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OUTPUT TRACKING OF BIOPROCESSES THROUGH RECIRCULATION WITH UNKNOWN INPUT CONCENTRATION [1]

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Abstract: In a recent work, a new regulator of the output of a continuous auto-catalytic bioprocess, by means of recirculation loop, has been presented. It was shown that controlling the recirculation flow rate allows the stabilization of the output in presence of an uncontrolled input flow rate. In the present paper, we extend this result when the input substrate concentration is unknown. For this purpose, we propose the design of an observer of the input concentration which, coupled with a slightly different control law reminiscent of the one used in the case where the input concentration is known, guarantees the regulation of the output.

Keywords: bioprocess, recirculation loop, output regulation, nonlinear control, unknown input observer.

1. INTRODUCTION

1.1 Context

In a recent work (cf. (Harmand *et al.*, 2005)), a new control law for regulating the output of a continuous auto-catalytic process was proposed. While most of the available studies of the literature use the input flow rate Q as the control variable, it was proposed to control the process through a recirculation loop. Among others, advantages are that no storage tank is needed anymore at the entrance of the process and that the flow rate has not to be known perfectly. The particular process configuration considered to do so is described in Fig. 1 where $\alpha \in [0, 1]$ and $\beta \geq 0$ are the manipulated variables.

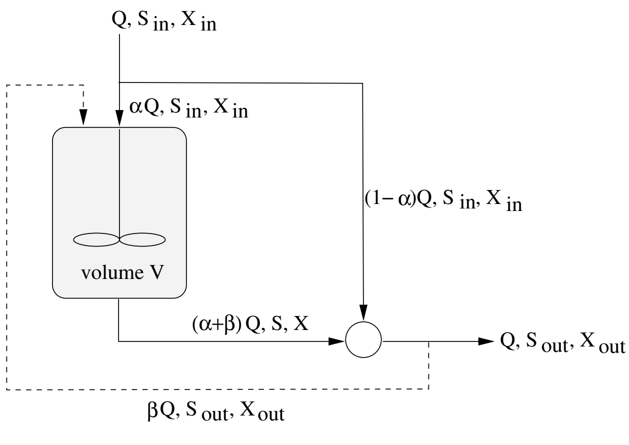

Fig. 1. General view of the bioprocess configuration under interest.

1.2 Modeling

The function $D(t) = Q(t)/V$ be given, the model of this system described in Figure 1 can be written as it follows:

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$$\begin{aligned}\dot{X} &= \mu(S)X + uD(t)\,(X_{in} - X)\\ \dot{S} &= -\frac{\mu(S)}{Y}X + uD(t)\,(S_{in} - S)\end{aligned} \quad (1)$$

where S and X stand for the biomass and the substrate concentrations (in mg/l) in the reactor and $u = \frac{\alpha+\beta}{1+\beta} \in [0,1]$ is the control variable. $\mu(S)$ is the reaction rate (in t^{-1}), Y the conversion yield (in mg of substrate consumed by mg of biomass formed) and V the volume of the reactor (in l). All these quantities are assumed to be known.

S_{in} and X_{in} are the unknown input substrate and biomass concentrations (in mg/l), possibly time varying but bounded:

$$(X_{in}(t), S_{in}(t)) \in [\underline{X}_{in}, \overline{X}_{in}] \times [\underline{S}_{in}, \overline{S}_{in}],\ \forall t \geq 0,$$

where $\overline{X}_{in} \geq \underline{X}_{in} \geq 0$ and $\overline{S}_{in} \geq \underline{S}_{in} > 0$ are known numbers.

The control problem investigated in the paper is the regulation of the output

$$S_{out} = uS + (1-u)S_{in} \quad (2)$$

even though S_{in} is unknown. We consider a time-varying reference trajectory to be tracked, that we denote $S^{\star}_{out}(\cdot)$, and we introduce the following hypothesis.

Hypothesis H0. There exist numbers $\overline{S}^{\star}_{out} \geq \underline{S}^{\star}_{out} > 0$ such that $S^{\star}_{out}(t) \in [\underline{S}^{\star}_{out}, \overline{S}^{\star}_{out}]$ for all $t \geq 0$, with

$$\overline{S}^{\star}_{out} < \underline{S}_{in}\ .$$

1.3 Regulation of S_{out} when S_{in} is known

In this section, we recall the result we obtained when S_{in} is perfectly known (Harmand *et al.*, 2005). We introduce usual assumptions on the growth function $\mu(\cdot)$.

