# Parametric Mismatch Detection and Isolation in Model Predictive Control System<sup>\*</sup>

Hong Wang<sup>\*</sup> Zhihuan Song<sup>\*</sup> Lei Xie<sup>\*\*</sup>

\* State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, Zhejiang, China(e-mail: {wanghong,zhsong}@iipc.zju.edu.cn)
\*\* Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, Zhejiang, China(e-mail: leix@csc.zju.edu.cn)

**Abstract:** Predictor that is built from the plant model, plays an important role in model predictive control system. The predictor should be updated timely to maintain certain calculation accuracy and performance optimality. In order to avoid unnecessary interruptions to production, however, updating should only be done when serious mismatch between the process and model appears. A novel method based on subspace approach is proposed to detect the mismatches using closed-loop operation data. The channels with mismatches in multi input multi output system are isolated. And some combinations of the mismatched parameters that have physical significance can be detected. These results provide useful information for the maintenance of model predictive control system. Simulations on a distillation process demonstrate the efficacy of the methodology.

Keywords: parametric mismatch, model predictive control, subspace approach

# 1. INTRODUCTION

Model predictive control (MPC) system can be found in numerous industrial applications, covering petroleum refinery, metal processing, aerospace and so on [1]. Predictor that is built from the plant model by first principles or system identification, is an essential part of MPC. It estimates the future plant outputs over a finite period. At one control interval, control actions over the finite horizon are calculated based on the current states and measurements and by minimizing some cost functions. But only the first calculated control action is implemented on the process in the following interval while the others are abandoned. Then the calculation and implementation procedure repeats which is known as receding horizon control. As an advanced control technology, MPC can move the outputs to the optimal values and prevent excessive movements of the manipulated variables. Output variation is drastically reduced.. Therefore, the performance of MPC depends on the accuracy of the plant model very much.

Mismatch detection and isolation is critical for the maintenance of MPC system. MPC has certain robustness to the mismatch. Usually the accuracy of the control model is good enough for designing a controller that achieves satisfying control performance. But the process is always changing over time such as mechanical wear in valves and scale formation in containers. Sometimes the changes get beyond the ability of the MPC and degrades the control performance a lot. Therefore re-identification is substantial in maintenance of the MPC system. But it's quite a demanding one because of long time plant tests and interruptions to the production [2]. As the scale of the modern plant grows larger, there may be hundreds of control loops to be taken care of in just one system. When the accuracy of the predictor deteriorates, it's always due to mismatches taking place in only partial input output channels [3]. Thus a full scale identification over hundreds of loops is obviously wasting. If the engineers know which channels the mismatches are located, precise identification can be done to cut the cost. In addition, the physical significance of the mismatch is informative and desired during the maintenance. For example, if a mismatch that happens on a chemical container is found to be related to the time constant, maybe it's caused by the scale formation which slows down the thermal conduction. This provides useful information to the engineers for checking the loops before the identification starts.

Mismatch in model based control loops has been a broad concern in recent years. Several methods have been proposed to detect the mismatch. Both partial correlation approach [5] and stepwise method [4] based on routine operation data are used to isolate the specific channels where the the mismatch appears. The former needs to identify several intermediate models while the latter uses indirect variable selection instead of exact identification. Three signatures relating to the state space matrices are utilized to detect model plant mismatch in multivariate systems in [6]. In [7] plant model ratio in the frequency domain is developed to identify the mismatch in corresponding components of a single input single output transfer function model. This method has the advantage of

 $<sup>\</sup>star$  This work is partially supported by China National 863 Project 2009AA04Z154. The authors would like to thank the reviewers for their comments.



Fig. 1. Model based control loop structure.

relating the mismatch to those parameters with physical significance. But some extensions are needed to deal with multivariate systems.

In this paper a procedure dealing with model plant mismatch is developed to improve the maintenance for MPC systems. Firstly the Markov parameters of the plant can be obtained by subspace approach in the context of closedloop. Then the input output channels which encounter changes can be isolated and some specific parametric combinations are diagnosed without calculating the exact values of these parameters.

The structure of the paper is organized as follows. Section 2 formulates the problem on mismatch. Section 3 gives a preliminary description about Markov parameters and subspace approach. The scheme of detection and isolation is presented in Section 4. A simulation study is demonstrated in Section 5 on a distillation plant followed by some remarks in Section 6. Section 7 makes the conclusion.

