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Reduced-order models for control of stratified flows in buildings

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Abstract— **Building heating and cooling systems have potential for energy savings by employing passive devices that exploit thermal stratification and buoyancy. The resulting thermal-fluid flow patterns from such systems tend to be sensitive to disturbances, and advanced flow control techniques are important to maintain occupant comfort. In this work, we employ Eigensystems Realization Algorithm (ERA) to obtain low-order models of airflow in buildings, which capture relevant dynamics and are amenable for control design. We present an alternative interpretation that allows one to obtain models using ERA, without resorting to lifting, an approach that is typically used to introduce boundary control and boundary disturbances explicitly in the reduced-order model. Using this reduced-order model we derive an optimal control law in closed form (which is composed of a feedforward and a feedback term) for rejecting a known disturbance, while minimizing a quadratic cost related to occupant discomfort and energy consumption. We demonstrate this approach using closed loop CFD simulations of airflow in a room with a passively cooled radiant ceiling and a displacement vent.**

I. INTRODUCTION

Residential and commercial buildings consume 40% of the energy in developed nations such as United States [1]. Enhancing building efficiency by low energy retrofits represents one of the most immediate and cost effective ways to reduce energy consumption. Typically, the low energy components/systems rely on functional integration between envelope, lighting, and heating, ventilation and air-conditioning (HVAC) systems to reduce energy consumption. Some example implementations include displacement ventilation [2], underfloor ventilation systems [3] and radiant cooling or chilled ceilings [4]. Such approaches are known to improve comfort and indoor air quality as well. While these strategies are promising, there are several challenges that prevent their broad application. In particular, the functional integration leads to coupling of components via nonlinear thermo-fluid interactions and dynamics at multiple spatial and temporal scales. Substantial disturbances (such as solar and occupancy heat gains) and their uncertain variation result in failure modes that erode energy reduction gains from early design. Studies [5] show losses of 10-20% in energy efficiency soon after occupancy; further, the newer technologies are more sensitive to these disturbances. For example, a passively ventilated building design will likely make use of low fan power and/or buoyancy to drive ventilation. In this application, buoyancy is the critical physics that ensures comfort and ventilation requirements are met. However, buoyancy is intrinsically unstable and sensitive to uncertainty. Furthermore, for newer HVAC systems, actuation authority is limited in dynamic range and bandwidth and as a result, disturbances are difficult to reject. Due to these fundamental differences in how the low energy buildings function, the physics and dynamics of the underlying phenomena are substantially different from those encountered in conventional buildings. Consequently, modeling and control of low energy buildings pose new challenges.

In this paper, we focus on the control problem of disturbance rejection for robust operation of low energy passive HVAC systems. Since the airflow in buildings is governed by the Boussinesq partial differential equation (PDE), one of the key challenges in model-based control design is the ability to capture computationally and in simple representations, the physics of the combined fluid and thermal system. The standard approach in the buildings community for modeling airflow is to use lumped models, typically based on energy balance over a large control volume [6]. These models essentially represent the air in a zone using a single node, and hence are inadequate for resolving spatial inhomogeneity. On the other extreme, high fidelity computational fluid dynamic (CFD) simulations are too complex and intractable for practical design, optimization or control.

We apply model reduction techniques to extract dynamics at temporal and spatial scales suitable for accurate analysis and control design of low-energy passive HVAC systems. In particular, we use eigensystem realization algorithm (ERA) to obtain input-output reduced models of the Boussinesq equations linearized about nominal operating conditions; see [7], [8]. Using this model, we design an optimal controller for rejecting a disturbance, assumed to be known over a future time horizon, by minimizing a quadratic cost related to occupant discomfort and control effort. Through closed-loop CFD simulations, we illustrate this approach for rejecting occupant heat gain in a model room problem equipped with displacement vent and passive chilled ceiling.