Hypothesis H1. The function $\mu(\cdot)$ is a non-negative Lipschitz continuous function with $\mu(0) = 0$ and $\mu(\underline{S}^{\star}_{out}) > 0$, that fulfills

$$\mu(S) \geq \mu(\underline{S}^{\star}_{out}), \quad \forall S \in [\underline{S}^{\star}_{out}, \overline{S}_{in}] \quad (3)$$

Contrary to usual approaches for which the input flow rate D is a manipulated variable, D is here imposed but we assume that it is bounded, with known bounds.

Hypothesis H2. There exist numbers $\underline{D} \leq \overline{D}$ and $T \geq 0$ such that $D(t) \in [\underline{D}, \overline{D}]$ for all $t \geq T$, with $\underline{D} > 0$ and

$$\overline{D} < \mu(\underline{S}^{\star}_{out})\frac{\underline{X}_{in} + Y(\underline{S}_{in} - \underline{S}^{\star}_{out})}{Y(\overline{S}_{in} - \underline{S}^{\star}_{out})}\ . \quad (4)$$

Then we have the following result (cf. (Harmand *et al.*, 2005)):

Proposition 1. Assume H0-H1-H2 are satisfied by the system (1). Then for any initial condition such that $X(0) > 0$ and $0 \leq S(0) < \overline{S}_{in}$, the feedback

$$u^{\star}(t, S, S_{in}) = \frac{S_{in} - S^{\star}_{out}(t)}{S_{in} - \min(S^{\star}_{out}(t), S)} \quad (5)$$

drives the output $S_{out}(\cdot)$, defined in (2), at $S^{\star}_{out}(\cdot)$ in finite time.

On the one hand, it is supposed here that the substrate concentration S inside the reactor: it is the case in many biotechnological industries, in particular in those where the input characteristics are not well known (biological Wastewater Treatment Plants for instance). Here, we consider the problem of controlling such bioprocesses. On the other hand, the measurement of X is also necessary: as it will be seen, it is rather a technical requirement and it can be argued that this measurement can be difficult to obtain in practice (it is part of the perspectives to develop a version of the proposed controller that will not need the measurement of X). Finally, when dealing with these biosystems, S_{in} is usually considered as an unknown input. While the above results were valid when S_{in} is measured, we show in the remaining part of the paper how to construct an observer for S_{in} under some mild assumptions (Section 2). Then we show how to couple this estimation with the control law (5) to achieve the regulation of the output (Section 3). Simulation results are presented and discussed in Section 4 while conclusions and perspectives are drawn in Section 5.

2. INPUT CONCENTRATION OBSERVER

In most of known approaches, the regulation of the substrate concentration at the output of a bioprocess requires the knowledge of the substrate concentration at the input of the process (in a sense, most of the available nonlinear approaches, as for example the well known adaptive controller by (Bastin and Dochain, 1990), should be considered as feedforward-feedback controllers rather than as feedback controllers). However, a number of practical and economical reasons makes the measurement of the input substrate concentration a difficult task. Thus, for control as well as for monitoring and diagnosis purposes, an accurate estimation of this exogenous input is appreciated. To our best knowledge, only very few approaches have been specifically proposed for estimating unknown input concentrations of biosystems (cf. (Aubrun *et al.*, 2001), (Sperandio and Queinnec, 2004), (Theilliol *et al.*, 2002) and (Theilliol *et al.*, 2003)).

We propose here a new observer, which is of interest by itself, independently of our control objective:

$$\begin{aligned}\dot{\widehat{S}} &= -\frac{\mu(S)}{Y}X + uD(t)(\widehat{S}_{in} - S) \\ &\quad + uD(t)(\theta+\theta^2)(S-\widehat{S}) \\ \dot{\widehat{S}}_{in} &= uD(t)\theta^3(S-\widehat{S})\end{aligned} \tag{6}$$

where $\theta > 1$ is a parameter to be tuned. The only assumption on the unknown function $S_{in}(\cdot)$ we require is to have a bounded first derivative.

Hypothesis H3. $S_{in}(\cdot)$ is differentiable and there exists $M < +\infty$ such that $|\dot{S}_{in}(t)| \leq M$, for any time t.

