FINITE TIME OBSERVER FOR NONLINEAR SYSTEMS

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Abstract: This paper proposes a nonlinear finite time convergent observer that does not require to compute any inverse coordinate transformation. The finite time estimate is recovered from two asymptotically convergent estimates which have linear error dynamics in transformed coordinates through the transformation Jacobian only. An extended version of this nonlinear finite time observer is next envisaged. The finite time estimate is obtained from two pseudo linear dynamic systems and require to compute only the Jacobian of two functions which are the solutions of two systems of partial derivative equations.

Keywords: State observers, time delay, nonlinear observers, finite time convergence

1. INTRODUCTION

Monitoring of the component concentrations is a key question for productivity and safety in the chemical industry. However it often requires very specific and expensive sensors that cannot be used in practice. Therefore the real-time estimation of component concentrations using a state observer is a very attractive option.

Since the first observers for linear systems have been developed by Kalman and Luenberger several decades ago, several different techniques have been proposed to deal with nonlinearities and model uncertainties. However these techniques give an estimate that reaches the real state asymptotically what may be a limitation for batch and fed-batch processes.

An observer that converges in finite time has been recently proposed (Engel and Kreisselmeier, 2002). The key idea is to use the present and delayed estimates provided by two independent classical observers to compute an estimate that converges exactly to the state after a predefinite time delay. The estimate formulation arises from solving a set of four equations linking the state and each of the two classical estimates at both present time and delayed time. The finite time observer performance relies on the linearity of the estimation error dynamics and its use is therefore restricted to linear time-invariant systems.

The field of application of this technique has been extended to linear time-varying systems (Menold *et al.*, 2003*b*). The use of the transition matrix of the system is introduced to compare the delayed and present estimates. The same authors have also extended the technique to nonlinear systems that can be transformed into the observer canonical form (Menold *et al.*, 2003*a*). Once the nonlinear system is transformed into its normal form, two observers with linear error dynamics can be developed and the finite time estimation can be carried out in these coordinates. The estimate in the original coordinates is then retrieved by the inverse transformation.

In this paper, we propose a finite time observer for nonlinear systems that does not require to

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compute the inverse coordinates transformation but only its Jacobian. The estimate is computed in transformed coordinates associated to a pseudo linear form of the system (Menold *et al.*, 2003*a*) and its expression comes from the set of equations used to estimate linear time invariant systems (Engel and Kreisselmeier, 2002). The estimate in original coordinates is obtained by differentiating the previous expression introducing the Jacobian of the coordinates transformation. A more general approach using two different changes of variables obtained from two systems of partial derivative equations is then presented. Also in this case, the estimation requires to compute the inverse Jacobian of each of the transformations only.

The paper is structured as follows. In Section 2 we present a finite time observer for nonlinear systems that can be transformed into pseudo linear system with nonlinearities depending on the input and output only. This section ends by an example of a numerical simulation. In Section 3 we propose a finite time observer which require to compute the Jacobian of two functions which are the solutions of two systems of partial derivative equations. This section is ended by an example of a numerical simulation.

2. FINITE TIME OBSERVER

Consider the following observable nonlinear system :

$$\begin{aligned} \dot{x} &= f(x,u),\ x(t_0) = x_0,\ t \geq t_0 \\ y &= h(x) \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}$. Assume that there exists a change of coordinates

$$z = \Psi(x) \tag{2}$$

allowing to transform the system (1) into the following observable pseudo linear system (Hou and Pugh, 1999),(Krener and Respondek, 1985) :

$$\begin{aligned} \dot{z} &= Az + \beta(y,u),\ z(t_0) = z_0,\ t \geq t_0 \\ y &= Cz \end{aligned} \tag{3}$$

where β is a known nonlinear function that only depends on the input and output. The observability involves that two gain matrices H_1 and H_2 can be computed so that both following matrices have desired eigenvalues with negative real parts :

$$F_1 = A - H_1 C \tag{4}$$
$$F_2 = A - H_2 C \tag{5}$$

implying that both following systems are observers for system (3):

$$\dot{\hat{z}}_1 = A\hat{z}_1 + \beta(y,u) + H_1(y - C\hat{z}_1) \tag{6}$$
$$\dot{\hat{z}}_2 = A\hat{z}_2 + \beta(y,u) + H_2(y - C\hat{z}_2) \tag{7}$$

Each of the reconstruction errors associated with the above observers is governed by a linear dynamics as follows:

$$\dot{\epsilon}_1 = F_1 \epsilon_1 \tag{8}$$
$$\dot{\epsilon}_2 = F_2 \epsilon_2 \tag{9}$$

This involves that for any time $t \geq D$, the following relations exist between the errors at different time instances:

$$\epsilon_1(t) = e^{F_1 D}\epsilon_1(t-D) \tag{10}$$
$$\epsilon_2(t) = e^{F_2 D}\epsilon_2(t-D) \tag{11}$$

