2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009



# System Identification Using a Retrospective Correction Filter for Adaptive Feedback Model Updating

M. A. Santillo[1], A. M. D'Amato[2], D. S. Bernstein[3]

***Abstract*— In this paper we use a retrospective correction filter (RCF) to identify MIMO LTI systems. This method uses an adaptive controller in feedback with an initial model. The goal is to adapt the closed-loop response of the system to match the response of an unknown plant to a known input. We demonstrate this method on numerical examples of increasing complexity where the initial model is taken to be a one-step delay. Minimum-phase and nonminimum-phase SISO and MIMO examples are considered. The identification signals used include zero-mean Gaussian white noise as well as sums of sinusoids. Finally, we examine the robustness of this method by identifying these systems in the presence of actuator noise.**

## I. Introduction

Identification of linear time-invariant systems is a fundamental problem in systems theory, and available methods range from frequency-domain techniques [1], to time-series methods [2], to state-space algorithms [3, 4]. These techniques assume different model structures, and the choice of model structure in practice may be guided by the ultimate intended use of the model.

Regardless of the desired model structure, the amount of available data and the quality of that data (that is, the level of noise that corrupts the data) directly impact the fidelity of the identified model. For data corrupted by stationary noise, we expect a fundamental tradeoff between the amount of available data and the noise level, where a weakness in quantity or quality can, at least to some extent, be offset by a strength in the other. In addition to the amount of the data, identification methods may be sensitive to additional issues, such as the type of noise, a priori estimates of the system order and relative degree, and, in the MIMO case, coupling strength between the inputs and outputs. All of these issues must be assessed within the context of the computational burden of competing algorithms.

A variation of the model identification problem is the case in which an initial model is available and data is used to refine the initial model to obtain an improved fit to the data. This problem has been extensively studied within the context of finite element modeling [5–7], and has received some attention within the systems and control literature [8–10]. In particular, various approaches to model refinement are considered in [10], including series, parallel, and feedback model-augmentation structures. The goal of the present paper is to further develop the feedback-model augmentation structure of [10]. Interestingly, this problem mimics the structure of model reference adaptive control, which shows that adaptive control can be used for system identification.

The adaptive control method used in [10] is based on [11], which was originally developed for adaptive disturbance rejection. More recent variations of this work are presented in [12–14], which consider adaptive disturbance rejection, adaptive stabilization, adaptive command following, and model reference adaptive control. In the present paper, we focus on the *retrospective correction filter-based* (RCF) adaptive control algorithm presented in [14] because of its ability to control nonminimum-phase systems. In particular, using an augmentation model structure that is a slight variation of the augmentation model structure used in [10], we present a series of numerical examples to investigate the effectiveness of the RCF algorithm for adaptive feedback model updating.

We consider a series of examples to explore several issues, namely, model properties, identification signals, and effect of noise. These examples are useful in assessing the effectiveness of RCF-based identification under a range of conditions.

## II. Problem Formulation

We seek to identify the SISO or MIMO plant $G$ by using a given initial model $G_0$ and an estimated feedback component $G_{\rm c}$. The objective is to determine $G_{\rm c}$ such that the resulting closed-loop model $G_{\rm cl}$ matches the true system $G$.

As shown in Figure 1, we use model reference adaptive control (MRAC) [15–17] as the feedback interconnection structure. To achieve model matching, we minimize the performance variable $z$ in the presence of the identification signal $w$. In particular, we use the retrospective correction filter (RCF) adaptive control algorithm given in [14]. The identification signal $w$ is assumed to be available to the controller as an additional measurement variable $y_w$. This problem setup is a minor variation of the approach of [10].

Consider a realization of the MIMO discrete-time system $G$ given by

$$x(k+1) = Ax(k) + Bw(k), \quad (1)$$

$$y_{\rm ref}(k) = Cx(k), \quad (2)$$

where $x(k) \in \mathbb{R}^n$, $y_{\rm ref}(k) \in \mathbb{R}^{l_{y0}}$, $w(k) \in \mathbb{R}^{l_w}$, and an initial

This work was sponsored by NASA under IRAC grant NNX08AB92A and the National Science Foundation under grant CNS-0539053.