# 2. PROBLEM FORMULATION

In the following work, a model based control structure in Fig.1 is used to demonstrate the methodology. As it's shown in Fig.1,  $P^0(s)$  is the transfer function matrix of control model that is used to design the controller. P(s)is the transfer function matrix of the current plant. C(s)is the controller and D(s) is the transfer function matrix of unmeasured disturbance model.  $\mathbf{r} \in \mathbb{R}^n$ ,  $\mathbf{u} \in \mathbb{R}^m$ ,  $\mathbf{v} \in \mathbb{R}^l$ ,  $\mathbf{y} \in \mathbb{R}^n$  and  $\mathbf{y}_m \in \mathbb{R}^n$  are the setpoint, controller output, white noise, plant output and predicted output respectively where m, n and l are the numbers of controller outputs, plant outputs and disturbance inputs.

Accordingly there are  $n \times m$  input output channels in the plant and corresponding  $n \times m$  sub-models in both  $P^{0}(s)$  and P(s). Unmeasured disturbance channels are not considered here as identification is not done in those channels. Denote the sub-model in the  $j^{th}$  input and  $i^{th}$  output channel as  $p_{ij}^0(s)$  and  $p_{ij}(s)$  in  $P^0(s)$  and P(s) respectively. When the plant changes significantly, there are serious mismatches between  $P^{0}(s)$  and P(s). Then to keep high performance of the control loop, re-identification is supposed to be done for updating the models timely. Generally, many plant tests should be carried out for each input output channel and thus normal production is frequently interrupted. Sometimes it takes several weeks to complete the work which is really a huge cost for the industry. Actually serious mismatch usually appears only in a few channels at a time for most of the cases, e.g. the first input and first output channel and the second input and first output channel in a  $10 \times 10$  system. And

identification is only necessary for these sub-models to update, e.g.  $p_{11}(s)$  and  $p_{12}(s)$ , instead of one hundred channels. Hence isolation of these significantly mismatched channels is desired.

First order plus time delay (FOTD) is one of the most typical model structures. It can approximate many physical systems in process industries. Assume the sub-model can be written in the form of FOTD as:

$$p_{ij}(s) = \frac{K_{ij}}{T_{ij}s + 1} e^{-\tau_{ij}s} \tag{1}$$

where  $K_{ij}$ ,  $T_{ij}$  and  $\tau_{ij}$  are the gain, time constant and time delay of the sub-model. These parameters may be attached with physical significance such as mass, thermal resistance or the friction factors in pipes. Suppose the model structures are fixed. The mismatches are thus caused by the changes of these parameters. For the maintenance engineers, physical parameters should be traced to figured out the reasons of changes in the field. It is of great importance to relating the mismatches to the physical parameters. Additionally these targets should be reached under closed-loop.

# 3. PRELIMINARY

# 3.1 Markov parameters

State space equation, transfer function and Markov parameters describe this plant in three different ways, but they can be transformed to each other accurately. Consider a time invariant stable plant P(s) written in an innovation form as:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{K}\boldsymbol{v}_k \\ \boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{D}\boldsymbol{u}_k + \boldsymbol{v}_k \end{cases}$$
(2)

where  $\boldsymbol{x} \in \mathbb{R}^{\gamma}$  is the state variables and  $\boldsymbol{K} \in \mathbb{R}^{\gamma \times l}$  is the Kalman filter gain.  $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$  and  $\boldsymbol{D}$  are the system matrices in the state space equation with appropriate dimensions. For the state space description in Eq.(2), let  $\boldsymbol{x}(0) = 0$  and  $\boldsymbol{v}(k) = 0$ . When the inputs are set to impulse signals, the Markov parameters matrices can be derived as:

$$\boldsymbol{H}_{n} = \begin{cases} \boldsymbol{D}, & n = 1\\ \boldsymbol{C}\boldsymbol{A}^{n-1}\boldsymbol{B}, & n > 1 \end{cases}$$
(3)

And Eq.(1) can also be written in Markov parameter or impulse response as follows:

$$p_{ij}(q^{-1}) = \sum_{k=0}^{\infty} h_{ij, k} q^{-k}$$
(4)

For a stable plant, a sufficient large number N can be selected that  $h_{ij, k} \stackrel{k>N}{\approx} 0$ . Then a truncated form can be used to describe the sub-model:

$$p_{ij}(q^{-1}) = \sum_{k=1}^{N} h_{ij, k} q^{-k}$$
(5)

where  $h_{ij, k}$  is the  $k^{th}$  element in Markov parameter vector  $h_{ij}$ .

