ROBUST MPC OF THE REFINING STAGE OF AN ELECTRIC ARC FURNACE

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Abstract: In this article a robust MPC controller is presented for the refining stage of an electric arc furnace. A reduced version of a generic EAF model will be used - it simplifies the controller. The controller's objective is to steer the temperature to the desired value before the carbon content reaches its target value. The controller design is verified through a simulation study. The controller behaves well even under extreme model mismatch when full state feedback is used, but is high dependent on the accuracy of the predictor under limited feedback conditions. With timely measurement, the error can be contained and even reduced if the predictor's parameters are adjusted when measurements are taken.

Keywords: Robust model predictive control, RMPC, electric arc furnace, EAF, steel making

1. INTRODUCTION

At the heart of recycling scrap metal into steel is the electric arc furnace (EAF). The electric arc furnace primarily uses an electric arc to generate the heat required to melt the scrap metal and refine it to steel. The process is still dominantly operator controlled, often resulting in suboptimal and inconsistent steel quality. The use of electric arc furnaces is growing world wide, and the process can benefit from increased automation in improving overall steel quality as well as improving the economics of the process. Automation could also improve the safety of this dangerous process, as well as reduce its negative effects on the environment (Bekker *et al.*, 2000).

In order to design and implement control, a suitable mathematical model is needed. There are different approaches to model the electric arc furnace process. The one approach is to model the process as containing equilibrium zones with mass transport transport between the zones (Cameron *et al.*, 1998; Matson and Ramirez, 1999; Modigell *et al.*, 2001). Some models tend to be proprietary (Morales *et al.*, 1997). An alternate approach was taken by Bekker *et al.* (1999) who derived a model from fundamental thermodynamic and kinetic relationships that resulted in a generic nonlinear model consisting of 17 ordinary differential equations (ODE). This generic model was then fitted to actual plant data by Rathaba (2004).

The following stage was to investigate the feasibility of control on this model. First efforts focused on the off-gas subsystem (Bekker *et al.*, 2000) by controlling the relative furnace pressure, CO emissions as well as the off-gas temperature. Oosthuizen *et al.* (2004) extended the scope by incorporating economic objectives into the control strategy.

The EAF process consists of three phases, the first two are meltdown phases and the last phase is called the refining phase, where the final grade of the steel is determined. During the refining stage, certain simplifying assumptions can be made with regards to the model in order to reduce it to only 5 ODEs (Rathaba, 2004). The reduced model is better suited for control, because it reduces the computational burden. Coetzee *et al.* (2005) applied nominal MPC to the reduced model, but the modelling effort by Rathaba (2004) revealed model uncertainties that could not be explicitly incorporated into the controller design.

The application of robust MPC to this problem was therefore investigated, as discussed in this paper. Two robust MPC methods were investigated for this paper. Firstly, a linear matrix inequalities (LMI) based robust MPC technique has been developed by Kothare *et al.* (1996). The technique was improved by Cuzzola *et al.* (2002) and Ding *et al.* (2004). This technique is limited to symmetric constraints on the inputs and use ellipsoid invariant sets, that makes it conservative. The on-line optimization is based on semi-definite programming (SDP) and is very slow compared to quadratic programming (QP). Secondly, a technique by Pluymers *et al.* (2005*b*,*a*) constructs a robust invariant set, that can accommodate asymmetric constraints and has a much bigger feasible are than their ellipsoid counterparts. This method uses a simple MPC algorithm similar to nominal MPC that is based on QP rather than SDP.

2. REDUCED EAF MODEL

The MPC controller uses an optimization algorithm to calculate the future control sequence. This requires a number of iterations to be done on the internal model. If the model is complex, as in the case of the generic EAF model (Bekker *et al.*, 1999), this will result in a large computation time. The reduced model reduces the computational burden in the refining stage where certain assumptions can be made to simplify the model.

The generic model (Bekker *et al.*, 1999) was reduced by Rathaba (2004) for the refining stage. During the first sixty to eighty percent of a tap, the process is often unpredictable due to delays, breakdowns and maintenance that invalidate the modelling assumption of process continuity. The advantage of using the refining stage for control is that after the initial measurement, except for deslagging, the process is mostly uninterrupted until the final measurement is made. This is typically a flat bath stage when all melting has occurred; the modeling assumption of homogeneity (Bekker *et al.*, 1999) is also valid. The bath temperature and carbon content become especially important during the refining stage just before tapping.

Process variables that undergo significant change during refining are bath temperature, carbon and silicon concentrations (masses), masses of SiO_2 and FeO in slag and all free-board gases. All masses of the bath and composite slag are at steady state - they can be treated as constants.