[1]National Defense Science and Engineering Graduate Fellow, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109-2140, santillo@umich.edu

[2]Graduate Student, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109-2140, amdamato@umich.edu

[3]Professor, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109-2140, dsbaero@umich.edu

978-1-4244-4524-0/09/$25.00 ©2009 AACC 

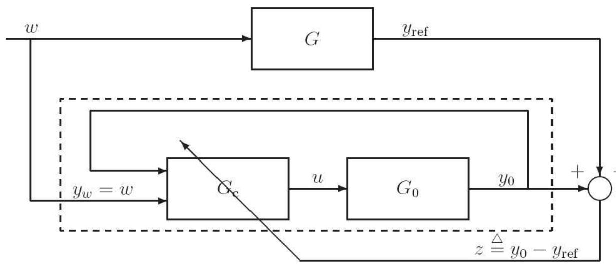


Fig. 1. Model reference adaptive control for system identification. The identified model resides in the dashed box. The diagonal arrow represents data-driven adaptation.

model $G_0$ with realization

$$x_0(k+1) = A_0 x_0(k) + B_0 u(k), \tag{3}$$

$$y_0(k) = C_0 x_0(k), \tag{4}$$

where $x_0(k) \in \mathbb{R}^{n_0}$, $y_0(k) \in \mathbb{R}^{l_{y_0}}$, and $u(k) \in \mathbb{R}^{l_u}$. Furthermore, define

$$y(k) \triangleq \begin{bmatrix} y_0(k) \\ y_w(k) \end{bmatrix} = \begin{bmatrix} C_0 x_0(k) \\ w(k) \end{bmatrix}, \tag{5}$$

$$z(k) \triangleq y_0(k) - y_{\mathrm{ref}}(k), \tag{6}$$

where $y_w(k) \in \mathbb{R}^{l_w}$, $y(k) \in \mathbb{R}^{l_y}$, and $z(k) \in \mathbb{R}^{l_z}$. Note that $l_y \triangleq l_{y_0} + l_w$ and $l_z = l_{y_0}$.

We thus seek an adaptive output feedback controller $G_{\mathrm{c}} \triangleq \begin{bmatrix} G_{\mathrm{c}_{y_0}} & G_{\mathrm{c}_{y_w}} \end{bmatrix}$ such that the performance variable $z$ is minimized in the presence of the identification signal $w$, and, hence, the closed-loop model

$$G_{\mathrm{cl}} \triangleq \left[I - G_0 G_{\mathrm{c}_{y_0}}\right]^{-1} G_0 G_{\mathrm{c}_{y_w}} \tag{7}$$

matches the true system $G$. Note that $y_0 = G_{\mathrm{cl}} w$.

## III. Controller Construction

In this section we give a brief overview of the RCF adaptive control algorithm for the control problem represented by (1)–(6). The algorithm is derived from [11] and [12]. The full details of the algorithm are presented in [14].

We use an exactly proper time-series controller of order $n_{\mathrm{c}}$, such that the control $u(k)$ is given by

$$u(k) = \sum_{i=1}^{n_{\mathrm{c}}} M_i(k) u(k-i) + \sum_{i=0}^{n_{\mathrm{c}}} N_i(k) y(k-i), \tag{8}$$

where $M_i \in \mathbb{R}^{l_u \times l_u}$, $i = 1, \ldots, n_{\mathrm{c}}$, and $N_i \in \mathbb{R}^{l_u \times l_y}$, $i = 0, \ldots, n_{\mathrm{c}}$, are given by an adaptive update law. The control can be expressed as

$$u(k) = \theta(k) \phi(k), \tag{9}$$

where

$$\theta(k) \triangleq \begin{bmatrix} N_0(k) & \cdots & N_{n_{\mathrm{c}}}(k) & M_1(k) & \cdots & M_{n_{\mathrm{c}}}(k) \end{bmatrix}$$

is the *controller parameter block matrix*, and the *regressor vector* $\phi(k)$ is given by

$$\phi(k) \triangleq \begin{bmatrix} y(k) \\ \vdots \\ y(k-n_{\mathrm{c}}) \\ u(k-1) \\ \vdots \\ u(k-n_{\mathrm{c}}) \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{c}} l_u + (n_{\mathrm{c}}+1) l_y}. \tag{10}$$

For positive integers $p$ and $\mu$, we define the *extended performance vector* $Z(k)$, and the *extended control vector* $U(k)$ by

$$Z(k) \triangleq \begin{bmatrix} z(k) \\ \vdots \\ z(k-p+1) \end{bmatrix}, \quad U(k) \triangleq \begin{bmatrix} u(k) \\ \vdots \\ u(k-p_{\mathrm{c}}+1) \end{bmatrix}, \tag{11}$$

where $p_{\mathrm{c}} \triangleq \mu + p$.