This paper is organized as follows: problem formulation and control objectives are first described in section II. The section also provides a brief review of model reduction techniques, a description of ERA and an interpretation of ERA using weak form, that allows model reduction of systems with boundary inputs. An optimal control design methodology in the discrete-time setting, to reject disturbances known over a future time-horizon, is provided in

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section III. The model reduction and control techniques are illustrated in section IV, using a model problem of airflow in a room with a displacement vent supplying conditioned air at the occupant level and a passive chilled ceiling. Finally, conclusions are provided in section V along with some future directions.

II. Problem statement and formulation

We consider dynamics of airflow in a room, shown schematically in figure 1, with the domain of interest $\mathbf{D}$ being the region defined by $X \in [-2,2], Y \in [0,3], Z \in [-2,2]$. The room is considered to be equipped with a displacement vent that supplies air near the floor and a chilled ceiling, that provides radiant and convective cooling. The vent is on the boundary $X = -2$ and defined by the region $Z \in [-2,2]$ and $Y \in [0,0.6]$. As described in the introduction, such a system relies on thermal stratification to provide occupant comfort while reducing energy consumption. For airflows in building applications, the coupled Navier-Stokes and energy equations can be approximated by Boussinesq equations

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla\mathbf{v} - \frac{1}{\rho_0}\nabla p' - \beta T\mathbf{g} + \nu\nabla^2\mathbf{v}, \tag{1}$$

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla T + \frac{\kappa}{\rho_0 C_p}\nabla^2 T, \tag{2}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{3}$$

where, the unknowns are the velocity $\mathbf{v} = \mathbf{v}(\mathbf{Z},t)$, temperature $T = T(\mathbf{Z},t)$, and pressure $p'(\mathbf{Z},t)$, $\mathbf{Z} = (X,Y,Z)^T$ is the spatial coordinate vector, $\mathbf{g}$ is acceleration due to gravity, ρ_0 is reference air density, β is its thermal coefficient of expansion, ν is kinematic viscosity, C_p is thermal capacitance and κ is the thermal conductivity. The boundary conditions for (1-3) are as follows: 1) inlet velocity and temperature are specified at the displacement vent, 2) chilled ceiling is modeled as a cooling source with uniformly distributed flux Q_{ceil} which can be controlled 3) the internal load due to occupants and equipment is represented as a floor-mat with a uniformly distributed heat flux Q_d and represents the disturbance and 4) the remaining boundary of $\mathbf{D}$ is assumed to be adiabatic. For nominal values of these boundary conditions (see section IV), the airflow in the room settles to a steady state. The resulting temperature contours over the central vertical slices through the room are shown in figure 1. The problem of interest is that, given the knowledge of the disturbance over a future time horizon, determine a minimal control effort (i.e. flux through the chilled ceiling), that maintains stable stratification. The assumption that the disturbance is known in future is not unrealistic; for example, it may be known a-priori that there is a meeting in the room with an expected attendance. Furthermore, we assume that localized temperature measurements are available at some given locations on the walls (say, from thermostats), which can be used in designing a control law for disturbance rejection.

The problem described above can be more formally stated as follows: for a known floor disturbance $d(t), 0 < t < T_f$,

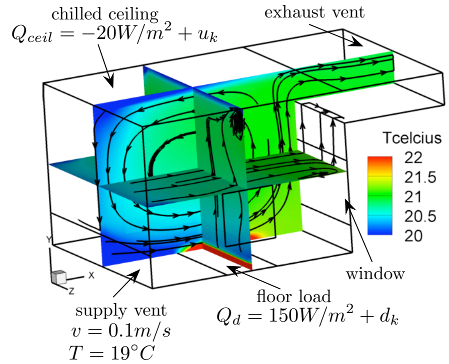

Fig. 1. Room equipped with a displacement vent, a chilled ceiling, and an internal load modeled as a floor-mat, with the boundary conditions as shown in the figure. The slices show contours of steady-state temperature field and projected stream-lines, obtained from a Fluent simulation.