Proposition 2. Under Hypothesis H3, for any control law $u(\cdot)$ such that $\inf_{t\geq 0} u(t)D(t) = \gamma > 0$ and any non-negative initial conditions of (1)–(6) such that $\widehat{S}(0) = S(0)$ and $\widehat{S}_{in}(0) \in [\underline{S}_{in}, \overline{S}_{in}]$, then the estimation of S_{in} provided by (6) fulfills the following inequality, for any $t \geq 0$

$$\begin{aligned}|\widehat{S}_{in}(t) - S_{in}(t)| \leq\ & \frac{2M}{\theta-1} \\ & + \frac{\theta}{\theta-1}(\overline{S}_{in} - \underline{S}_{in})e^{-\gamma\theta t}.\end{aligned} \tag{7}$$

Remark 3. When the unknown S_{in} is constant ($M = 0$), the convergence (7) of the observer is exact. In face of unknown variations of $S_{in}(\cdot)$, the convergence (7) is practical (by practical, it is meant that, tuning parameters, one can ensure that the error variables enter an arbitrary small neighborhood of the origin).

Proof. Define the error variables $e_S = \widehat{S} - S$ and $e_{S_{in}} = \widehat{S}_{in} - S_{in}$, whose dynamics can straightforwardly be written as follows

$$\frac{d}{dt}\begin{bmatrix} e_S \\ e_{S_{in}} \end{bmatrix} = u(t)D(t)A\begin{bmatrix} e_S \\ e_{S_{in}} \end{bmatrix} - \begin{bmatrix} 0 \\ \dot{S}_{in} \end{bmatrix}$$

with

$$A = \begin{bmatrix} -\theta-\theta^2 & 1 \\ -\theta^3 & 0 \end{bmatrix}$$

One can easily check that the eigenvalues of A are $-\theta$ and $-\theta^2$. Remark also that due the choice of initial conditions of (6), one has

$$e_S(0) = 0, \quad |e_{S_{in}}(0)| \leq \overline{S}_{in} - \underline{S}_{in} \tag{8}$$

Consider the time parameterization

$$\tau := \int_0^t u(s)D(s)ds \geq \gamma t, \quad t \geq 0 \tag{9}$$

and define the function

$$\psi(\tau) = \frac{\frac{dS_{in}}{dt}(\tau)}{u(\tau)D(\tau)} \in \left[-\frac{M}{\gamma}, \frac{M}{\gamma}\right]. \tag{10}$$

This leads to write the dynamics

$$\frac{d}{d\tau}\begin{bmatrix} e_S \\ e_{S_{in}} \end{bmatrix} = A\begin{bmatrix} e_S \\ e_{S_{in}} \end{bmatrix} - \begin{bmatrix} 0 \\ \psi(\tau) \end{bmatrix} \tag{11}$$

Consider the change of variables

$$z_1 = \theta e_S - e_{S_{in}}, \quad z_2 = -\theta^2 e_S + e_{S_{in}}. \tag{12}$$

One can readily check that

$$\frac{dz_1}{d\tau} = -\theta z_1 + \psi(\tau), \quad \frac{dz_2}{d\tau} = -\theta^2 z_2 - \psi(\tau),$$

which implies the following inequalities, using (10)

$$|z_i(\tau)| \leq \frac{M}{\gamma\theta^i} + |z_i(0)|e^{-\theta^i\tau} \quad (i = 1,2) \tag{13}$$

From equations (12) and (13), one obtains easily the inequality

$$|e_{S_{in}}(\tau)| \leq \frac{(\theta+\theta^{-1})\frac{M}{\gamma} + \theta^2(|z_1(0)| + |z_2(0)|)e^{-\theta\tau}}{\theta^2-\theta}$$

Finally, from (8), one has $|z_i(0)| \leq \overline{S}_{in} - \underline{S}_{in}$ ($i = 1,2$), and the announced estimation of the error is guaranteed

$$|\widehat{S}_{in}(t) - S_{in}(t)| \leq \frac{2M}{\theta-1} + \frac{\theta}{\theta-1}(\overline{S}_{in} - \underline{S}_{in})e^{-\gamma\theta t}.$$

3. COUPLING THE OBSERVER WITH THE FEEDBACK CONTROL LAW

Let us first define the saturation function $sat_{[\underline{S}_{in},\overline{S}_{in}]}$ as follows

$$sat_{[\underline{S}_{in},\overline{S}_{in}]}(\sigma) = \max(\underline{S}_{in}, \min(\overline{S}_{in}, \sigma)).$$

Then, coupling the feedback law (5) with the observer (6) leads to the following result.