From (9), it follows that the extended control vector $U(k)$ can be written as

$$U(k) \triangleq \sum_{i=1}^{p_{\mathrm{c}}} L_i \theta(k-i+1) \phi(k-i+1), \tag{12}$$

where

$$L_i \triangleq \begin{bmatrix} 0_{(i-1) l_u \times l_u} \\ I_{l_u} \\ 0_{(p_{\mathrm{c}}-i) l_u \times l_u} \end{bmatrix} \in \mathbb{R}^{p_{\mathrm{c}} l_u \times l_u}. \tag{13}$$

We define the *surrogate performance vector* $\hat{Z}(\hat{\theta}(k), k)$ by

$$\hat{Z}(\hat{\theta}(k), k) \triangleq Z(k) - \bar{B}_{zu} \left( U(k) - \hat{U}(k) \right), \tag{14}$$

where $\hat{U}(k) \triangleq \sum_{i=1}^{p_{\mathrm{c}}} L_i \hat{\theta}(k) \phi(k-i+1)$, $\hat{\theta}(k) \in \mathbb{R}^{l_u \times [n_{\mathrm{c}} l_u + (n_{\mathrm{c}}+1) l_y]}$ is the *surrogate controller parameter block matrix*, and the block-Toeplitz *surrogate control matrix* $\bar{B}_{zu}$ is given by

$$\bar{B}_{zu} \triangleq \begin{bmatrix} 0_{l_z \times l_u} & \cdots & 0_{l_z \times l_u} & H_d & \cdots & \cdots & H_\mu & 0_{l_z \times l_u} & \cdots & 0_{l_z \times l_u} \\ 0_{l_z \times l_u} & \ddots & & \ddots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \ddots & & \ddots & 0_{l_z \times l_u} & \\ 0_{l_z \times l_u} & \cdots & 0_{l_z \times l_u} & 0_{l_z \times l_u} & \cdots & \cdots & 0_{l_z \times l_u} & H_d & \cdots & H_\mu \end{bmatrix}, \tag{15}$$

where the *relative degree* $d$ is the smallest positive integer $i$ such that the $i$th Markov parameter $H_i \triangleq C_0 A_0^{i-1} B_0$ is nonzero. The leading zeros in the first row of $\bar{B}_{zu}$ account for the nonzero relative degree $d$. The algorithm places no constraints on either the value of $d$ or the rank of $H_d$ or $\bar{B}_{zu}$.

The adaptive update law presented in [14] depends on a time-varying weighting parameter $\alpha(k) \in (0, \infty)$, referred to as the *learning rate* since it affects convergence speed of the adaptive control algorithm. As $\alpha(k)$ is increased, convergence speed is lowered. Likewise, as $\alpha(k)$ is decreased, convergence speed is raised.

The novel feature of the adaptive control algorithm (9) is the use of the retrospective correction filter (14). The RCF provides an inner loop to the adaptive control law by modifying the performance variable $Z(k)$ based on the difference between the actual past control inputs $U(k)$ and the recomputed past control inputs based on the current control law $\hat{U}(k)$.

## IV. Numerical Examples: Minimum Phase

We now present a series of examples that demonstrate the RCF algorithm for identifying plants of increasing complexity. We consider minimum-phase systems with zero-mean Gaussian white noise identification signals.

*Example 4.1 (SISO, $G_0 = 1/z$, $w =$ white noise):* Consider the SISO plant with poles $\{0.34 \pm 0.87\jmath, -0.3141 \pm 0.9\jmath, 0.05 \pm 0.3122\jmath, -0.6875\}$ and zeros $\{0.14 \pm 0.97\jmath, -0.12 \pm 0.62\jmath, -0.89\}$. We take $n_c = 9$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = 1/z$. The identification signal is zero-mean Gaussian white noise. The controller performance is shown in Figure 2. The RCF adaptive algorithm is initiated at $t = 0\,\text{sec}$, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 3. ■

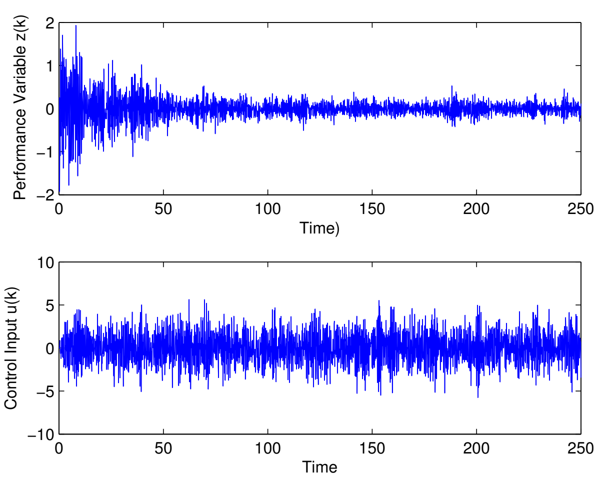


Fig. 2. Identification performance for a minimum-phase SISO plant with $G_0 = 1/z$. The RCF adaptive algorithm is initiated at $t = 0\,\text{sec}$. The controller order is $n_c = 9$ with parameters $p = 1, \mu = 1, \alpha = 1$.

