintained at that level. However, for security reasons, this task must be performed in such a way that, during the power ascent, the instantaneous period must always be greater than or equal to a lower limit value. In the event that this constraint is not complied, a scram (automatic shut down of the reactor) occurs. Currently, this reactor is controlled manually by an expert operator or automatically by a PID-type scheme with constrained control action. Although this scheme has been designed without any model of the reactor, its tuning process is time-consuming. Besides, regarding the capabilities of the reactor, the performance provided by this controller can be still improved, for example, reducing the transient time to get different levels of power. Given the constraints associated with the reactor control problem, it could be considered that, in this case, the natural solution consists of employing the classical optimal control theory. Nonetheless, the direct application of Pontryagin's maximum principle or Bellman's dynamic programming to find online the optimal control could be very costly from a computational point of view. Moreover, these techniques tolerate neither modeling errors nor unforeseen perturbations. To overcome these drawbacks, some works have been reported in which the basic operation of the reactor is carried out automatically and in a minimum time satisfying, at the same time, the period constraint without resorting to the direct use of the optimal control theory. For example, in [2], it was established that the unique optimal solution to the reactor control problem is to achieve the control action in two stages: 1) During the transient stage, the instantaneous period of the reactor must be maintained equal to a minimum allowable limit. 2) Starting from the time in which the specified power level is attained, the neutron power must remain constant. Using this principle, Aleksakov developed a controller based on the physical model of the reactor. In a completely independent way using another approach, in [3], the same principle of the Aleksakov's work was found. With this principle, Bernard developed the MIT-SNL Period-Generated Minimum Time Control Laws approach. This approach needs the dynamic period equation which is derived from the physical model. Unfortunately, for our particular case, the last two approaches are not effective because we do not know the current values of the Mexican TRIGA reactor's parameters (the last time these values were computed was in 1994 [13]). On the other hand, in [4], it was proposed an associative stochastic automaton for reactor power ascent. Although this system does not require any physical model of the plant, its performance is inferior with respect to Bernard's controller. In addition, a long previous training process is required. In pursuit of a model-free controller (but still with a relatively simple structure) able to work satisfactorily during long periods of time and with a very short off-line learning phase, we are particularly interested in applying a special kind of artificial neural networks (ANN), the so called differential

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neural networks (DNN). For the aforementioned control problem of the reactor, this approach is particularly attractive and promising due to its capability of robust universal approximation. Roughly speaking, ANN can be classified as static ones, using the, so called, back-propagation technique [5] or as recurrent ones [6]. In the first kind of networks, the system dynamics is approximated by a static mapping; therefore, the output of these networks is uniquely determined by the current inputs and weights. These networks have two major disadvantages: a slow learning rate and a high sensitivity to the training data. On the other hand, the second approach incorporates feedback in its structure. Thus, recurrent neural networks overcome many problems associated with the first ones such as global extrema search and consequently they have better approximation properties. Depending on their structure, recurrent neural networks can be classified as (discrete-time) difference ones or (continuous-time) differential ones. Although there exist previous works which report the usage of ANN for nuclear reactor control [7-11], generally the networks employed have been static or discrete multilayer recurrent with a long preliminary off-line learning phase or else, they lack a rigorous proof of the stability of the corresponding closed-loop system based on the ANN. Surprisingly, to the best of our knowledge, no author has considered the possibility of applying DNN to control nuclear processes. Thereby, in this paper, the reactor control problem subjected to the period constraint is solved by the following way: First, to avoid the use of observers, and since the internal dynamics associated with the non measurable variables is stable, a single layer second order DNN is used to accomplish the identification of the uncertain measurable dynamics using only the external reactivity, the fuel temperature, and the neutron power. Second, based on the obtained DNN model, the general control strategy proposed in [2] is implemented. The convergence for each control stage is assured by the use of the power derivative which can be calculated from the period measurement. The workability of the suggested approach is illustrated by a simulation example in which an eighth order nonlinear model of the Mexican TRIGA reactor considered as a black box is used instead of the real plant.