Proposition 4. Under Hypotheses H0-H1-H2-H3, for any initial condition $X(0) > 0$, $\widehat{S}(0) = S(0) \in [0, \overline{S}_{in}[$ and $\widehat{S}_{in}(0) \in [\underline{S}_{in}, \overline{S}_{in}]$, the dynamic output feedback law

$$\widetilde{u}^*(t, S, \widehat{S}_{in}) := u^*(t, S, sat_{[\underline{S}_{in},\overline{S}_{in}]}(\widehat{S}_{in})) \tag{14}$$

where $u^*(\cdot)$ is defined in (5) and $\widehat{S}_{in}$ is given by (6) possesses the following property

$$\limsup_{t\geq 0} |S_{out}(t) - S^*_{out}(t)| \leq \Omega$$

with

$$\Omega = \frac{2M}{\theta - 1}\left(1 - \frac{\underline{S}_{in} - \overline{S}^*_{out}}{\overline{S}_{in}}\right).$$

Remark 5. Notice that $\widetilde{u}^*(\cdot)$ is well defined because of the saturation and Hypothesis H0.

Proof. From Hypothesis H1, it is immediate to check that the domain $\mathcal{D} = \Re^*_+ \times [0, \overline{S}_{in}[$ is invariant under the dynamics (1), for any non-negative control law $u(\cdot)$. Fix an initial condition $(X(0), S(0)) \in \mathcal{D}$ and denote $(X(\cdot), S(\cdot))$ the solution of system (1) with the dynamic output feedback (14). Denote also

$$\widetilde{u}(t) = \widetilde{u}^*(t, S(t), \widehat{S}_{in}(t))$$

Observe that from assumptions H0 and H2 one has the inequality

$$\widetilde{u}(t) \geq \underline{u} = (1 - \overline{S}^*_{out}/\underline{S}_{in}) > 0.$$

Consequently, there exists $T' > 0$ such that $S(t) \leq S^*_{out}(t)$ for any $t \geq T'$ (see Lemma 1 in Appendix). Posit $\widetilde{S}_{in}(t) = sat_{[\underline{S}_{in}, \overline{S}_{in}]}(\widehat{S}_{in}(t))$ and $\widetilde{e}_{S_{in}}(t) = \widetilde{S}_{in}(t) - S_{in}(t)$ and it follows, for any $t \geq T'$,

$$\begin{aligned} S_{out}(t) &= \frac{\widetilde{S}_{in}(t) - S^*_{out}(t)}{\widetilde{S}_{in}(t) - S(t)}(S(t) - S_{in}(t)) \\ &\quad + S_{in}(t) \\ &= S^*_{out}(t) \\ &\quad - \widetilde{e}_{S_{in}}(t)\underbrace{\left(1 - \frac{\widetilde{S}_{in}(t) - S^*_{out}(t)}{\widetilde{S}_{in}(t) - S(t)}\right)}_{\Gamma(t)} \end{aligned}$$

where $0 \leq \Gamma(t) \leq \overline{\Gamma} = 1 - (\underline{S}_{in} - \overline{S}^*_{out})/\overline{S}_{in}$. Then

$$|S_{out}(t) - S^*_{out}(t)| \leq |\widehat{S}_{in}(t) - S_{in}(t)|\overline{\Gamma}, \quad (15)$$

for all $t \geq T'$. Finally, notice that $u(t)D(t) \geq \gamma = \underline{u}\,\underline{D} > 0$, for $t \geq T$, and one conclude from Proposition 2:

$$|S_{out}(t) - S^*_{out}(t)| \rightarrow \left[0, \frac{2M}{\theta - 1}\overline{\Gamma}\right] \text{ as } t \rightarrow +\infty.$$

4. SIMULATIONS

Numerical simulations were performed using the control law presented here above, with a Monod