*Example 4.2 (2x2, $G_0 = (1/z)I_{2\times 2}$, $w =$ white noise):* Consider the two-input, two-output plant with poles $\{0.9, -0.5 \pm 0.5\jmath, -0.6 \pm 0.09\jmath, 0.8\}$ and transmission zeros $\{0.2, 0\}$. We take $n_c = 18$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = (1/z)I_{2\times 2}$. The identification signals are zero-mean Gaussian white noise. The controller performance is shown in Figure 4. The RCF adaptive algorithm is initiated at $t = 0\,\text{sec}$, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 5. ■

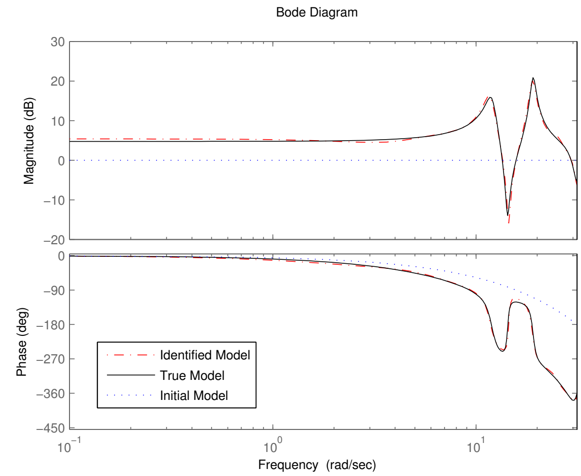


Fig. 3. Frequency response comparison of the true, initial, and identified models.

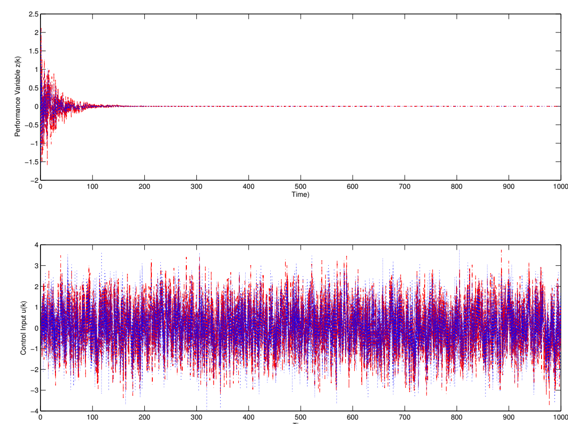

Fig. 4. Identification performance for a minimum-phase 2x2 MIMO plant with $G_0 = (1/z)I_{2\times 2}$. The RCF adaptive algorithm is initiated at $t = 0\,\text{sec}$. The controller order is $n_c = 18$ with parameters $p = 1, \mu = 1, \alpha = 1$.

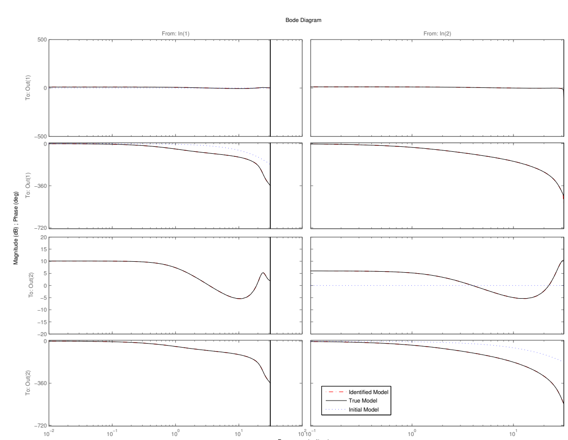

Fig. 5. Frequency response comparison of the true, initial, and identified models.

## V. Numerical Examples: Nonminimum Phase

We now present a series of examples that demonstrate the RCF algorithm for identifying nonminimum-phase plants of increasing complexity. We consider nonminimum-phase systems with random and harmonic identification signals.