determine a control input $u(t)$ that maintains the average temperature in a specified region $\mathbf{D}_o \subset \mathbf{D}$ of the room at a desired value T_{avg} while minimizing the control effort or equivalently, energy consumption. This trade-off can be represented as a quadratic cost functional:

$$J(u,T) = \int_0^{T_f} q\,(\overline{T}(t) - \overline{T}_{avg})^2 + ru^2(t)\,dt, \tag{4}$$

where q and r are given positive constants, and $\overline{T}$ represents the average of $T(Z)$ over the region $\mathbf{D}_o$. Here, we think of $\overline{T}_{avg}$ being the average temperature of the steady-state obtained at nominal values of the boundary conditions. One approach to solving this optimal control problem is the design-then-reduce paradigm [9], where for instance, the PDE (1,2) linearized about nominal operating conditions is used to formulate an infinite-dimensional linear quadratic regulator (LQR) problem [10]. The feedback control gain operator is then obtained as a solution of an infinite-dimensional Riccati equation. This approach, while introduces approximations only after control design, requires solution of an adjoint system. Thus, it becomes difficult to apply this methodology where one has to rely on use of commercial software for numerical simulations, since such software in general do not provide adjoint solvers. In this paper, we follow a reduce-then-design paradigm, which can alleviate this difficulty. In this approach, we first seek an appropriate reduced-order model to capture essential dynamics of the full system, and use that model for control design.

A. Model Reduction

As pointed out in the introduction, reduced-order lumped nodal models, widely used in the buildings community, are inadequate for capturing thermal stratification. We seek a data-driven model reduction approach which extracts dynamics relevant to the problem. One of the most popular methods has been the proper orthogonal decomposition (POD) and Galerkin projection [11]. In this method, a low-dimensional approximation of the original system is obtained by an orthogonal projection onto a set of an energetically optimal basis functions determined from empirical data. In the context of building airflows, this method has been applied for real-time estimation of airflows in buildings [12], [10]. The standard POD-Galerkin approach suffers from several limitations: it is sensitive to the choice of inner product [13], and models near stable equilibrium points can even be unstable [14]. An alternate snapshot-based approach, inspired from control theory, is the approximate balanced truncation [14]; it is particularly suitable for control design since it accurately captures input-ouput dynamics of the full system. This method is applicable to stable linear input-output systems, and results in balanced models, with guaranteed error bounds. The resulting models are superior as compared to POD/Galerkin, in the sense that they preserve stability of the original system and fewer modes are required to capture the original dynamics. However, compared to POD, this approach is only applicable to linear systems and requires solution of the associated adjoint system. Recently, it was shown that for a discrete-time input-output system, a system identification method known as eigensystem realization algorithm (ERA) is equivalent to balanced POD [7], [8]. The advantage of ERA is that it has a significantly lower computational cost, and it does not require adjoint simulations, thus making it applicable for model reduction using experimental data and simulation data from commercial software. In this work, we use ERA to obtain reduced-order model of the Boussinesq equations linearized about a steady state obtained using nominal values for the boundary conditions.

B. Model reduction using the Eigensystem Realization Algorithm (ERA)

ERA is a method for model reduction of discrete-time, stable, linear time-invariant systems of the form

$$x_{k+1} = Ax_k + Bu_k \tag{5}$$

$$y_k = Cx_k, \tag{6}$$

where, $x_k \in R^n, y_k \in R^m$. In the context of the paper, one can think of (5, 6) as being obtained from a spatio-temporal discretization of the linearized Boussinesq equations. The discretization timestep Δt is assumed to be a constant, and the index k is used to represent time $t = k\Delta t$. ERA begins by computing the impulse response of (5, 6), and the resulting outputs y_k can be compactly described by the Markov parameters as $\mathbf{y}_k = CA^kB$, where $\mathbf{y}_k \in \mathbb{R}^{q\times p}$ is a matrix with elements y^{ij} which represent the i^{th} output from an impulse on the j^{th} input. The Markov parameters are sampled every timestep:

$$\begin{pmatrix}\mathbf{y}_0 & \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_{m_c+m_o}\end{pmatrix} \tag{7}$$

$$= \begin{pmatrix}CB & CAB & CA^2B & \dots & CA^{m_c+m_o}B\end{pmatrix}, \tag{8}$$

and these outputs are used to assemble the Hankel matrix H as follows:

$$H = \begin{pmatrix} \mathbf{y}_0 & \mathbf{y}_1 & \dots & \mathbf{y}_{m_c} \\ \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_{m_c+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{m_o} & \mathbf{y}_{m_o+1} & \dots & \mathbf{y}_{m_c+m_o} \end{pmatrix} \tag{9}$$

The reduced-order model is obtained by computing the SVD of $H = U\Sigma V^*$. Let U_r and V_r be the leading columns of U and V, and $\Sigma_r \in \mathbb{R}^{r\times r}$ contain the leading rows and columns of Σ, then the reduced model of (5, 6) is given by

$$a_{k+1} = A_r a_k + B_r u_k, \tag{10}$$

$$y_k = C_r a_k \tag{11}$$

where, $A_r = (\Sigma_r^{-\frac{1}{2}} U_r^*) H_1 (V_r \Sigma_r^{-\frac{1}{2}})$, $B_r = (\Sigma_r^{-\frac{1}{2}} U_r^*)\,\mathrm{Col}_{\text{first}}(H)$, and $C_r = \mathrm{Row}_{\text{first}}(H)(V_r\Sigma_r^{-\frac{1}{2}})$, where $\mathrm{Col}_{\text{first}}(H)$ and $\mathrm{Row}_{\text{first}}(H)$ represent the first block column and row of H (9) respectively, and

$$H_1 = \begin{pmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_{m_c+1} \\ \mathbf{y}_2 & \mathbf{y}_3 & \dots & \mathbf{y}_{m_c+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{m_o+1} & \mathbf{y}_{m_o+2} & \dots & \mathbf{y}_{m_c+m_o+1} \end{pmatrix}. \tag{12}$$

C. ERA for systems with boundary control

Model reduction using ERA assumes that the given system is in the state-space form (5, 6), i.e. the inputs appears explicitly on the right hand side. In our model problem, the inputs (both, control and disturbance) appear as boundary conditions. For such systems, a lifting method can be used to transform the system into the standard form (5, 6); see [15]. We briefly describe this method here, point out the main difficulty in using it for boundary control problem and propose an alternative view-point based on the weak form of the PDE that alleviates this problem.

1) Lifting: We describe the lifting approach using the following linear PDE:

$$\dot{\phi} = \mathscr{L}\phi, \quad \phi = \phi(\mathbf{Z},t), \quad \mathbf{Z} \in \Omega \tag{13}$$

$$\phi(\mathbf{Z},t) = u(t), \quad \mathbf{Z} \in \delta\Omega, \tag{14}$$

where $\mathscr{L}$ is a linear operator defined over the domain Ω with boundary given by $\delta\Omega$, and $u(t)$ is the control. In this approach, the state $\phi(Z,t)$ is expressed as a sum of a homogenous part and a particular part:

$$\phi(\mathbf{Z},t) = \phi_h(\mathbf{Z},t) + \phi_p(\mathbf{Z})u(t) \tag{15}$$

where, the particular solution $\phi_p(\mathbf{Z})$ is obtained by solving:

$$\mathscr{L}\phi_p = 0, \quad \text{with} \quad \phi_p(\mathbf{Z},t) = 1, \quad \mathbf{Z} \in \delta\Omega. \tag{16}$$