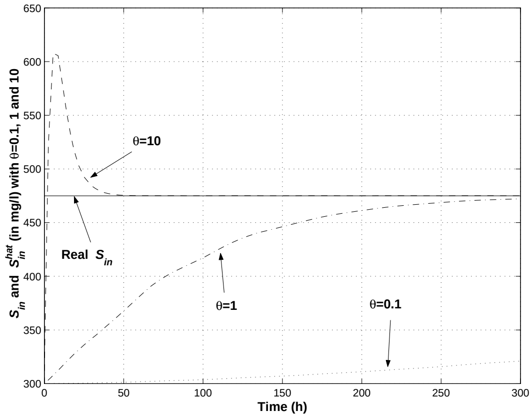

Fig. 2. S_{in} is an unknown constant: S_{in} and estimations with different values of θ.

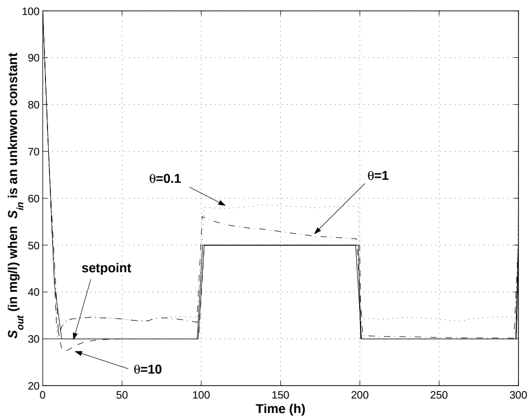

Fig. 3. S_{in} is an unknown constant: S_{out} with different estimations of S_{in}.

growth function: $\mu(S) = \mu_{\max}S/(K_S + S)$. Variables S and X were measured online. The following model parameters were used: $\mu_{max} = 0.045$, $K_S = 10$, $Y = 0.05$, $V = 40$, D is the sum of a constant ($\bar{D} = 0.02\ 1/h$) and of three other signals:

i) a sinusoid of magnitude 0.0025 and of frequency 0.02,
ii) a sinusoid signal of magnitude 0.001 and of frequency 0.0002,
iii) a square signal of magnitude 0.0015 and of frequency 0.015.

Thus, at $t = 100$, a set point step was simulated. The input substrate concentration is measured and is built as follows. It consists of the sum of:

i) a constant equal to 475 mg/l
ii) a sinusoid of magnitude 25 and of frequency 0.01
iii) a sinusoid of magnitude 15 and of frequency 0.005

The objective is to regulate the output substrate concentration S_{out} between $S^\star_{out} = 30\ mg/l$ and $50\ mg/l$. First, it was verified that condition (4) holds given the extreme expected values of D, S_{in} and $S^\star_{out}$. The simulations were performed over

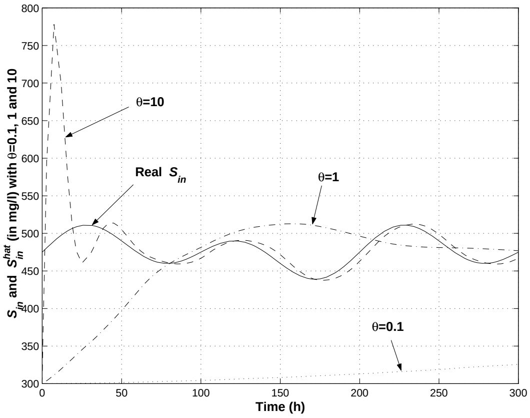

Fig. 4. S_{in} is time-varying: S_{in} and estimations with different values of θ.

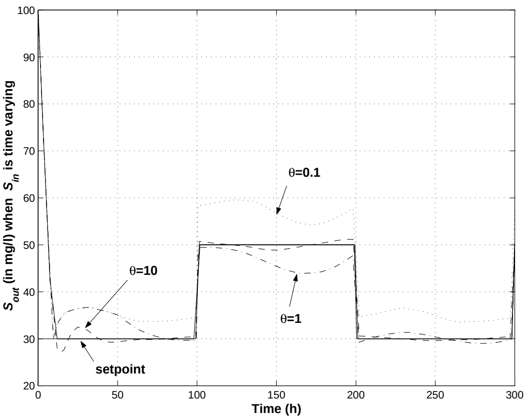

Fig. 5. S_{in} is time-varying: S_{out} with different estimations of S_{in}.