*Example 5.1 (SISO, $G_0 = 1/z$, $w =$ white noise):* Consider the SISO plant with poles $\{0.34 \pm 0.87\jmath, -0.3141 \pm 0.9\jmath, 0.05 \pm 0.3122\jmath, -0.6875, 0.1\}$ and zeros $\{0.14 \pm 0.97\jmath, -0.12 \pm 0.62\jmath, -0.89, 3\}$. We take $n_{\rm c} = 9$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = 1/z$. The identification signal is zero-mean Gaussian white noise. The controller performance is shown in Figure 6. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 7. ■

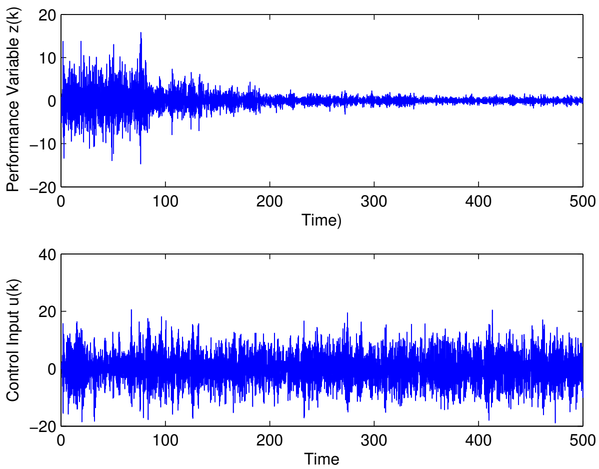


Fig. 6. Identification performance for a minimum-phase SISO plant with $G_0 = 1/z$. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$. The controller order is $n_{\rm c} = 9$ with parameters $p = 1, \mu = 1, \alpha = 1$.

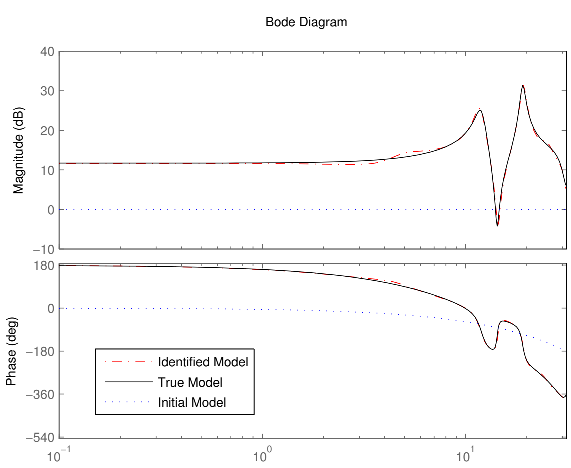


Fig. 7. Frequency response comparison of the true, initial, and identified models.

*Example 5.2 (SISO, $G_0 = 1/z$, $w =$ sum of sinusoids ):* Consider the SISO plant with poles $\{0.34 \pm 0.87\jmath, -0.3141 \pm 0.9\jmath, 0.05 \pm 0.3122\jmath, -0.6875, 0.1\}$ and zeros $\{0.14 \pm 0.97\jmath, -0.12 \pm 0.62\jmath, -0.89, 3\}$. We take $n_{\rm c} = 10$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = 1/z$. The identification signal is a sum of 12 sinusoids. The controller performance is shown in Figure 8. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 9. ■

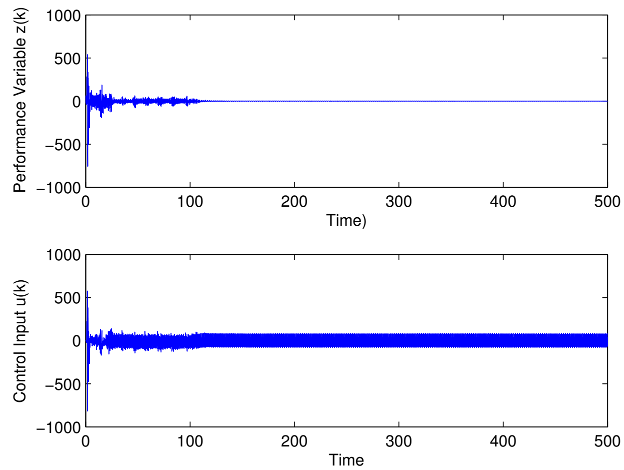


Fig. 8. Identification performance for a minimum-phase SISO plant with $G_0 = 1/z$. The identification signal is a sum of 12 sinusoids. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$. The controller order is $n_{\rm c} = 10$ with parameters $p = 1, \mu = 1, \alpha = 1$.