II. Physical Model of a TRIGA Reactor

One of the simplest ways of describing the dynamic behavior of a nuclear reactor is to use the following eighth order nonlinear model with six groups of delayed neutron precursors and one thermal feedback mechanism [12]

$$\dot{n}_t = \frac{\alpha T_o - \beta}{\Lambda} n_t + \sum_{i=1}^{6} \lambda_i C_{i,t} - \frac{\alpha}{\Lambda} n_t T_t + \frac{1}{\Lambda} n_t v_t \quad (1)$$

$$\dot{C}_{i,t} = \frac{\beta_i}{\Lambda} n_t - \lambda_i C_{i,t}, \qquad i = 1, \ldots, 6 \quad (2)$$

$$\dot{T}_t = K(n_t - n_o) - \gamma (T_t - T_o) \quad (3)$$

where constant parameters are defined as: Λ is the effective prompt neutron lifetime (s), λ_i is the radioactive decay constant of ith group neutron precursor (s^{-1}), β_i is the fraction of ith group delayed neutrons, β is the total delayed neutron fraction ($\beta = \Sigma^6_{i=1}\beta_i$), α is the negative temperature reactivity coefficient (°C^{-1}), K is the reciprocal of the reactor heat capacity (°C/($W \cdot s$)), γ is the reciprocal of mean time for heat transfer to the coolant (s^{-1}), T_0 is the initial average temperature, n_0 is the initial power. Likewise, the model variables are defined as: n_t is the neutron power (W) (from $1W$ to $1.1MW$ for the Mexican TRIGA reactor), $C_{i,t}$ is the power of the ith group delayed neutron precursor (W), T_t is the fuel average temperature (°C), and $\rho_{ext,t}$ is the external reactivity which ranges from 0 to 1.4354 (dimensionless). The external reactivity is associated with the displacement of the control rods. The relationship between these variables can be inferred as a static mapping by correlations determined empirically and off-line. Since we do know this relationship, from now on the external reactivity can be considered as the control input of the nuclear system and will be denoted as v_t. The nominal parameters [13], corresponding to Mexico TRIGA MARK III reactor are as follows: $\alpha=0.01359875\,°C^{-1}$, $\beta_1=0.240\times10^{-3}$, $\beta_2=1.410\times10^{-3}$ $\beta_3=1.255\times10^{-3}$, $\beta_4=2.525\times10^{-3}$, $\beta_5=0.737\times10^{-3}$, $\beta_6=0.266\times10^{-3}$, $\beta=6.433\times10^{-3}$, $\lambda_1=0.0124s^{-1}$, $\lambda_2=0.0305s^{-1}$, $\lambda_3=0.1140s^{-1}$, $\lambda_4=0.3013s^{-1}$, $\lambda_5=1.1360s^{-1}$, $\lambda_6=3.0130s^{-1}$, $\Lambda=38\mu s$, $\gamma=0.2s^{-1}$, $K=1/5.21045\times10^4$ $°C/(W\cdot s)$. Due to the assumptions employed to deduce it, discrepancies between the actual plant and the physical model here studied are inevitable. Next, some limitations and drawbacks associated with the use of this model are summarized:

- Although the parameters were assumed to be constants, in fact, they vary according to changes in operating conditions.
- The effect of sensors and actuators is not considered.
- Due to instruments for measuring power of 6 groups of delayed neutron precursors in a nuclear reactor are not available, a control design based on this model could result in complex structured closed loop system since a robust nonlinear observer is required.

Finally, two facts should be noted: First, if the output of the system is n_t then the relative degree of the physical model is *1*. Second, although we do not know the current values of the reactor's parameters we can assure that these ones are always positive. Consequently, the dynamics associated with the ith group delayed neutron precursor power is stable. This can be seen from the equation (2).

A. Period Constraint

The instantaneous period of the reactor can be defined as

$$\tau_t := \frac{n_t}{\dot{n}_t} \quad (4)$$

This variable is generally available for measurement and plays a very important role in the plant control. For security, the power ascent must be achieved in such a way that the instantaneous period must be always maintained greater than or equal to a minimum allowable value, that is, $\tau_t \geq \overline{\tau}_{\min}$. For Mexico TRIGA reactor, the theoretical minimum allowable period is $\overline{\tau}_{\min} = 3s$. However, how the experienced operators of this reactor know, due to the limitations associated with real instrumentation, to avoid frequent scrams is suitable to maintain the instantaneous period at levels greater than or equal to *10s*. Thus, in terms of this practical minimum value, the period constraint can be expressed as

$$\tau_t \geq \overline{\tau}^*_{\min} \tag{5}$$

where $\overline{\tau}^*_{\min} = 10s$.