The reduced model (Rathaba, 2004) is given by

$$\dot{x_3} = -k_{dC}\left(X_C - X_C^{eq}\right) \tag{1}$$

$$\dot{x_4} = -k_{dSi}\left(X_{Si} - X_{Si}^{eq}\right) \tag{2}$$

$$\dot{x_7} = \frac{2M_{FeO}d_1}{M_{O_2}} - \frac{x_7 k_{gr} M_{Fe} d_5}{\left(m_{T(slag)} + x_7 + x_8\right)M_C} + 0.13d_2 \tag{3}$$

$$\dot{x_8} = \frac{M_{SiO_2}}{M_{Si}} k_{dSi}\left(X_{Si} - X_{Si}^{eq}\right) + 0.045d_2 \tag{4}$$

$$\dot{x_{12}} = (p_t + \eta_{ARC} d_4 - k_{VT}(x_{12} - T_{air})) / \left[\frac{m_{T(Fe)} C_{p(FeL)}}{M_{Fe}} + \frac{2m_{T(slag)} + 2x_7 + 3x_8}{M_{slag}} C_{p(slag_{(L)})}\right] \tag{5}$$

where the molar concentrations are given by

$$X_c = \frac{x_3/M_C}{m_{T(Fe)}/M_{Fe} + x_3/M_C + x_4/M_{Si}} \tag{6}$$

$$X_{FeO} = \frac{x_7/M_{FeO}}{m_{T(slag)}/M_{slag} + x_7/M_{FeO} + x_8/M_{SiO_2}} \tag{7}$$

$$X_C^{eq} = k_{XC}\left(\frac{m_{T(slag)} M_{FeO}}{x_7 M_{slag}} + \frac{x_8 M_{FeO}}{x_7 M_{SiO_2}} + 1\right) \tag{8}$$

$$X_{Si} = \frac{x_4/M_{Si}}{m_{T(Fe)}/M_{Fe} + x_3/M_{Si} + x_4/M_{Si}} \tag{9}$$

$$X_{Si}^{eq} = k_{XSi}\left(\frac{m_{T(slag)} M_{FeO}}{x_7 M_{slag}} + \frac{x_8 M_{FeO}}{x_7 M_{SiO_2}} + 1\right)^2 \tag{10}$$

The reduced equations for the heat balance are:

$$p_2 = \left(-2H_{FeO} d_1 / M_{O_2}\right)\eta_{FeO} \tag{11}$$

$$p_5 = \frac{d_1}{M_{O_2}}\left(x_{12} - T_{O_2}\right)C_{P(O_2)} \tag{12}$$

$$p_{11} = \frac{x_7 k_{gr} d_5 (\Delta H_{FeO} - \Delta H_{CO})}{\left(m_{T(slag)} + x_7 + x_8\right)M_C} \tag{13}$$

$$p_t = p_2 + p_5 + p_{11} \tag{14}$$

where k_{dC} and k_{dSi} are the constants for removal of carbon and silicon from the bath; k_{gr} is the graphite reactivity constant; η_{ARC} and η_{FeO} are the efficiencies of arc energy input and bath oxidation; $m_{T(Fe)}$ and $m_{T(slag)}$ are the total masses of the slag formers and bath - both are assumed constant; M_C, M_{Fe}, M_{FeO}, M_{Si}, M_{SiO_2} and M_{slag} are the molar masses of the different elements. The states and inputs are described in table 1.

3. LINEARIZED MODEL

For the robust MPC controller (Pluymers *et al.*, 2005*b*,*a*), linear models are required to describe the uncertain polytope. The reduced model of section 2 was fitted to actual plant data (Rathaba, 2004), with resulting confidence intervals on the parameter values. The parameters were varied to determine the extreme deviations from the nominal plant. The reduced model was linearized with different parameter values, resulting in different linear models that form the polytopic uncertainty description used to synthesise the robust MPC controller. In order to reduce complexity of the controller, only the most significant uncertainties were included in the uncertainty description. The nominal model is shown below with the maximum variation of the input matrix. The states of the linear models were reduced to 2 (Temperature and FeO), because these are the states that are of primary interest. The inputs were reduced to three (oxygen injection rate, electric power and graphite injection rate). Slag and DRI are not added during the refining stage and therefor removed from the model. To compensate for the reduced dynamics in the linearised model, the controller uses the full nonlinear model of section 2 as a predictor. The operating point around which the linearization is done is the average values from measured tap data as summarized in table 1. The total iron mass $m_{T(Fe)}$ is 80000 kg and the slag mass $m_{T(slag)}$ is 6917.8 kg.