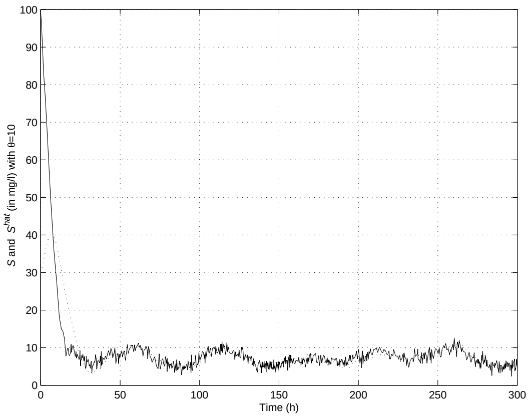

Fig. 6. S and $\hat{S}$ in presence of a measurement noise (S_{in} is time-varying).

a period of 300 hours. The results are shown in Figures 2 to 5.

Obviously, simulation results are in accordance with theoretical developments. It should be noticed that the regulation exhibits good performance whatever the case investigated: S_{in} constant (in this case, the convergence of the observer is exact) and S_{in} variable (in which case the convergence is practical).

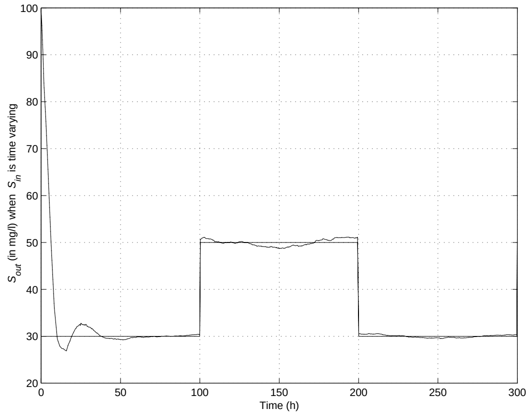

Fig. 7. S_{out} in presence of measurement noise on S (S_{in} is time-varying).

In presence of a 10 % measurement noise on S (cf. Figure 6), it can be seen in Figure 7 that the performance of the controller remains correct, even when S_{in} is time-varying.

5. CONCLUSIONS AND PERSPECTIVES

This paper proposed a new observer of an unknown input substrate concentration. It was shown that it exhibits an exact convergence property if S_{in} is constant while it is practical when S_{in} is time varying a bounded first derivative. The coupling of this estimator to a control law first proposed in (Harmand *et al.*, 2005), allows us to track any bounded reference trajectory. However, it should be stressed that, while in its original form only S and S_{in} were necessary, in the present paper, the measurement of S and X are required. Thus, extension of the present approach to cases where X is unmeasured is under investigation.

6. APPENDIX

Lemma 6. Under Hypotheses H0-H1-H2, for any initial condition $(X(0), S(0)) \in \Re_+^* \times [0, \overline{S}_{in}[$ and any control law such that $\underline{u} = \inf_{t \geq 0} u(t) > 0$, S stays below $\underline{S}^*_{out}$ after a finite time.

Proof. From Hypothesis H1, we immediately deduce that the domain $\Re_+^* \times [0, \overline{S}_{in}[$ is invariant by the dynamics (1), for any non-negative control law $u(\cdot)$. Make the change of variable $(S, Z) = (S, X + YS)$:

$$\begin{aligned} \dot{S} = F(t, S, Z) &= u(t)D(t)(S_{in}(t) - S) \\ &\quad - \frac{\mu(S)}{Y}(Z - YS) \\ \dot{Z} &= -u(t)D(t)(Z - Z_{in}(t)) \end{aligned}$$

where $Z_{in} = X_{in} + YS_{in}$. Remark that, for any $t \geq 0$, $Z_{in}(t) \in [\underline{Z}_{in}, \overline{Z}_{in}]$, where $\underline{Z}_{in} =$

$\underline{X}_{in} + Y\underline{S}_{in}$ and $\overline{Z}_{in} = \overline{X}_{in} + Y\overline{S}_{in}$. Thus, from $u(t)D(t) \geq \underline{u}\,\underline{D} > 0$, we infer that $Z(t)$ converges exponentially towards $[\underline{Z}_{in}, \overline{Z}_{in}]$.