Substituting (15) in the original equation (13) results in a homogenous PDE for ϕ_h,

$$\dot{\phi}_h = \mathscr{L}\phi_h - \phi_p\dot{u}, \quad \text{with} \quad \phi_h(\mathbf{Z},t) = 0, \quad \mathbf{Z} \in \delta\Omega, \tag{17}$$

where, now the time derivative of the control input $u(t)$ appears explicitly in the equations. Numerical discretization of (17) would result in an input-output system of the form (5), suitable for model reduction using ERA. The main disadvantage of the lifting approach is that, while using similar procedure for the boundary disturbance $d(t)$, one obtains homogeneous PDE of the form

$$\dot{\phi}_h = \mathscr{L}\phi_h + \mathscr{B}\dot{u} + \mathscr{G}\dot{d}, \tag{18}$$

where, similar to that for control, a time derivative of disturbance appears on the right-hand-side. The above equation is not in a standard input-output form, making the control design and implementation cumbersome.

2) Interpretation using weak form: We now provide an alternative approach to deal with boundary inputs, that result in equations in which, instead of their time-derivatives, the control inputs explicitly appear in the linearized dynamics. This approach involves the weak form [16] of the original PDE (13,14). Define the space of weighting functions $w(\mathbf{Z})$:

$$\mathscr{W} = \{w(\mathbf{Z}) \in H(\Omega), | \, w(\mathbf{Z}) = 0, \forall \mathbf{Z} \in \delta\Omega\}, \tag{19}$$

where $H(\Omega)$ is an appropriate Hilbert space endowed with an inner-product $\langle \cdot, \cdot \rangle$. The weak form of (13, 14) seeks to find $\phi(\mathbf{Z}, t)$ such that,

$$\langle w, \dot{\phi} - \mathscr{L}\phi \rangle = 0, \qquad \forall w \in \mathscr{W}. \tag{20}$$

Integrating (20) by parts (which may have to be carried out multiple times) results in a weak form,

$$\langle w, \dot{\phi} \rangle = \langle \mathscr{L}^* w, \phi \rangle + \mathscr{B}u(t) + \mathscr{G}d(t). \tag{21}$$

where $\mathscr{L}^*$ is the adjoint of $\mathscr{L}$, and $\mathscr{B}$ and $\mathscr{G}$ are operators that represent the boundary control and disturbance, respectively. The explicit forms of these operators depend on the type of the PDE and the nature of boundary conditions. Their derivation is however not required for model reduction, since typically, numerical PDE solvers obtain discretized system of equations from an appropriate weak form (21). Hence, one can assume that the equations solved by a PDE solver are already in the form (5). By choosing appropriate outputs y_k from the simulations, one naturally obtains the system in the form (5,6) suitable for model reduction using ERA.

III. Optimal control design for discrete-time systems with known disturbance

In this section, we consider an inhomogeneous LQR problem, for rejection of a disturbance known over a future time-horizon. Consider the following linear system obtained, for instance, by numerical discretization of the weak form (21):

$$x_{k+1} = Ax_k + Bu_k + Dd_k, \tag{22}$$

$$s_k = C_1 x_k, \tag{23}$$

$$z_k = C_2 x_k, \tag{24}$$

where the index k represents the time-instant $t_k = k\Delta t$. The outputs s_k are considered to be the outputs that represent sensor measurements, while the outputs z_k are considered to be the outputs that represent quantities of interest for control, such as the average temperature in the occupied region of a room, and are used to define a cost function. The objective is to find a control law u_k such that the output z_k tracks a reference trajectory r_k, while minimizing a quadratic cost function of the form:

$$J(z_k, u_k) = \frac{1}{2}\Big[\sum_{k=0}^{N-1} \big((z_k - r_k)^T Q (z_k - r_k) + u_k^T R u_k\big) + (z_N - r_N)^T Q (z_N - r_N)\Big], \tag{25}$$

where, $Q > 0, R > 0$ are positive-definite weighing matrices. Note that due to the disturbance term d_k appearing in (22), this is not a standard LQR problem [17]. The solution to this problem in continuous time setting has been considered, for instance, in [18]. We are not aware of any known solution approach in literature for the discrete-time setting considered here. We solve this problem below using the method of constrained Lagrangian.