The linearized model is described as a nominal model, or a model with values in the middle of the uncertainty interval, and matrices that describes the extreme deviations, found using Monte Carlo type simulations, from the nominal model. The notation used is as in MATLAB, where 3.03e-6 represents 3.03×10^{-6}. The nominal linear model is

$$\begin{aligned} A_{nominal} &= \begin{bmatrix} 9.99e{-1} & 0 \\ -7.87e{-6} & 1.00 \end{bmatrix} \\ B_{nominal} &= \begin{bmatrix} 11.99 & 0 & -1.17 \\ 0.41 & 6.07e{-6} & -1.67e{-2} \end{bmatrix} \\ C_{nominal} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D_{nominal} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{15}$$

with extreme deviations from nominal on the inputs

$$B_{dev} = \begin{bmatrix} 0 & 0 & 0 \\ 0.11 & 3.03e{-6} & 1.42e{-2} \end{bmatrix} \tag{16}$$

The polytopic uncertainty produces a total of eight linear models used to construct the robust invariant set for the controller as described in section 4.1.

4. CONTROL STRATEGY

In general, the steel grade is determined by the carbon content. It is desirable to have the steel at a specific tap temperature for down-stream processing. The carbon content and temperature should be controlled without creating too much undesirable material such as FeO in the slag. The controller should contain the FeO content, controllable through the oxygen injection rate, to less than 40% of the total slag mass. Oxygen injection influences the decarburization rate depending on the level of the bath carbon. Under high bath carbon levels, oxygen injection leads to high decarburization, while only a limited influence is observed in the late stages of refining. The speed of the reaction cannot be accelerated through control, because of the carbon content being weakly controllable. (The carbon content is controllable, but the constraints on the inputs limit the influence over the decarburization rate to practically none.) The aim of the controller would therefore be to steer the temperature to the desired value before the carbon reaches its target.

4.1 Robust model predictive controller

The controller proposed by Pluymers *et al.* (2005*b*) uses an optimized control sequence over the prediction horizon N after which a global stabilizing state feedback gain K is used. The system $\tilde{x}(k+1) = \Phi(k)\tilde{x}(k)$ uses an augmented description with vertices of the uncertainty polytope given by:

$$\Phi_i = \begin{bmatrix} A_i - B_i K & B_i & 0 \\ 0 & 0 & I_{((N-1).n_u,(N-1).n_u)} \\ 0 & 0 & 0 \end{bmatrix} \tag{17}$$

for $i = 1, 2, \dots, L$, where L is the number of models, subject to constraints $A_{\tilde{x}}\tilde{x}(k) \leq b_{\tilde{x}}, k = 0, ..., \infty$ with $A_{\tilde{x}}$ and $b_{\tilde{x}}$ defined as:

$$A_{\tilde{x}} = \begin{bmatrix} A_x & 0 & 0 \\ -A_u K & A_u & 0 \end{bmatrix} \; b_{\tilde{x}} = \begin{bmatrix} b_x \\ b_u \end{bmatrix} \tag{18}$$

where the state constraints are $A_x x \leq b_x$ and the input constraints are $A_u u \leq b_u$.

To calculate the robust invariant set from $A_{\tilde{x}}$ and $b_{\tilde{x}}$ the algorithm from Pluymers *et al.* (2005*a*) is used to construct $S_{aug} = \{\tilde{x} | A_S \tilde{x} \leq b_S\}$.

The implemented input vector is the combination of the state feedback gain and the first block of the optimized sequence of free control moves.

$$u(k) = -Kx(k) + c^o(k|k) \tag{19}$$

The optimized sequence of free control moves $c^o_N(k)$ are determined from a quadratic program

Table 1. States, inputs and operating point.

State	Operating point	Input	Operating point
x_3 Dissolved Carbon	480 kg	d_1 Oxygen injection rate	1 kg/s
x_4 Dissolved Silicon	24 kg	d_2 DRI addition rate	0 kg/s
x_7 FeO in bath	4250.6 kg	d_3 Slag addition rate	0 kg/s
x_8 SiO_2 in bath	1405 kg	d_4 Arc power	40000 kW
x_{12} Bath temperature	1600 ^0C	d_5 Graphite injection rate	0.5 kg/s

subject to the polyhedral set constraints of the form $A_S\tilde{x} \leq b_S$ that form S_{aug}.