III. Single Layer Neural Identifier

A. Uncertain Dynamics and Basic Assumptions

Suppose that the *N*-order global dynamics of the reactor under consideration is stable. Likewise, consider that the *n*-order uncertain measurable dynamics ($n \leq N$) of this system can be described, in general, as

$$\begin{aligned} \dot{x}_t &= f(x_t, v_t, t) \\ x_t &\in \Re^n, v_t \in \Re^q, \quad n \geq q \end{aligned} \tag{6}$$

Notice that an alternative representation for (6) always could be done as follows:

$$\dot{x}_t = Ax_t + W^*\sigma(x_t) + Bv_t + \Delta f(x_t, v_t, t) \tag{7}$$

where $A \in \Re^{n\times n}$ is a Hurwitz matrix, $W^* \in \Re^{n\times n}$ is a constant matrix, $B \in \Re^{n\times q}$ is the input matrix, $\sigma(\cdot)$ is the activation vector-function with sigmoidal components, that is, $\sigma(\cdot):=[\sigma_1(\cdot), \ \ldots, \ \sigma_n(\cdot)]^{\mathrm{T}}$

$$\sigma_j(x) := a_{\sigma j}\left[1 + b_{\sigma j}\exp\left(-\sum_{j=1}^{n} c_{\sigma j}x_{j,t}\right)\right]^{-1} \quad \text{for } j = 1,\ldots,n \tag{8}$$

and

$$\Delta f(x_t, v_t, t) := f(x_t, v_t, t) - Ax_t - W^*\sigma(x_t) - Bv_t$$

Hereafter we consider that the following assumptions are complied:

A.1) System (6) satisfies the (uniform on *t*) Lipschitz condition, that is,

$$\begin{aligned} \|f(x,u,t) - f(z,v,t)\| &\leq L_1\|x - z\| + L_2\|u - v\| \\ x,z \in \Re^n;\ u,v &\in \Re^q;\ 0 \leq L_1, L_2 < \infty \end{aligned} \tag{9}$$

A.2) The function $\sigma(\cdot)$ satisfies sector conditions:

$$\tilde{\sigma}_t^T\Lambda_\sigma\tilde{\sigma}_t \leq \Delta_t^T D_\sigma \Delta_t$$

where

$$\Delta_t := \hat{x}_t - x_t \tag{10}$$

$$\tilde{\sigma}_t := \sigma(\hat{x}_t) - \sigma(x_t) \tag{11}$$

and $\Lambda_\sigma \in \Re^{n\times n}$, $D_\sigma \in \Re^{n\times n}$ are known constant positive definite matrices.

A.3) Admissible controls are bounded, to be precise, $U^{adm} := \left\{v \ : \ \|v_t\|^2 \leq \overline{v} < \infty\right\}$. Besides, v_t is such that does not violate the existence of the solution to ODE (6).

A.4) Unmodeled dynamics is bounded by

$$\|\Delta f(x_t, v_t, t)\|^2_{\Lambda_f} \leq \overline{\eta}$$

where $\Lambda_f \in \Re^{n\times n}$ is a constant positive definite matrix.

A.5) The matrix W^* is bounded in the following sense

$$W^*\Lambda_\sigma^{-1}W^{*T} \leq \overline{W}$$

where $\overline{W} \in \Re^{n\times n}$ is a known positive definite matrix.

A.6) There exits a strictly positive defined matrix Q_0 such that if the matrices *R* and *Q* are defined as

$$\begin{aligned} R &:= \overline{W} + \Lambda_f^{-1} \\ Q &:= Q_0 + D_\sigma \end{aligned} \tag{12}$$

then the following matrix Riccati equation

$$A^T P + PA + PRP + Q = 0 \tag{13}$$

has a positive solution *P* (In [15] there are given conditions for matrices *A*, *R* and *Q* which guarantees the existence of $P>0$).

It is worth mentioning that the preceding assumptions are not unusual. On the contrary, they are generally met for physically meaningful dynamic systems and a nuclear reactor is not the exception.