Assuming the sampling time is  $T_s$ , we can discretize Eq.(1) as:

$$p_{ij}(z^{-1}) = \frac{K_{ij} z^{-\tau_{ij}/T_s}}{T_{ij} \left(1 - e^{-T_s/T_{ij}} z^{-1}\right)}$$
(6)

By series expansion the corresponding coefficients can be derived as:

$$h_{ij, k} = \begin{cases} 0 & \text{if } 0 \le k \le \theta_{ij} \\ \frac{K_{ij}}{T_{ij}} e^{-(k-\theta_{ij}-1)T_{s}/T_{ij}} & \text{otherwise} \end{cases}$$
(7)

where  $\theta = \tau_{ij}/T_s$ . Here  $\theta$  is time delay with the unit of the sampling time and thus is reasonable to round the delay into integers. Then Eqs.(5) and (7) can be utilized to identify the corresponding changes in the parameters of FOTD.

# 3.2 Subspace Approach

Subspace approach has been intensively used to identify plant models and design controllers such as MPC in the past few decades. By proper iterations, extensions and ensembles, a subspace equation can be derived:

$$\begin{cases} \mathbf{Y}_{\mathrm{f}} = \mathbf{L}_{W} \mathbf{W}_{\mathrm{p}} + \mathbf{L}_{u} U_{\mathrm{f}} + \mathbf{L}_{v} \mathbf{V}_{\mathrm{f}} \\ D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N-2}B & CA^{N-3}B & CA^{N-4}B & \cdots & D \end{pmatrix} \\ \mathbf{L}_{v} = \begin{pmatrix} \mathbf{I} & 0 & 0 & \cdots & 0 \\ CK & \mathbf{I} & 0 & \cdots & 0 \\ CAB & CK & \mathbf{I} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N-2}K & CA^{N-3}K & CA^{N-4}K & \cdots & \mathbf{I} \end{pmatrix} \\ \mathbf{W}_{\mathrm{p}} = \begin{pmatrix} \mathbf{Y}_{\mathrm{p}} \\ \mathbf{U}_{\mathrm{p}} \end{pmatrix}$$
(8)

where subscripts p and f indicate the data of the past and future respectively; Y, U and V are the block Hankel matrices of plant output, controller output and white noise;  $L_W$ ,  $L_u$  and  $L_v$  are the coefficient matrices with corresponding dimensions; N is the number of blocks in the subspace matrices as same as the selected order above.

Assume C(s) is the linear time invariant controller in the closed-loop, the controller output can be described by:

$$\boldsymbol{u}_t = C(s)(\boldsymbol{r}_t - \boldsymbol{y}_t) \tag{9}$$

The state apace equation form of Eq.(9) can be written as:

$$\begin{cases} \boldsymbol{x}_{k+1}^{c} = \boldsymbol{A}^{c} \boldsymbol{x}_{k}^{c} + \boldsymbol{B}^{c} (\boldsymbol{r}_{k} - \boldsymbol{y}_{k}) \\ \boldsymbol{u}_{k} = \boldsymbol{C}^{c} \boldsymbol{x}_{k}^{c} + \boldsymbol{D}^{c} (\boldsymbol{r}_{k} - \boldsymbol{y}_{k}) \end{cases}$$
(10)

where superscript c denotes the controller and  $A^{c}$ ,  $B^{c}$ ,  $C^{c}$ and  $D^{c}$  are coefficient matrices with appropriate dimensions. Then another subspace equation is also derived in the same way as Eq.(8):

$$U_{\rm f} = \boldsymbol{L}_W^{\rm c} \boldsymbol{W}_{\rm p}^{\rm c} + \boldsymbol{L}_r^{\rm c} \boldsymbol{R}_{\rm f} - \boldsymbol{L}_y^{\rm c} \boldsymbol{Y}_{\rm f}$$
(11)

Combining Eq.(8) with Eq.(11) an equation can be formulated as:

$$\boldsymbol{Y}_{\mathrm{f}} = \boldsymbol{L}_{w}\boldsymbol{W}_{\mathrm{p}} + \boldsymbol{L}_{u}(\boldsymbol{L}_{w}^{\mathrm{c}}\boldsymbol{W}_{\mathrm{p}}^{\mathrm{c}} + \boldsymbol{L}_{r}^{\mathrm{c}}\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{L}_{y}^{\mathrm{c}}\boldsymbol{Y}_{\mathrm{f}}) + \boldsymbol{L}_{v}\boldsymbol{V}_{\mathrm{f}} \quad (12)$$