Using Lagrange multipliers λ_k for the constraints (22), the Lagrangian can be expressed as

$$L(x_k, u_k) = J(x_k, u_k) + \sum_{k=0}^{N-1} \lambda_{k+1}^T (Ax_k + Bu_k + Dd_k - x_{k+1}),$$

leading to following Kuhn-Tucker optimality conditions

$$\lambda_k = A^T \lambda_{k+1} - C_2^T Q (r_k - C_2 x_k) \tag{26}$$

$$u_k = -R^{-1} B^T \lambda_{k+1}, \tag{27}$$

for $k = 0, 1, \ldots, N-1$, with $\lambda_N = -C_2^T Q (r_N - C_2 x_N)$. To obtain the control u_k in closed form, we use the notion of backward sweeping to express the Lagrange multipliers as

$$\lambda_k = P_k x_k + n_k, \qquad k = 0, 1, \ldots, N-1. \tag{28}$$

Further manipulations (details not included here) result in the following recursion relations for P_k and n_k:

$$P_k = A^T P_{k+1} A - A^T P_{k+1} B (R + B^T P_{k+1} B)^{-1} B^T P_{k+1} A + C_2^T Q C_2$$

$$n_k = -A^T P_{k+1} B (R + B^T P_{k+1} B)^{-1} B^T (P_{k+1} D d_k + n_{k+1}) + A^T P_{k+1} D d_k + A^T n_{k+1} - C_2^T Q r_k$$

with the terminal conditions, $P_N = 0$ and $n_N = 0$. If the disturbance d_k is known over the horizon $k = 0, 1, \ldots, N$, the above recursions can be solved for $P_k, n_k, k = 0, \cdots, N-1$ and the control law computed using

$$u_k = -R^{-1} B^T (P_k x_k + n_k) = K_k^{fb} x_k + u_k^{ff}, \tag{29}$$

where, $K_k^{fb} = -R^{-1} B^T P_k$ is the feedback gain and $u_k^{ff} = -R^{-1} B^T n_k$ is the feedforward term incorporating the knowledge of the disturbance d_k. Note that, when the entire state x_k is not accessible for control, but only sensor measurements s_k given by (23) are available, one can design an observer for the system (22, 23) to obtain an estimate $\hat{x}_k$ to be used in place of x_k in the expression (29).

IV. Application to control of building system environment

In this section we apply the reduced order based control design for the room problem described in section II. For CFD simulations we use ANSYS® FLUENT, a commercially available software. The maximum grid size is limited to 0.2m, 0.15m and 0.2m along the X, Y and Z axes respectively, leading to 66,205 mesh points in the domain $\mathbf{D}$ comprising the room. A constant integration time-step of 2 seconds is used for all the simulations. To obtain nominal operating conditions for the room, we use constant heat flux values for the boundary conditions, obtained by simple energy balance calculation, so that the chilled ceiling can compensate for roughly 50% of the heat input through the floor-mat. Under these conditions, the flow settles to a steady state, as shown in the figure 1.

A. *Reduced-order model*

We apply ERA to compute the reduced-order model of the airflow linearized about the above computed steady state. The outputs considered are as follows:

1) Sensed outputs s_k, are the temperatures averaged over two regions on the walls $Z=-2$ and $Z=2$ (the walls to the left and right of the supply vent), bounded by $Z=[-0.25,0.25]$ and $Y=[0.25,0.75]$. This output is used for feedback control.
2) Controlled output z_k is the volume average of the temperature $T(\mathbf{Z},t)$ field over the occupied region $\mathbf{D}_o \equiv [-1.5,1.5]\times[0.25,1.25]\times[-1.5,1.5]$. This output is used in the cost function (25).

The control input u_k is the chilled ceiling flux, perturbed about its steady-state value of $-20W/m^2$, while the disturbance d_k is the floormat flux, perturbed about its steady-state value of $150W/m^2$. Similarly, the outputs defined in (6) are the perturbations from their steady states.