B. Differential neural network

Consider the neural identifier with the following structure

$$\frac{d}{dt}\hat{x}_t = A\hat{x}_t + W_t\sigma(\hat{x}_t) + Bv_t \tag{14}$$

where $\hat{x}_t \in \Re^n$ is the state of the neural network and $W_t \in \Re^{n\times n}$ is the weight matrix which is adjusted on-line by a learning law to minimize the identification error Δ_t. The neural network (14) can be classified as a Hopfield-type one [14]. In spite of its simple structure, the neural network (14) is adequate for our purposes since excepting the pulsed mode and accidents, the reactor dynamics change relatively slowly and therefore it can be sufficiently well approximated by the aforementioned neural identifier. In addition, the controller design based on this structure is considerably simpler. Next, the basic result on the identification process of measurable dynamics (6) by the neural network (14) is formulated:

Theorem 1: If the assumptions **A.1-A.6** are satisfied and the weight matrix W_t of the neural identifier (14) is adjusted by the differential learning law

$$\dot{W}_t = -KP\Delta_t\sigma(\hat{x}_t)^T, \ \ W_0 = W^* \tag{15}$$

where *K* is a positive definite matrix and *P* is the solution of matrix Riccati equation given by (13) then, the "averaged" identification error has the following upper bound

$$\limsup_{T\to\infty}\frac{1}{T}\int_0^T \Delta_t^T Q_0 \Delta_t dt \leq \overline{\eta} \tag{16}$$

Proof: First, the dynamics of the identification error must be determined. From (10), the first derivative of Δ_t is

$$\dot{\Delta}_t = \dot{\hat{x}}_t - \dot{x}_t \tag{17}$$

Substituting (14) and (7) into (17) yields

$$\dot{\Delta}_t = A\Delta_t + W_t\sigma(\hat{x}_t) - W^*\sigma(x_t) - \Delta f \tag{18}$$

Adding and subtracting the term $W^*\sigma(\hat{x}_t)$ and taking into account equation (11), (18) can be expressed as

$$\dot{\Delta}_t = A\Delta_t + \widetilde{W}_t\sigma(\hat{x}_t) + W^*\tilde{\sigma}_t - \Delta f \qquad (19)$$

where $\widetilde{W}_t := W_t - W^*$. On the other hand, the Lyapunov function candidate is selected as

$$V_t := \Delta_t^T P\Delta_t + tr\left[\widetilde{W}_t^T K^{-1}\widetilde{W}_t\right] \qquad (20)$$

The first derivative of V_t is

$$\dot{V}_t = 2\Delta_t^T P\dot{\Delta}_t + 2tr\left[\dot{\widetilde{W}}_t^T K^{-1}\widetilde{W}_t\right] \qquad (21)$$

Since $\widetilde{W}_t := W_t - W^*$ then $\dot{\widetilde{W}}_t = \dot{W}_t$. But $\dot{W}_t$ is given by (15). Consequently,

$$2tr\left[\dot{\widetilde{W}}_t^T K^{-1}\widetilde{W}_t\right] = -2\Delta_t^T P\widetilde{W}_t\sigma(\hat{x}_t) \qquad (22)$$

Therefore substituting (19), and (22) into (21) yields

$$\dot{V}_t = 2\Delta_t^T PA\Delta_t + 2\Delta_t^T PW^*\tilde{\sigma}_t - 2\Delta_t^T P\Delta_f$$

We need to find an upper bound for $\dot{V}_t$. To accomplish this task, first we consider the term $2\Delta_t^T PW^*\tilde{\sigma}_t$. Since this term is an scalar, it is possible to express it alternatively as

$$2\Delta_t^T PW^*\tilde{\sigma}_t = \Delta_t^T PW^*\tilde{\sigma}_t + \tilde{\sigma}_t^T W^{*T} P\Delta_t$$

From assumptions A.2 and A.5 and using the matrix inequality proved in [15]

$$X^TY + Y^TX \le X^T\Gamma^{-1}X + Y^T\Gamma Y \qquad (23)$$

which is valid for any $X, Y \in \Re^{n\times k}$ and for any positive definite matrix $0 < \Gamma = \Gamma^T \in \Re^{n\times n}$, $2\Delta_t^T PW^*\tilde{\sigma}_t$ can be bounded by

$$2\Delta_t^T PW^*\tilde{\sigma}_t \le \Delta_t^T P\overline{W}P\Delta_t + \Delta_t^T D_\sigma\Delta_t \qquad (24)$$

Likewise, it is possible to demonstrate that from (23) and assumption A.4

$$-2\Delta_t^T P\Delta_f \le \Delta_t^T P\Lambda_f^{-1}P\Delta_t + \overline{\eta} \qquad (25)$$