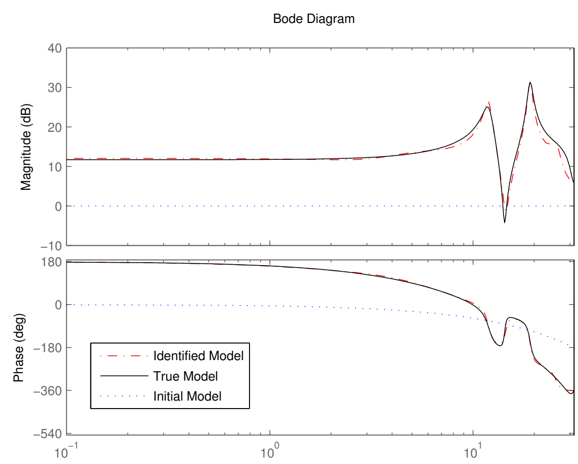


Fig. 9. Frequency response comparison of the true, initial, and identified models.

*Example 5.3 (2x2, $G_0 = (1/z)I_{2\times 2}$, $w =$ white noise):* Consider the two-input, two-output plant with poles $\{0.9, -0.5 \pm 0.5\jmath, -0.6 \pm 0.09\jmath, 0.8\}$ and transmission zeros $\{2, 0\}$. We take $n_{\rm c} = 18$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = (1/z)I_{2\times 2}$. The identification signals are zero-mean Gaussian white noise. The controller performance is shown in Figure 10. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 11. ■

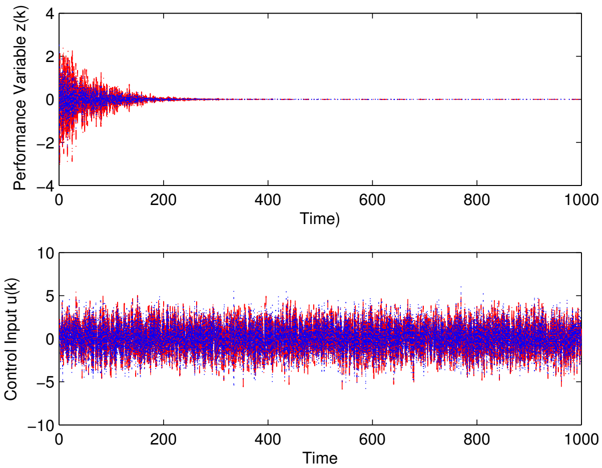


Fig. 10. Identification performance for a nonminimum-phase 2x2 MIMO plant, with $G_0 = (1/z)I_{2\times 2}$. The RCF adaptive algorithm is initiated at $t = 0\,$sec. The controller order is $n_c = 18$ with parameters $p = 1, \mu = 1, \alpha = 1$.

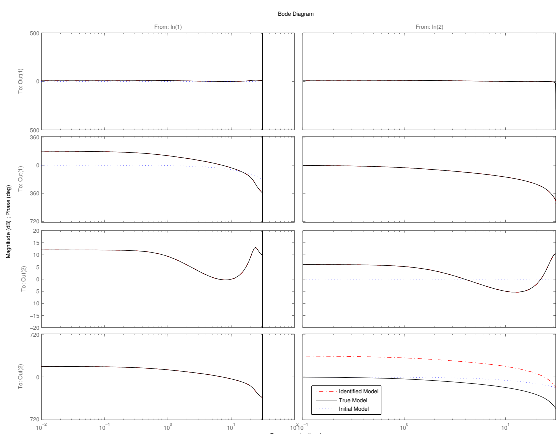

Fig. 11. Frequency response comparison of the true, initial, and identified models.

## VI. Numerical Examples: Off-Nominal Nonminimum-Phase Cases

In the following examples, plants presented in the previous section are identified with an unknown Gaussian white noise input to the true model $G$, therefore the input to $G$ becomes $w + \delta$ where $\delta$ is input sensor noise. We set the signal to noise ratio (SNR) of the identification signal to the input sensor noise equal to 5 for each case. In all cases, both the identification signal $w$ and the input sensor noise $\delta$ are taken to be uncorrelated zero-mean Gaussian white noise.