Two regression equations can be decomposed from Eq.(12) as:

$$\begin{cases} \boldsymbol{Y}_{\mathrm{f}} = \boldsymbol{L}_{y}^{\mathrm{CL}} \boldsymbol{W}_{\mathrm{p}}^{\mathrm{CL}} + \boldsymbol{L}_{yr}^{\mathrm{CL}} \boldsymbol{R}_{\mathrm{f}} + \boldsymbol{L}_{yv}^{\mathrm{CL}} \boldsymbol{V}_{\mathrm{f}} \\ \boldsymbol{U}_{\mathrm{f}} = \boldsymbol{L}_{u}^{\mathrm{CL}} \boldsymbol{W}_{\mathrm{p}}^{\mathrm{CL}} + \boldsymbol{L}_{ur}^{\mathrm{CL}} \boldsymbol{R}_{\mathrm{f}} + \boldsymbol{L}_{uv}^{\mathrm{CL}} \boldsymbol{V}_{\mathrm{f}} \end{cases}$$
(13)

where R is the block Hankel matrix for setpoint and superscript CL indicates the closed-loop with  $W_{\rm p}^{\rm c}$  =  $\begin{pmatrix} \boldsymbol{Y}_{\mathrm{p}}^{\mathrm{T}} \ \boldsymbol{U}_{\mathrm{p}}^{\mathrm{T}} \ \boldsymbol{R}_{\mathrm{p}}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$ . Apparently  $\boldsymbol{R}_{\mathrm{f}}$  and  $\boldsymbol{V}_{\mathrm{f}}$  are not correlated so that the closed-loop problem are decomposed into two open-loop problems in Eq.(13). The coefficient matrices  $\boldsymbol{L}_{y}^{\mathrm{CL}}, \boldsymbol{L}_{yr}^{\mathrm{CL}}, \boldsymbol{L}_{u}^{\mathrm{CL}}$  and  $\boldsymbol{L}_{ur}^{\mathrm{CL}}$  can be calculated by least squares method. The block Hankel matrix for Markov parameters of plant model is derived as:

$$\boldsymbol{L}_{u} = \boldsymbol{L}_{yr}^{\mathrm{CL}} (\boldsymbol{L}_{ur}^{\mathrm{CL}})^{-1}$$
(14)

The Markov parameter vectors in each channel of the plant model can be extracted as  $h_{ij}$  from  $L_u$ . Step by step derivations and explanations are saved duo to space limit, please see more in [8].

#### 4. DETECTION AND ISOLATION PROCEDURE

Nowadays there are plenty of routine operation data in the database from industry that are unused. These data are generated from daily operations such as setpoint changes and load changes, instead of being designed for systematic identification from numerous intrusive tests. But if we select those have enough changes, they can provide sufficient information that can be used to detect the mismatches. Here the subspace approach is adopted to obtain the Markov parameters matrices from these informative routine operation data. Then the Markov parameter vector of each input output channel can be extracted, based on which the channels contain mismatches are isolated. After that, the combinations of the gain, time constant and time delay in FOTD are detected and physical explanations on the mismatches can be given to aid the maintenance.

#### 4.1 Mismatched Channel Detection

As presented above, the Markov parameter vector of each sub-model in the current plant can be obtained, e.g.  $h_{ij}$  in the  $j^{th}$  input and  $i^{th}$  output channel. Let the corresponding Markov parameter vector of the control model be  $h_{ij}^0$ . Then the mismatch can be described as:

$$\boldsymbol{\Delta}_{ij} = \boldsymbol{h}_{ij} - \boldsymbol{h}_{ij}^0 \tag{15}$$

An area measure [9] is used to quantify the Markov parameter vector as:

$$\Omega_{\text{MPM}\_ij} = \sum_{k=1}^{N} |\Delta_{ij, k}|$$
(16)

$$\Omega_{p_{ij}^0} = \sum_{k=1}^N \left| h_{ij,\ k}^0 \right| \tag{17}$$

where  $\Delta_{ij, k}$  and  $h_{ij, k}^{0}$  are the  $k^{th}$  elements in  $\boldsymbol{\Delta}_{ij}$  and  $\boldsymbol{h}_{ij}$  respectively.  $\Omega_{\text{MPM}\_ij}$  and  $\Omega_{p_{ij}^{0}}$  are the corresponding area measures of mismatch and control model. Then a scaled index  $\eta_{ij}$  is developed to asses the significance of the mismatch in this channel:

$$\eta_{ij} = \frac{\Omega_{\text{MPM}\_ij}}{\Omega_{p_{ij}^0}} \tag{18}$$

It has the following properties:

(P1)  $0 \le \eta_{ij}$ ; (P2)  $\eta_{ij} = 0$  if and only if  $\boldsymbol{\Delta}_{ij} = 0$ ;

(P3) The greater  $\eta_{ij}$  is, the more significant the mismatch is.