Using (24) and (25) we can find that $\dot{V}_t$ is bounded by

$$\dot{V}_t \le 2\Delta_t^T PA\Delta_t + \Delta_t^T P\overline{W}P\Delta_t + \Delta_t^T D_\sigma\Delta_t + \Delta_t^T P\Lambda_f^{-1}P\Delta_t + \overline{\eta}$$

Adding and subtracting $\Delta_t^T Q_0\Delta_t$ into the right-hand side of the last inequality, the expression $A^TP + PA + P(\overline{W} + \Lambda_f^{-1})P + D_\sigma + Q_0$ is formed. However, this expression, in accordance with assumption A.6, is equal to zero. Then

$$\dot{V}_t \le -\Delta_t^T Q_0\Delta_t + \overline{\eta}$$

Integrating both sides of this inequality from $t = 0$ up to $t = T$, we obtain

$$\int_0^T \Delta_t^T Q_0\Delta_t dt \le V_0 - V_T + \overline{\eta}T \le V_0 + \overline{\eta}T$$

Dividing both sides of the last inequality by T yields

$$\frac{1}{T}\int_0^T \Delta_t^T Q_0\Delta_t dt \le \frac{V_0}{T} + \overline{\eta}$$

Finally, taking lim sup when $T \to \infty$, the conclusion of theorem 1 is proved. ■

Since we will utilize only the external reactivity v_t, the fuel temperature T_t and the neutron power n_t then $x_t \in \Re^2$ and $v_t \in \Re$. Because of the different ranges of the values associated with each reactor variable, these ones must be first normalized (each one is divided by its corresponding maximum value). Thus,

$$x_{1,t} = \frac{n_t}{g_1}, \quad x_{2,t} = \frac{T_t}{g_2},$$

where $g_1 = \max(n_t)$, $g_2 = \max(T_t)$. Such normalization does not affect the identification results. Instead, it permits to DNN (14) works satisfactorily. On the other hand, according to the structure of physical model and for convenience, it is reasonable to select $B = [1\ \ 0]^T$.

IV. Control via the Neural Identifier

In this section, on the basis of the structure provided by the neural identifier (14), the control strategy established in [2] is implemented. Before presenting the controller two additional assumptions must be done:

A.7) Considering that the reactor output is n_t, the relative degree of system (6) is 1.

A.8) System (6) is controllable and its internal dynamics is stable.

Both assumptions are based on the structure of the physical model presented in section II. However, as it will be seen, actually none knowledge of the parameters of this model will be required to control to the reactor.

Proposition 2: A control law which achieves the power ascent of (6) from an initial level R_0 to a desired level R_1 ($R_1 > R_0$) constrained to (5) in a minimum time is given by

$$v_t = \begin{cases} v_{1,t} & \text{for transient stage} \\ v_{2,t} & \text{for regulation stage} \end{cases} \qquad (26)$$

where

$$v_{1,t} = \frac{x_{1,t}}{\overline{\tau}^*_{\min}} - a_{11}\hat{x}_{1,t} - a_{12}\hat{x}_{2,t} - W_t^{11}\sigma_1(\hat{x}_{1,t}) - W_t^{12}\sigma_2(\hat{x}_{2,t}) + \dot{\hat{x}}_{1,t} - \dot{x}_{1,t} \qquad (27)$$

$$v_{2,t} = c(R_1 - x_{1,t}) - a_{11}\hat{x}_{1,t} - a_{12}\hat{x}_{2,t} - W_t^{11}\sigma_1(\hat{x}_{1,t}) - W_t^{12}\sigma_2(\hat{x}_{2,t}) - \dot{x}_{1,t} + \dot{\hat{x}}_{1,t} \qquad (28)$$

a_{11} and a_{12} are elements of the matrix A, W_t^{11} and W_t^{12} are elements of the matrix W_t, and c is a positive constant.