Remark that $S \in [\underline{S}^*_{out}, \overline{S}_{in}[$ implies the following inequality, for any $t \leq T$

$$F(t,S,Z) \leq (\overline{D} - \mu(\underline{S}^\star_{out}))(\overline{S}_{in} - S) \\ -\frac{\mu(S)}{Y}(\underline{Z}_{in} - Y\overline{S}_{in}) + \frac{\mu(S)}{Y}(\underline{Z}_{in} - Z) \quad (16)$$

At $S = \underline{S}^*_{out}$, one has

$$F(t, \underline{S}^*_{out}, Z) \leq -\delta + \frac{\mu(\underline{S}^*_{out})}{Y}(\underline{Z}_{in} - Z)$$

where $\delta = \mu(\underline{S}^\star_{out})(\underline{Z}_{in} - Y\underline{S}^\star_{out})/Y - \overline{D}(\overline{S}_{in} - \underline{S}^\star_{out})$ (notice that condition (4) ensures $\delta > 0$). But the convergence of Z towards the interval $[\underline{Z}_{in}, \overline{Z}_{in}]$ implies the existence of $T_1 \geq T$ such that

$$F(t, \underline{S}^*_{out}, Z) \leq -\frac{\delta}{2} < 0, \quad t \geq T_1$$

Then, the existence of a finite time $T_2 \geq T_1$ such that $S(T_2) \leq \underline{S}^*_{out}$ implies that the variable $S(t)$ stays below $\underline{S}^*_{out}$ for any future time $t \geq T_2$. We show now that such a time T_2 necessarily exists.

If such a time T_2 does not exist, then $S(t) > \underline{S}^*_{out}$ for any $t \geq T_1$. We distinguish three cases:

Case 1. $\underline{Z}_{in} > Y\overline{S}_{in}$ and $\overline{D} \leq \mu(\underline{S}^\star_{out})$ (note that condition (4) is necessarily fulfilled), then from (16) and the convergence of $Z(\cdot)$ towards $[\underline{Z}_{in}, \overline{Z}_{in}]$, we deduce the existence of $T_3 \geq T_1$ such that

$$F(t, S, Z(t)) \leq -\frac{1}{2}\frac{\mu(\underline{S}^\star_{out})}{Y}(\underline{Z}_{in} - Y\overline{S}_{in}) < 0$$

for all $t \geq T_3$. We conclude that $S(\cdot)$ reaches $\underline{S}^\star_{out}$ in finite time, thus a contradiction.

Case 2. If $\underline{Z}_{in} > Y\overline{S}_{in}$ and $\overline{D} > \mu(\underline{S}^\star_{out})$, we obtain the following inequality from (16) and (4)

$$F(t, S, Z) \leq -\delta + \frac{\mu(S)}{Y}(\underline{Z}_{in} - Z) \ .$$

The asymptotic properties of $Z(\cdot)$ allow then to write

$$F(t, S, Z(t)) \leq -\frac{\delta}{2} < 0, \quad t \geq T'_3$$

for a certain $T'_3 \geq T_1$. Thus we obtain again a contradiction.

Case 3. If $\underline{Z}_{in} \leq Y\overline{S}_{in}$, then condition (4) ensures $\overline{D} < \mu(\underline{S}^\star_{out})$. One can then write, from (1) and (3), the following inequalities for all $t \geq T_1$

$$\dot{X} \geq (\mu(\underline{S}^\star_{out}) - \overline{D})X \ , \\ \dot{S} \leq -(\mu(\underline{S}^\star_{out}) - \overline{D})\frac{X}{Y} + \frac{\overline{D}}{Y}(Y\overline{S}_{in} - \underline{Z}_{in}) \\ +\frac{\overline{D}}{Y}(\underline{Z}_{in} - Z(t)).$$

Since $X(T_1)$ is positive, $X(\cdot)$ is increasing, and there exist $\gamma > 0$, $T'_1 \geq T_1$ such that $(\mu(\underline{S}^\star_{out}) - \overline{D})X(t) > \overline{D}(Y\overline{S}_{in} - \underline{Z}_{in}) + \gamma$ for all $t \geq T'_1$. From the convergence of $Z(\cdot)$, we deduce that there exists $T''_3 \geq T'_1$ such that

$$\dot{S}(t) \leq -\frac{\gamma}{2Y} < 0, \quad t \geq T''_3$$

that leads again to a contradiction.

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