Proposition 1. Assume N is selected satisfying:  $\forall \epsilon > 0$ ,  $|h_{ij, N+1}| < \epsilon$ . The submodel in the  $j^{th}$  input and  $i^{th}$  output channel of the plant is mismatched if and only if  $\eta_{ij} > 0$ .

#### 4.2 Mismatched Parameter Isolation

To identify the specified mismatched parameters of the FOTD model, three signatures are developed. Assume the signs of the parameters are not changed. First we check the situation of time delay. Denote the time index of the first nonzero element in  $\boldsymbol{h}_{ij}^0$  and  $\boldsymbol{h}_{ij}$  as  $\lambda_{ij}^0$  and  $\lambda_{ij}$ . Therefore the time delay shift can be identified by the difference of  $\lambda_{ij}^0$  and  $\lambda_{ij}$  as  $\sigma_{ij} = \lambda_{ij} - \lambda_{ij}^0$ .

Proposition 2. The time delay in FOTD model is mismatched if and only if  $\sigma_{ij} \neq 0$ . A positive  $\sigma_{ij}$  indicates an increase of time delay while a negative one indicates decrease.

Then we eliminate the influences of time delay mismatch for other parametric mismatch isolation.  $\boldsymbol{h}_{ij}^0$  and  $\boldsymbol{h}_{ij}$  are modified into new vectors so that their first element are all nonzero. Let  $\zeta_{ij} = \max\{\lambda_{ij}^0, \lambda_{ij}\}$  and  $\kappa_{ij} = N - \zeta_{ij}$ . Only their first  $(\kappa_{ij} + 1)$  nonzero elements are kept. Two corresponding new vectors are obtained as  $\tilde{\boldsymbol{h}}_{ij}^0 = \boldsymbol{h}_{ij}^0(\lambda_{ij}^0 :$  $(\kappa_{ij} + \lambda_{ij}^0))$  (this operation means that elements from the  $(\lambda_{ij}^0)^{\text{th}}$  to the  $(\kappa_{ij} + \lambda_{ij}^0)^{\text{th}}$  in  $\boldsymbol{h}_{ij}^0$  are extracted to form a new vector  $\tilde{\boldsymbol{h}}_{ij}^0$ ) and  $\tilde{\boldsymbol{h}}_{ij} = \boldsymbol{h}_{ij}(\lambda_{ij} : (\kappa_{ij} + \lambda_{ij}))$ . Linear regression is done to check the slope  $(\alpha_{ij} + 1)$  and intercept  $\beta_{ij}$  where  $\tilde{\boldsymbol{h}}_{ij} = (\alpha_{ij} + 1)\tilde{\boldsymbol{h}}_{ij}^0 + \beta_{ij}\boldsymbol{I}_{N\times 1}$ . When  $\beta_{ij} = 0$ , the time constant isn't mismatched. If  $\alpha_{ij} = 0$ , the gain remains the same; if  $\alpha_{ij} > 0$  the gain increases; if  $-1 < \alpha_{ij} < 0$  the gain decreases. When  $\beta_{ij} \neq 0$  the time constant is mismatched. Here we can't tell if the gain is mismatched or not without calculating the exact value.

Proposition 3. The time constant in FOTD model is mismatched if and only if  $\beta_{ij} \neq 0$ .

Proposition 4. When  $\beta_{ij} = 0$  and  $\alpha_{ij} = 0$ , the gain in FOTD model is not mismatched; When  $\beta_{ij} = 0$  and  $\alpha_{ij} \neq 0$ , the gain is mismatched; When  $\beta_{ij} \neq 0$ , more information is needed to judge if the gain is mismatched or not.

## 4.3 Practical Modification

In practice, mismatches are very common due to error from noise and limited samples. The criterions given above should be loosen to deal with this situation. That is to improve the robustness of the proposed method by changing the thresh values. For example, as only those significant mismatches are our concern, thresh values  $\eta_{ij}^0$  other than zero can be chosen to separate significant mismatches from non-significant mismatches. Usually it's selected based on maintenance requirements. Smaller thresh values are more sensitive to the noises. For similar purpose, interval  $\|\beta_{ij}\| \leq \beta_{ij}^0$  replaces  $\beta_{ij} = 0$  while  $\|\beta_{ij}\| > \beta_{ij}^0$  replaces  $\beta_{ij} \neq 0$  for time constant mismatch isolation. Nonzero value $\alpha_0$  should be used instead of zero. In this paper  $\alpha_{ij}^0$  and  $\beta_{ij}^0$  are chosen based on the calculated values from normal case.