Recall from section II-B, that ERA based model reduction requires computation of the impulse response of the system (5, 6) to obtain the Markov parameters (8). Reduced-order representations of both C_1 and C_2, defined in (23, 24) are obtained by lumping the two outputs together: that is, we define $y_k \equiv \begin{pmatrix} s_k & z_k \end{pmatrix}^T = \begin{pmatrix} C_1 & C_2 \end{pmatrix}^T x_k \equiv Cx_k$. Since it is difficult to numerically subject a black-box simulator like FLUENT to an impulsive input, we alternatively compute the step response. The step response is obtained by gradually changing the boundary inputs (fluxes at chilled ceiling and floor-mat) from their nominal value linearly to a perturbed value over 30 time-steps (i.e., 1 minute of simulation time). If the step-response of (5, 6) is denoted by y_k^{step}, the Markov parameters can be obtained by simply computing the differences $y_k = y_{k+1}^{step} - y_k^{step}$. We then assemble the Hankel matrix (9), compute its SVD, and use the singular values and eigenvectors to compute the reduced-order model (10, 11). The performance of the model is tested against the data from the original step-responses. A comparison is shown in figure 2, which shows that the model accurately predicts the controlled output z_k.

B. *Controller design and performance*

The reduced model derived using ERA is used to develop a controller that rejects a floor disturbance known over a time horizon $(0,T_f)$. The floor disturbance is considered to be of the form shown in figure 4. In the absence of the control, the average temperature in the room rises by about $4°C$, as shown in the resulting flow-field in the figure 4. The control design approach described in section III is used to develop a controller that suppresses the deviations of the averaged temperature from its steady-state value. We define $Q=qI$ (where I is the identity) in the cost function (25) and consider different values of q, while fixing $R=1$. We use Kalman filter as a reduced-order observer for state estimation based on the two temperature measurements to compute the feedback term in the control law (29). The observer gains are computed by assuming a Gaussian noise in the actuator inputs and sensor measurements. The resulting observer-based control is implemented in FLUENT using user-defined functions (UDFs) to test its performance in the full CFD simulation; a schematic is shown in figure 3. The results are shown in 5 for $q=5$ and $q=50$, where it is evident that the controller suppresses the deviation of the temperature from its steady value, thus maintaining occupant comfort. For the larger value of $q=50$, the controller completely suppresses the effect of the disturbance, but the required control effort is almost twice as large as that required using $q=5$. The figure also shows the performance of the controller, with an imperfect knowledge of the disturbance, when the actual disturbance is twice that assumed in control design. The controller still suppresses the temperature rise, due to feedback, but the performance deteriorates.

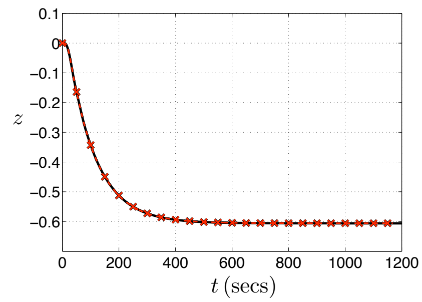

Fig. 2. Response of the full system (black, solid line) and a 10-mode reduced-order model (red, dashed line, crosses) to a $-10W/m^2$ step in the chilled ceiling flux. Plots compare the controlled outputs z_k, that is, perturbations from the steady state of temperature averaged over the occupied region $\mathbf{D}_o$.

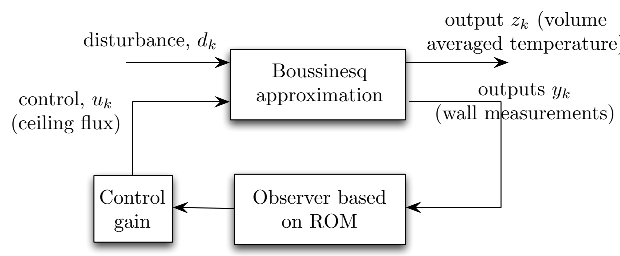

Fig. 3. Schematic of controller implementation in the full simulation.