*Example 6.1 (SISO, $G_0 = 1/z$, SNR = 5):* Consider the SISO plant with poles $\{0.34 \pm 0.87\jmath, -0.3141 \pm 0.9\jmath, 0.05 \pm 0.3122\jmath, -0.6875, 0.1\}$ and zeros $\{0.14 \pm 0.97\jmath, -0.12 \pm 0.62\jmath, -0.89, 3\}$. We take $n_c = 9$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = 1/z$. The identification signal is a sum of 12 sinusoids. The controller performance is shown in Figure 12. The RCF adaptive algorithm is initiated at $t = 0\,$sec, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 13. ■

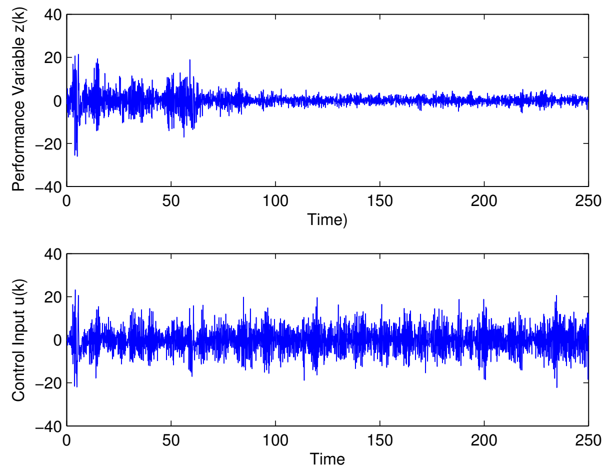


Fig. 12. Identification performance for a nonminimum-phase SISO plant with $G_0 = 1/z$ and SNR=5. The RCF adaptive algorithm is initiated at $t = 0\,$sec. The controller order is $n_c = 9$ with parameters $p = 1, \mu = 1, \alpha = 1$.

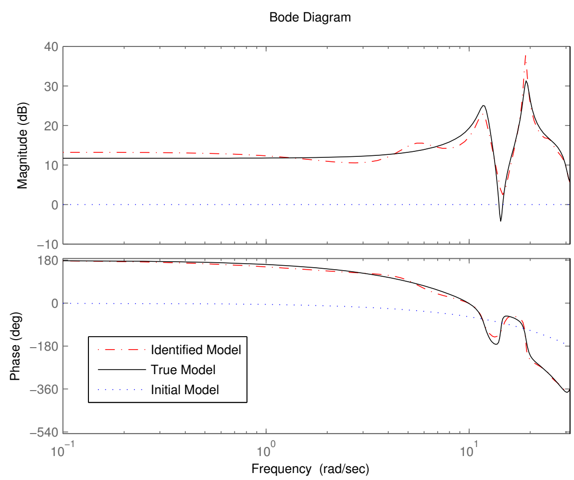


Fig. 13. Frequency response comparison of the true, initial, and identified models.

*Example 6.2 (2x2, $G_0 = (1/z)I_{2\times 2}$, SNR=5):* Consider the two-input, two-output plant with poles $\{0.9, -0.5 \pm 0.5\jmath, -0.6 \pm 0.09\jmath, 0.8\}$ and transmission zeros $\{2, 0\}$. We take $n_c = 18$, $p = 1$, $\mu = 1$, and $\alpha = 1$. We initialize $G_0 = (1/z)I_{2\times 2}$. The identification signals are zero-mean Gaussian white noise. The disturbance signals are also zero-mean Gaussian white noise. The controller performance is shown in Figure 14. The RCF adaptive algorithm is initiated at $t = 0\,$sec, and the convergence of the performance variable indicates that the output of the true plant is approximately matched by the closed-loop controller and initial model. The frequency response of the closed-loop initial model and adapted controller are compared with the true model in Figure 15. ■

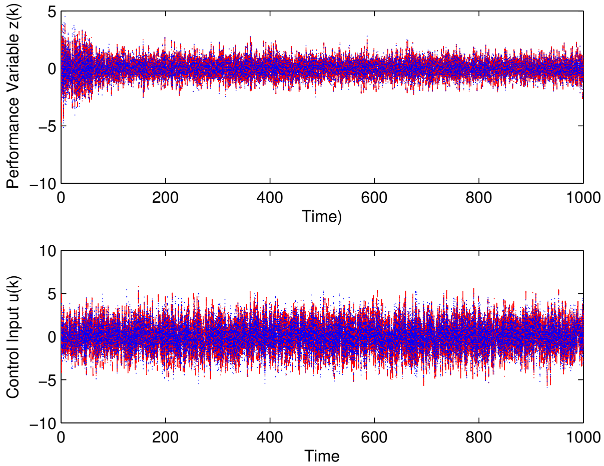


Fig. 14. Identification performance for a nonminimum-phase 2x2 MIMO plant with $G_0 = (1/z)I_{2\times 2}$ and SNR=5. The RCF adaptive algorithm is initiated at $t = 0\,\mathrm{sec}$. The controller order is $n_c = 18$ with parameters $p = 1, \mu = 1, \alpha = 1$.