Normalized indices are established:

$$I_{ij}^{x} = \frac{x_{ij} - x_{ij}^{0}}{x_{ij} + x_{ij}^{0}}$$
(19)

where x represents  $\eta$ ,  $\alpha$  or  $\beta$  and  $0 \leq I_{ij}^x \leq 1$ . These normalized ones can be used as significance level indicators where monitored case is compared with nominal case. Bigger positive values of  $I_{ij}^x$  indicate more significant changes in monitored case while negative ones indicate less significant. Thresh values for significance  $I_{ij}^1$ ,  $I_{ij}^2$  and  $I_{ij}^3$  can also be chosen based on the maintenance requirements, instead of the nominal case.

In other words,  $\eta_{ij}$ ,  $\alpha_{ij}$ ,  $\beta_{ij}$  and  $\sigma_{ij}$  are absolute indicators while  $I_{ij}^x$  is a relative indicator. The latter can tell the significance level comparing with nominal case or maintenance requirement.

# 5. SIMULATION STUDY: DISTILLATION PROCESS

A pilot scale binary distillation column for a methanol water mixture developed by Wood and Berry [10] is slightly modified to demonstrate the efficacy of the method. This plant has two controlled variables  $y_{\rm T}$  and  $y_{\rm B}$ , two manipulated variables R and S and one unmeasured disturbance variable F. The transfer function matrix for control model is:

$$\begin{bmatrix} y_{\rm T} \\ y_{\rm B} \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R \\ S \end{bmatrix} + \begin{bmatrix} \frac{-3.8e^{-8s}}{14.9s+1} \\ \frac{4.9e^{-3s}}{13.2s+1} \end{bmatrix} F$$
(20)

where  $y_{\rm T}$ ,  $y_{\rm B}$ , R, S and F are the top composition, bottom composition, reflux flow rate, steam flow rate and feed flow rate respectively. MPC is used to control the plant and the sampling time is 1 minute. The setpoints for  $y_{\rm T}$  and  $y_{\rm B}$ are changed to simulate the real industrial situations. Fis set to be unmeasured disturbance. The deviated input signals which are changed based on the operating points are plotted in Fig.2.  $p_{11}$ ,  $p_{12}$ ,  $p_{21}$  and  $p_{22}$  are the channels of  $R - y_{\rm T}$ ,  $S - y_{\rm T}$ ,  $R - y_{\rm B}$  and  $S - y_{\rm B}$  respectively. TC and TD indicate time constant and time delay correspondingly in the tables. The standard deviation of the noise is 0.032.

#### 5.1 Nominal Case

The plant model is identified by subspace approach as shown in Fig.3. As we can see, the identified results are very consistent with the plant dynamics.

In Table 1 the signatures are calculated under normal conditions. There are differences when comparing with the theoretical results which should all be zeroes. But with limited samples of routine operation data, these errors are tolerable and the following analysis will take this circumstance into account. For example,  $\eta_{21}^0 = 0.2188$  in  $p_{21}$  is significant in absolute value. That's why we analyze the following results with significance level too.  $\eta_{ij}^0$ ,  $\alpha_0$  and  $\beta_0$  are selected as thresh values for different channels. The values with significant changes comparing with Table 1 are in bold front in the other tables. Several combinations of parametric mismatch are set to test the method.

8th IFAC Symposium on Advanced Control of Chemical Processes Furama Riverfront, Singapore, July 10-13, 2012



Fig. 2. Setpoint and white noise signals.



Fig. 3. Comparison of Markov parameters: true plant (solid line) and identified results (dashed line).

Table 1. No mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
Mismatch	0%	0%	0%	0%
$\eta_{ij}^0$	0.0793	0.0747	0.2188	0.1277
$\sigma_{ij}^{0}$	0	0	0	0
$\alpha_{ij}^{0}$	-0.0234	-0.0292	-0.0555	-0.0389
$\beta_{ij}^{0}$	-0.0075	0.0053	-0.0105	0.0133

Table 2. Gain mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
Gain mismatch	20%	5%	50%	2%
$\eta_{ij}$	0.1456	0.0647	0.4315	0.1118
$\sigma_{ij}$	0	0	0	0
$lpha_{ij}$	0.1615	0.0233	0.3912	-0.0180
$\beta_{ij}$	-0.0067	0.0075	-0.0074	0.0153

# 5.2 Gain Mismatch Case

Different magnitudes of gain mismatch are set to the four input output channels of the plant. The settings are shown in Table 2. Comparing the index  $\eta_{ij}$  with those in Table 1, it can be found that channels  $p_{11}$  and  $p_{12}$  have significant mismatches as  $I_{11}^{\eta} = 29\%$  and  $I_{21}^{\eta} = 33\%$ . For  $\sigma_{ij}$  and  $\beta_{ij}$ there are no big differences, which means no time delay or time constant mismatch appears. Obvious changes for  $\alpha_{ij}$ in channels  $p_{11}$  and  $p_{12}$  indicates that gain mismatches exist in both channels with  $I_{11}^{\alpha} = 75\%$  and  $I_{21}^{\alpha} = 75\%$ . This is consistent with the settings.