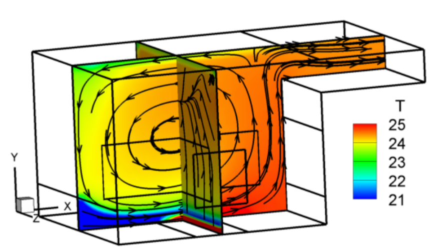

Fig. 4. Flow-field when the disturbance reaches its maximum value.

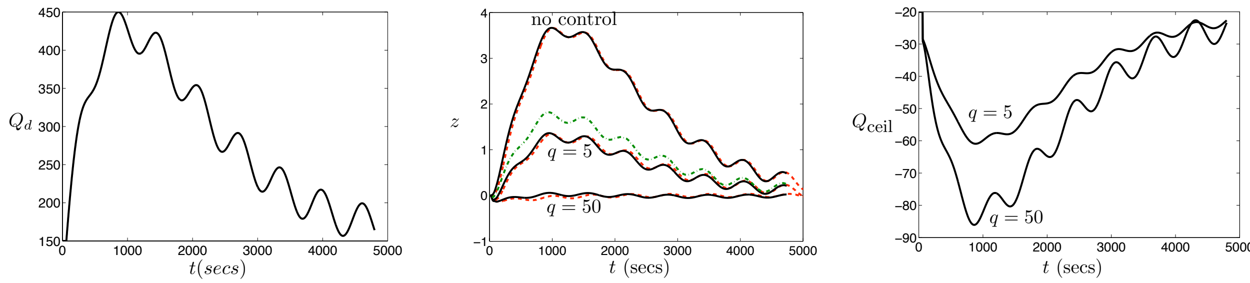

Fig. 5. (a) Disturbance input (floor heat flux, in W/m^2) as a function of time. (b) The controlled outputs z_k (temperature, averaged over the occupied zone), under this disturbance, for different control gains obtained using $q = 5, 50$. Also shown is the response when the control is off. The response of the full simulation (black, solid line) is compared with the observer reconstruction (red, dashed line). The green dash-dotted line shows the response of the full system, using a controller with imperfect knowledge of the future disturbance, assuming half the actual value. (c) Control inputs (chilled ceiling flux in W/m^2) for the two control gains.

V. CONCLUSIONS

We presented a method for developing controller, based on reduced-order airflow model, to maintain comfort in an indoor building environment under boundary disturbances. The reduced order model was obtained by applying ERA to the linearized Boussinesq equations. We presented an interpretation of ERA using the weak form of the PDE, that circumvents the use of lifting, an approach commonly used to introduce control and disturbances explicitly in the reduced model. Using the constrained Lagrangian approach, we derived a control law for discrete time state-space systems that reject a disturbance known over a future time horizon, while maintaining occupant comfort, and minimizing the control effort. The model reduction and control design techniques were demonstrated on a model room problem, equipped with displacement vent and chilled ceiling. Through closed loop CFD simulation, it was demonstrated that the resulting controller is capable of rejecting a known floor disturbance, and suppresses the undesirable temperature rise within the room, while minimizing the control effort.

Several challenges remain to be addressed. The approach outlined in the paper needs to be evaluated in more realistic building applications and experimentally validated. In the room problem considered in the paper, the nominal airflow pattern turned out be steady. In general, the nominal behavior can be time-periodic or exhibit general time dependence, in which case the linearized dynamics will not be time invariant. Recently, ERA has been extended to time periodic linear systems [19]; it would be useful to extend the approach developed here to this more general setting. The control law we derived assumed that the disturbance is known or can be predicted over a future time horizon. We are currently considering a robust control design under worst case disturbance conditions, and will compare the robust control performance with that derived in this paper to evaluate the value of disturbance prediction.

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