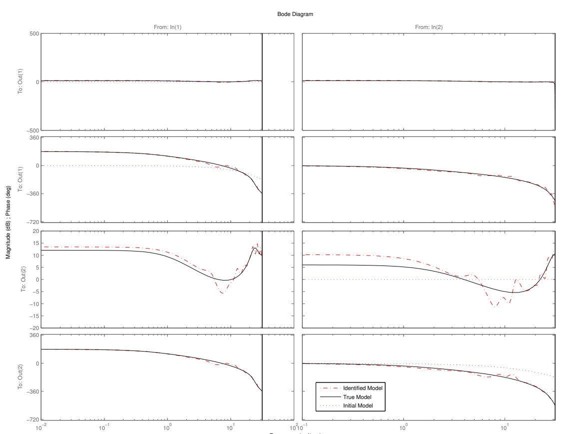


Fig. 15. Frequency response comparison of the true, initial, and identified models.

## VII. Conclusions

In this paper we applied the retrospective correction filter approach to discrete-time adaptive control to a model reference adaptive control problem that effectively identifies LTI systems. The model structure is a closed-loop system consisting of an initial model and feedback correction. The initial models were one-step delays that have no relation to the true dynamics. The approach was demonstrated with both white noise and harmonic inputs, as well as with input actuator noise. SISO and MIMO examples were considered, including both minimum-phase and nonminimum-phase cases. Future work will focus on comparing the accuracy of this technique with alternative methods such as subspace techniques.

## References

[1] R. Pintelon and J. Schoukens, *System Identification: A Frequency Domain Approach*. Piscataway, NJ: IEEE Press, 2001.

[2] L. Ljung, *System Identification: Theory for the User, 2nd ed.* Upper Saddle River, NJ: Prentice-Hall Information and Systems Sciences, 1999.

[3] J. N. Juang, *Applied System Identification*. Upper Saddle River, NJ: Prentice-Hall, 1993.

[4] T. Katayama, *Subspace Methods for System Identification*. Englewood Cliffs, New Jersey: Springer, 2005.

[5] C. Minas and D. Inman, "Matching finite element models to modal data," *J. Vibration Acoust.*, vol. 112, pp. 84–92, 1990.

[6] J. B. Carvalho, B. N. Datta, W. Lin, and C. Wang, "Symmetry preserving eigenvalue embedding in finite-element model updating of vibrating structures," *J. Sound Vibration*, vol. 290, pp. 839–864, 2006.

[7] M. I. Friswell and J. E. Mottershead, *Finite Element Model Updating in Structural Dynamics*. Kluwer, Dordrecht, 1995.

[8] S. Mijanovic, G. E. Stewart, G. A. Dumont, and M. S. Davies, "A controller perturbation technique for transferring closed-loop stability between systems," *Automatica*, vol. 39, pp. 1783–1791, 2003.

[9] S. O. R. Moheimani, "Model correction for sampled-data models of structures," *J. Guidance, Contr., and Dynamics*, vol. 24(3), pp. 634–637, 2001.

[10] H. J. Palanthandalam-Madapusi, E. L. Renk, and D. S. Bernstein, "Data-based model refinement for linear and hammerstein systems using subspace identification and adaptive disturbance rejection," in *Proc. Conf. Cont. App.*, Toronto, Canada, August 2005, pp. 1630–1635.

[11] R. Venugopal and D. S. Bernstein, "Adaptive disturbance rejection using ARMARKOV/Toeplitz models," *IEEE Trans. Contr. Sys. Tech.*, vol. 8, pp. 257–269, 2000.

[12] J. B. Hoagg, M. A. Santillo, and D. S. Bernstein, "Discrete-time adaptive command following and disturbance rejection with unknown exogenous dynamics," *IEEE Trans. Autom. Contr.*, vol. 53, pp. 912–928, 2008.

[13] M. A. Santillo and D. S. Bernstein, "Inherent robustness of minimal modeling discrete-time adaptive control to flight anomalies," in *Proc. Guid. Nav. Contr. Conf.*, Honolulu, HI, August 2008, AIAA-2008-7289.

[14] ——, "A retrospective correction filter for discrete-time adaptive control of nonminimum phase systems," in *Proc. Conf. Dec. Contr.*, Cancun, Mexico, December 2008, pp. 690–695.

[15] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, New Jersey: Prentice Hall, 1989.

[16] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*. Englewood Cliffs, New Jersey: Prentice Hall, 1989.

[17] P. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall, 1996.