Proof: As it was established in [2], during the transient stage, the instantaneous period of the reactor must be maintained equal to minimum allowable period to achieve the change of the power in a minimum time, that is,

$$\tau_t = \overline{\tau}^*_{\min}$$

but in accordance with the definition of instantaneous period (4), we can obtain

$$\dot{x}_{1,t} = \frac{x_{1,t}}{\overline{\tau}^*_{\min}} \tag{29}$$

The key question is how to find $v_{1,t}$ such that (29) is guaranteed. Notice that system (6) can be represented alternatively as

$$\dot{x}_t = Ax_t + W_t\sigma(x_t) + Bv_t + \delta_t \tag{30}$$

where δ_t is the modeling error. Considering that $B=[1 \quad 0]^T$ and in accordance with (30), the first component of $\dot{x}_t$ is

$$\begin{aligned}\dot{x}_{1,t} &= a_{11}x_{1,t} + a_{12}x_{2,t} + W_t^{11}\sigma_1(x_{1,t}) \\ &\quad + W_t^{12}\sigma_2(x_{2,t}) + v_t + \delta_{1,t}\end{aligned} \tag{31}$$

Substituting (31) into (29) and after some algebraic operations, we can obtain

$$\begin{aligned}v_t &= \frac{x_{1,t}}{\overline{\tau}^*_{\min}} - a_{11}x_{1,t} - a_{12}x_{2,t} - W_t^{11}\sigma_1(x_{1,t}) \\ &\quad - W_t^{12}\sigma_2(x_{2,t}) - \delta_{1,t}\end{aligned} \tag{32}$$

Now we must find an expression for $\delta_{1,t}$. Since δ_t is continuously being minimized by the learning law (15), it is reasonable to consider that $\|\delta_t\| \le \|\Delta f(x_t, v_t, t)\|$ (in other case, to use (14) would be meaningless). But, in accordance with **A.4,** $\Delta f(x_t, v_t, t)$ is bounded and consequently δ_t is also bounded. If (14) is subtracted from (30) then δ_t is given by

$$\delta_t = A(\hat{x}_t - x_t) + W_t\{\sigma(\hat{x}_t) - \sigma(x_t)\} + \dot{x}_t - \dot{\hat{x}}_t$$

and in particular the first component of δ_t that is, $\delta_{1,t}$ can be expressed as

$$\begin{aligned}\delta_{1,t} &= a_{11}(\hat{x}_{1,t} - x_{1,t}) + a_{12}(\hat{x}_{2,t} - x_{2,t}) \\ &\quad + W_t^{11}\{\sigma_1(\hat{x}_{1,t}) - \sigma_1(x_{1,t})\} \\ &\quad + W_t^{12}\{\sigma_2(\hat{x}_{2,t}) - \sigma_2(x_{2,t})\} + \dot{x}_{1,t} - \dot{\hat{x}}_{1,t}\end{aligned} \tag{33}$$

Since τ_t is available for measurement, $\dot{x}_{1,t}$ can be calculated using (4) and consequently $\delta_{1,t}$ can be completely determined. Finally, if (33) is substituted into (32) then (27) can be deduced. On the other hand, when the desired power level R_1 is attained, the control action is switched to a regulation stage. The regulation error is defined as

$$e_t = x_{1,t} - R_1 \tag{34}$$

Now, the idea is to maintain the following error dynamics

$$\dot{e}_t + ce_t = 0 \tag{35}$$

Differentiating (34) yields

$$\dot{e}_t = \dot{x}_{1,t} \tag{36}$$

But if (33) is substituted into (31) and next (36) is substituted into (35), after some algebraic operations, finally (28) is obtained. Thus, whenever $c > 0$ the asymptotic stability of the regulation error can be guaranteed. ■

V. Simulation Results

In this section, the identification and control process proposed in this work is illustrated by simulation. Instead of the real plant, an eighth order model of a TRIGA reactor is used with the nominal parameters given in section II. During the simulation, such model is, in reality, considered as a black box since apart from the assumptions on the stability, controllability and relative degree equal to 1, none previous knowledge of this model is required to achieve satisfactorily the control process. Due to, in real situations, variables such as the *i*th group delayed neutron precursor power are not available for measurement, here the identification is accomplished by a single layer second order DNN using only the external reactivity, the fuel temperature and the neutron power. The main parameters of the neural identifier (14) are selected as follows:

$$A = \begin{bmatrix} -1.4 & 0 \\ 0 & -0.3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, P = \begin{bmatrix} 63 & 16 \\ 16 & 52 \end{bmatrix}$$

$$W^* = \begin{bmatrix} 0.00078 & -0.00099 \\ 0.025 & 0.029 \end{bmatrix}, K = \begin{bmatrix} 21 & 0 \\ 0 & 16 \end{bmatrix}$$

The variables n_t and T_t are normalized by g_1=1000000W and g_2=122.55 °C, respectively. The constant c is selected equal to 40. The results of the control process are displayed in Fig. 1-2. In Fig. 1 it can be observed that the neutron power is increased first from an initial level of 10 kW to 100kW and later from 100kW to 1MW. Because the ample range of variation, a logarithmic scale is employed. As it can be clearly appreciated (Fig. 1), a constant period is obtained during the transient stages and no overshoot occurs. In Fig. 2, it is showed the control signal which is in general smooth. Finally, in Fig. 3, the components of the weight matrix are presented.