Table 3. Time constant mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
TC mismatch	5%	50%	5%	5%
$\eta_{ij}$	0.0897	0.2622	0.2272	0.1396
$\sigma_{ij}$	0	0	0	0
$\alpha_{ij}$	-0.0642	-0.3720	-0.0965	-0.0742
$\beta_{ij}$	-0.0011	-0.0637	-0.0066	0.0044

Table 4. Time delay mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
TD mismatch	0%	33.33%	0%	0%
$\eta_{ij}$	0.0771	0.1113	0.2111	0.1235
$\sigma_{ij}$	0	1	0	0
$lpha_{ij}$	-0.0318	-0.0198	-0.0713	-0.0295
$\beta_{ij}$	-0.0045	0.0079	-0.0063	0.0166

Table 5. Gain and time constant mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
Gain mismatch	20%	5%	50%	2%
TC mismatch	5%	50%	5%	5%
$\eta_{ij}$	0.1284	0.2371	0.4122	0.1203
$\sigma_{ij}$	0	0	0	0
$lpha_{ij}$	0.1159	-0.3346	0.3441	-0.0507
$\beta_{ij}$	0.0002	-0.0653	-0.0036	0.0069

## 5.3 Time Constant Mismatch Case

The mismatches and the detection results for time constant mismatch scenario are shown in Table 3.  $\eta_{12}$  is changed significantly by 56% indicating that mismatch appears in channel  $p_{12}$ . The corresponding  $\sigma_{12}$  is 0 while  $\alpha_{12}$ and  $\beta_{12}$  have both big variations by 85%. A time constant mismatch is included with possible gain mismatch.

## 5.4 Time Delay Mismatch Case

The simulation data for time delay mismatch are presented in Table 4. Channel  $p_{12}$  is accurately isolated as significantly time delay mismatched since  $\eta_{12}$  is changed obviously by 20% and the corresponding  $\sigma_{12}$  is 1.

#### 5.5 Gain and Time Constant Mismatch Case

Scenario of gain and time constant mismatch gives results in Table 5. Three channels are isolated as evidently mismatched ones.  $p_{11}$  and  $p_{21}$  are diagnosed with gain mismatch while  $p_{12}$  with time constant mismatch and possible gain mismatch according to  $I_{12}^{\eta} = 52\%$ ,  $I_{12}^{\alpha} = 84\%$  and  $I_{12}^{\beta} = 85\%$ .

## 5.6 Gain and Time Delay Mismatch Case

The data for gain and time delay mismatch are demonstrated in Table 6. Channel  $p_{12}$  is accurately isolated as significantly time delay mismatched since  $\eta_{12}$  is changed obviously from 0.0747 to 0.1317 and the corresponding  $\sigma_{12}$ is 1. Both  $p_{11}$  and  $p_{21}$  are diagnosed as mismatches in gain.

# 5.7 Time Constant and Time Delay Mismatch Case

As shown in Table 7, the detection procedure accurately isolate the seriously mismatched channel  $p_{12}$ . For the parametric mismatch, results suggest that time constant mismatch and time delay mismatch appear in channel  $p_{12}$ .

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
Gain mismatch	20%	5%	50%	2%
TD mismatch	0%	33.33%	0%	0%
$\eta_{ij}$	0.1418	0.1317	0.4252	0.1111
$\sigma_{ij}$	0	1	0	0
$lpha_{ij}$	0.1630	0.0277	0.3899	-0.0119
$\beta_{ij}$	-0.0082	0.0081	-0.0091	0.0165

Table 6. Gain and time delay mismatch scenario

 
 Table 7. Time constant and time delay mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
TC mismatch	5%	50%	5%	5%
TD mismatch	0%	33.33%	0%	0%
$\eta_{ij}$	0.0926	0.2797	0.2370	0.1374
$\sigma_{ij}$	0	1	0	0
$\alpha_{ij}$	-0.0635	-0.3703	-0.0975	-0.0700
$\beta_{ij}$	-0.0022	-0.0647	-0.0079	0.0043