$$\min_{c_N(k)} J(x(k),c_N(k)) \tag{20}$$

subject to

$$\left[\, x(k)^T \;\; c_N(k)^T \,\right]^T \in S_{aug} \tag{21}$$

where the objective function is

$$\begin{aligned} J(x(k),c_N(k)) = {} & \left[\, x(k)^T \;\; c_N(k)^T \,\right] P \\ & \times \left[\, x(k)^T \;\; c_N(k)^T \,\right]^T \end{aligned} \tag{22}$$

with

$$P = P^T \in \Re^{(n_x+N.n_u)\times(n_x+N.n_u)} \tag{23}$$

satisfying

$$P - \Phi_i^T P \Phi_i > \Gamma_x^T Q \Gamma_x + \Gamma_u^T R \Gamma_u,\; i=1,\dots,L \tag{24}$$

where $\Gamma_x = \left[\, I_{(n_x,n_x)} \;\; 0 \,\right]$, $\Gamma_u = \left[\, -K \;\; I_{(n_u,n_u)} \;\; 0 \,\right]$ and $\Phi_i,\; i = 1,...,L$. The R matrix is the weighting on the inputs and Q the weighting on the states. The P matrix can be obtained by doing convex optimization

$$\min_{P=P^T>0} \text{trace}(P) \tag{25}$$

subject to (24).

4.2 Controller design

There are a few parameters that have to be chosen in order to construct the controller. These are the prediction horizon, weights on the state deviation, and the weights on the control actions. The constraints on the input and states are determined by the process and actuator limitations as summarized in table 2. The dimension of the constraint set grows exponentially with the prediction horizon. A prediction horizon of 4 gives adequate forward prediction, without resulting in an excessively large constraint set. The aim is to control the temperature of the bath without any penalty on the FeO content, except when it reaches the constraint. The weighting is expressed in the Q value of table 3. The FeO content in the slag should be kept below 40% of the total slag mass. This requirement is enforced by a state constraint on FeO as stipulated in table 2. In order for the process to reach the desired set-point as fast as possible and reduce the steady-state offset, cheap control is used, as expressed in the R value of table 3. The structure of the Q and R matrices are diagonal, because only the individual states and inputs are penalized.

Table 2. Constraints on manipulated and controlled variables.

Variable	Minimum	Maximum
Oxygen injection rate	0 kg/s	1 kg/s
Power	0 kW	40 000 kW
Graphite injection rate	0 kg/s	1 kg/s
FeO amount	0% of Slag	40% of Slag

Table 3. Controller parameters

Parameter	Value
N	4
Q	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
R	$\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$

5. SIMULATION AND RESULTS

In this section, a simulation study is done to determine the effectiveness of the control on the system. For these simulations a predictor, the reduced nonlinear model of section 2, is used to interpolate the missing state-data and a second nonlinear model is used to represent the actual plant. Model mismatches are introduced between the predictor and real plant, by manipulating the parameters of the "real" plant. The model mismatch has the greatest influence on the temperature, because of the uncertainty in the effectivity coefficients (ECs), η_{ARC} of equation (5) and η_{FeO} of equation (11). The ECs will be used to describe the parameter set being used.

In the first scenario, full state feedback is assumed, and the parameters of the "real" plant are set to the two extreme points of the uncertainty intervals. The controller is able to handle the extreme cases of model mismatches as seen in figure 1. In the scenario where the ECs are lower than nominal, the rate of change in temperature decreases because the FeO constraint was met; oxygen could no longer be used as an energy source.

In the second scenario, a more practical approach is taken. In the real process, it is difficult to get measurements. Each time a temperature measurement is taken, the slag layer must be removed from the metal through a process called deslagging. For deslagging, the electrical power is reduced and a probe is dropped into the molten metal by an operator. The probe is burnt away as part of the measurement process, making measurements costly. Only one measurement is available for feedback from the real plant during the refining stage. A predictor, the reduced nonlinear model of section 2, is used to interpolate the missing process state-data between measurements. For this scenario, nominal plant parameters are assumed for the predictor, while the real plant uses the extreme points as before. In this case (figure 2), the controller was not able to steer the temperature to the desired set-point and could not even steer it to within 10 degrees of the set-point, which is an acceptable temperature variation. In practice this variation is often much larger.

In the third scenario, the predictor parameters, that have the greatest influence on the temperature response, are manipulated with the difference in predicted and measured temperature, to better approximate the real plant. With the corrections to the predictor, the controller is able to steer the process closer to the desired set-point (figure 2). For this simulation, the model mismatch was assumed to be constant, but unknown. The predictor was initiated with nominal plant parameters.