VI. Conclusions

In this work, it has been presented the application of an indirect adaptive controller based on a single layer second order DNN to solve the power ascent problem of a nuclear research reactor when only the external reactivity, neutron power and fuel temperature are available for measurement. The developed controller has a relatively simple structure. In addition, the tuning of the identifier-controller parameters can be accomplished quickly and easily. To verify its effectiveness, this controller was tested by simulation. Instead of the real plant, an eighth order nonlinear model of a TRIGA reactor considered as a black box was used. The simulation results confirm the workability of the suggested approach.

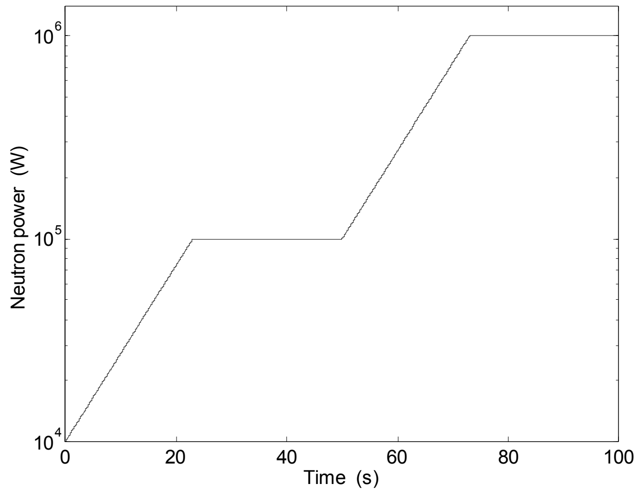

Fig. 1. Control process result for power ascent.

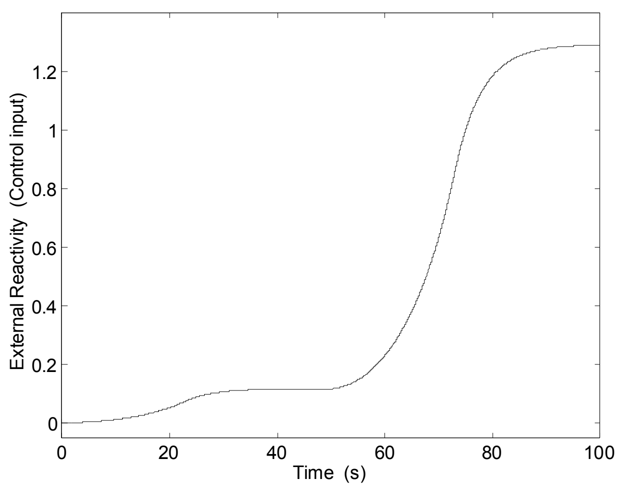

Fig. 2. The control signal v_t (external reactivity).

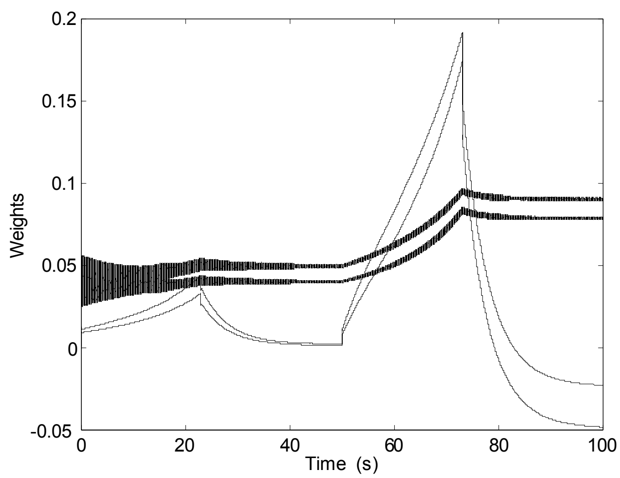

Fig 3. Time evolution of the components of the weight matrix W_t.

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