 Table 8. Gain, time constant and time delay mismatch scenario

Types	$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$
Gain mismatch	20%	5%	50%	2%
TC mismatch	5%	50%	5%	5%
TD mismatch	0%	33.33%	0%	0%
$\eta_{ij}$	0.1267	0.2602	0.4114	0.1195
$\sigma_{ij}$	0	1	0	0
$lpha_{ij}$	0.1164	-0.3347	0.3442	-0.0479
$eta_{ij}$	-0.0003	-0.0662	-0.0042	0.0071

5.8 Gain, Time Constant and Time Delay Mismatch Case

When the three parameters change at the same time, mismatch index can isolate the seriously mismatched channels accurately. Both  $p_{11}$  and  $p_{21}$  are found to have gain mismatch while  $p_{12}$  is definitely with time delay and time constant mismatch.

# 6. REMARKS

Though this methodology focuses on MPC system, it can deal with other model-based multivariate control system too. Some aspects related are discussed in brief.

6.1 Index Usage

Due to space limit, the specified significance levels are not shown here. The four measures  $\eta_{ij}$ ,  $\sigma_{ij}$ ,  $\alpha_{ij}$  and  $\beta_{ij}$  and the corresponding significance levels  $I_{ij}^x$  should together be used for explanation of the results. Monte carlo can be used to show the efficacy of the method too.

# 6.2 Signal Check

Sufficient excitation is critical for the success of all mismatch detection problems. The focus in this paper is to utilize the untouched data for guiding the maintenance. And the sufficiency should be concerned during the procedure. One method is to check the rank of the closed-loop matrices in Eq.(14) [8]. And for those MPCs which receive setpoints from upper optimization level, this method should be carefully used for possible correlation between setpoint and disturbance.

# 6.3 Measured Disturbance

Measured disturbance channels can also be handled by this methodology. After extending the subspace equations, subspace approach can estimate the corresponding Markov parameters. But for the routine operation data, these channels may have limited excitation. This needs to collect sufficiently excited data to ensure the efficacy of this procedure.

# 7. CONCLUSION

In this paper a method mining the potential of plenty of unused data in industry is developed. Subspace approach can estimate the Markov parameter vector of each submodel. Then we can detect the significantly mismatched input output channels based on the area index. The corresponding contributing parameters of FOTD sub-models in MPC system are isolated by three signatures too. Significance levels provide more practical guidelines for industrial usages. These are of great help to improve the efficiency of system maintenance.

## REFERENCES

- [1] S. J. Qin, and T. A. Badgwell (2003). "A survey of industrial model predictive control technology." Control Engineering Practice 11(7): 733-764.
- [2] J. S. Conner, and D. E. Seborg (2005). "Assessing the need for plant re-identification." Industrial and Engineering Chemistry Research 44(8): 2767-2775.
- [3] M. Kano, and M. Ogawa (2010). "The state of the art in chemical plant control in Japan: Good practice and questionnaire survey." Journal of Process Control 20(9): 969-982.
- [4] M. Kano, Y. Shigi, S. Hasebe, and S. Ooyama (2010). "Detection of Significant Model-Plant Mismatch from Routine Operation Data of Model Predictive Control System." The 9th International Symposium on Dynamics and Control of Process Systems (DYCOPS 2010). Leuven, Belgium: pp.677-682.
- [5] A. S. Badwe, R. D. Gudi, R. S. Patwardhan, S. L. Shah, and S.C. Patwardhan (2009). "Detection of model-plant mismatch in MPC applications." Journal of Process Control 19(8): 1305-1313.
- [6] H. Jiang, W. Li, and S.L. Shah. "Detection and isolation of model-plant mismatch for multivariate dynamic systems." In 6th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, volume 6, pages 1396C1401, 2006.
- [7] S. Selvanathan, and A. K. Tangirala (2010). "Diagnosis of poor control loop performance due to modelplant mismatch." Industrial and Engineering Chemistry Research 49(9): 4210-4229.
- [8] B. Huang, and R. Kadali, Eds. (2008). "Dynamic Modeling, Predictive Control and Performance Monitoring: A Data-driven Subspace Approach," Springer Verlag.
- [9] H. Wang, R. Fu, L. Xie, and Z. Song (2011). "Model plant mismatch detection algorithm based on subspace approach." Journal of Nanjing University of Science and Technology 35(SUPPL. 2): 279-283.
- [10] R. K. Wood, and M. W. Berry (1973). "Terminal composition control of a binary distillation column." Chemical Engineering Science 28(9): 1707-1717.