From this results we see that robust MPC does not guarantee offset free tracking. Integral action or robust MPC techniques that specifically address this problem (Pannocchia, 2004; Wang and Rawlings, 2004*a*,*b*) will remedy the problem and will be considered for future work. The third scenario where corrections are made to the predictor, show optimistic results. The assumption of unknown but constant model mismatch might not be valid. All these scenarios investigate the worst case where the model mismatch between the predictor and actual plant is at the extreme. The performance of the controller should yield better results if the actual plant behaviour is closer to the nominal case.

6. CONCLUSION

This paper, through a simulation study, found robust model predictive control feasible to improve the quality of the steel produced in an EAF. This is provided that the predictor is a suitably accurate predictor of the real process. The robust model predictive controller could explicitly take the model uncertainties into account as part of the synthesis process. The focus was on the refining stage, because it allowed for the use of the reduced model of the process, which is better suited for on-line calculations. The speed of the process could not be increased, because the carbon is only weakly controllable. The only improvement possible is to control the temperature to the desired set-point before the carbon reaches its target value. This will result in fewer delays attributed to incorrect tap temperature. The controller performed well with full state feedback, but in practice, the performance is dependant on the accuracy of the predictor.

An ad-hoc method was used to update the parameters of the predictor in order to determine if it will yield better results than a predictor without parameter update. The updating of the parameters of the predictor show promise, but further study is needed where a more systematic method is followed.

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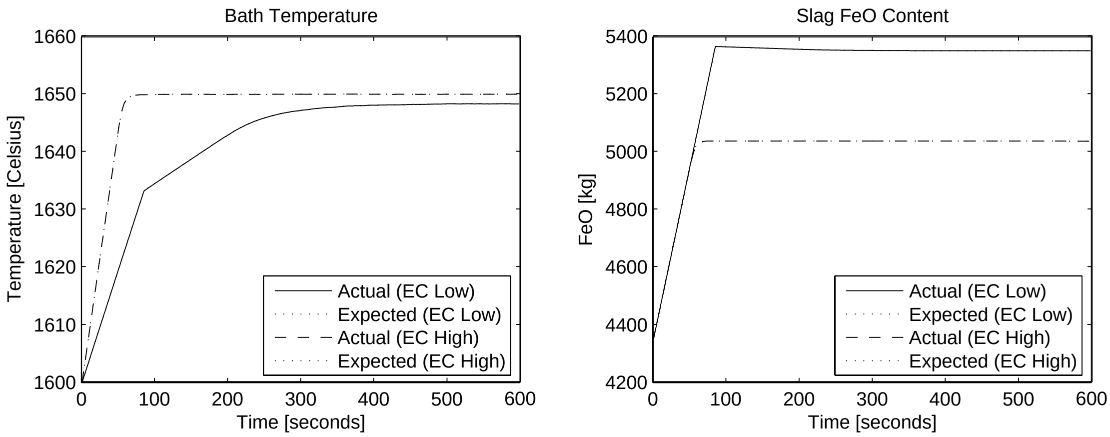
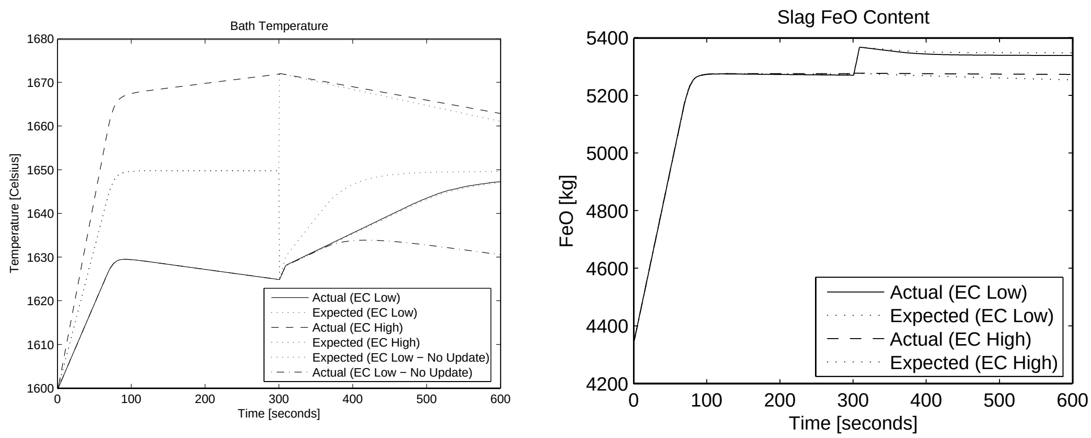


(a) Temperature (b) FeO Content

Fig. 1. Process with full state feedback



(a) Temperature (b) FeO Content

Fig. 2. Process with one measurement at time $t = 300s$

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