This leads to the following set of four equations :

$$\hat{z}_1(t) = z(t) + \epsilon_1(t) \tag{12}$$
$$\hat{z}_2(t) = z(t) + \epsilon_2(t) \tag{13}$$
$$\hat{z}_1(t-D) = z(t-D) + e^{-F_1 D}\epsilon_1(t) \tag{14}$$
$$\hat{z}_2(t-D) = z(t-D) + e^{-F_2 D}\epsilon_2(t) \tag{15}$$

which, when it is solved for $z(t)$, gives rise to the following observer that converges within the predefinite time D (Engel and Kreisselmeier, 2002), (Menold *et al.*, 2003*a*):

$$\begin{aligned} \hat{z}(t) = {} & \left(e^{-F_1 D} - e^{-F_2 D}\right)^{-1}\left(e^{-F_1 D}\hat{z}_1(t)\right. \\ & \left. -\hat{z}_1(t-D) - e^{-F_2 D}\hat{z}_2(t) + \hat{z}_2(t-D)\right) \end{aligned}$$

The observer proposed in this paper comes from the same set of equations. However, as the ultimate goal is to estimate the state in the original coordinates, the set of equations will be transformed to depend on x explicitly. This is achieved by differentiating each of the equations, using Equation (2), it becomes:

$$\dot{\hat{z}}_1(t) = \frac{\partial \Psi}{\partial x}\dot{x}(t) + \dot{\epsilon}_1(t) \tag{16}$$
$$\dot{\hat{z}}_2(t) = \frac{\partial \Psi}{\partial x}\dot{x}(t) + \dot{\epsilon}_2(t) \tag{17}$$
$$\dot{\hat{z}}_1(t-D) = \frac{\partial \Psi}{\partial x}\dot{x}(t-D) + e^{-F_1 D}\dot{\epsilon}_1(t) \tag{18}$$
$$\dot{\hat{z}}_2(t-D) = \frac{\partial \Psi}{\partial x}\dot{x}(t-D) + e^{-F_2 D}\dot{\epsilon}_2(t) \tag{19}$$

The expression for the observer proposed in the following theorem arises from solving the above set of equations for $\dot{x}(t)$.

Theorem 1. Assume that the change of coordinates $z = \Psi(x)$ that transforms the nonlinear system (1) into the pseudo linear system (3) exists and that its Jacobian is invertible. Furthermore assume that the systems (6) and (7) are observers for (3) designed so that for any positive constant D, the following term is invertible :

$$e^{-F_1 D} - e^{-F_2 D} \tag{20}$$

Then the following dynamical system:

$$\begin{aligned}\dot{\hat{x}} = & \left(\frac{\partial \Psi}{\partial x}\right)^{-1}_{x=\hat{x}} \left(e^{-F_1 D} - e^{-F_2 D}\right)^{-1} \\ & \Big(e^{-F_1 D}\dot{\hat{z}}_1(t) - \dot{\hat{z}}_1(t-D) - \\ & e^{-F_2 D}\dot{\hat{z}}_2(t) + \dot{\hat{z}}_2(t-D)\Big)\end{aligned} \tag{21}$$

with

$$\Psi(\hat{x}(t_0)) = \hat{z}_1(t_0) = \hat{z}_2(t_0)$$

is a finite time observer for the system (1) in the sense that the estimation $\hat{x}$ converges exactly to the state x after the time delay D.

Proof

Let us choose an arbitrary positive constant D. As the systems (6) and (7) are observers that converge to z with linear error dynamics; furthermore, as they have the same intial conditions; the following system is a finite time observer for (3):

$$\begin{aligned}\hat{z} = & \left(e^{-F_1 D} - e^{-F_2 D}\right)^{-1} \big(e^{-F_1 D}\hat{z}_1(t) \\ & -\hat{z}_1(t-D) - e^{-F_2 D}\hat{z}_2(t) + \hat{z}_2(t-D)\big)\end{aligned} \tag{22}$$

and for any time $t \geq t_0 + D$, we have :

$$\hat{z}(t) = z(t) = \Psi(x(t)) \tag{23}$$

Therefore, integration of the following dynamical system:

$$\begin{aligned}\dot{\hat{z}} = & \left(e^{-F_1 D} - e^{-F_2 D}\right)^{-1} \Big(e^{-F_1 D}\dot{\hat{z}}_1(t) \\ & -\dot{\hat{z}}_1(t-D) - e^{-F_2 D}\dot{\hat{z}}_2(t) - \dot{\hat{z}}_2(t-D)\Big)\end{aligned} \tag{24}$$

with the initial conditions:

$$\hat{z}(t_0) = \hat{z}_1(t_0) = \hat{z}_2(t_0) \tag{25}$$

leads to a finite time observer for $z(t)$. Let us define the estimate $\hat{x}$ so that

$$\hat{z} = \Psi(\hat{x}) \tag{26}$$

Then, the following dynamical system

$$\dot{\hat{x}}(t) = \left(\frac{\partial \Psi}{\partial x}\right)^{-1}_{x=\hat{x}} \dot{\hat{z}}(t) \tag{27}$$

with the following initial conditions:

$$\Psi(\hat{x}(t_0)) = \hat{z}(t_0) \tag{28}$$

is an observer for system (1) that converges exactly to the state within the predefined time delay D. $\square$

This one-step approach is different from the two-step one adopted in (Menold *et al.*, 2003*a*). Their technique consists in transforming the system into a linear one by an appropriate transformation and to make a finite time estimation in these coordinates. Then the estimate in the original coordinates is retrieved through the inverse transformation. The observer proposed in this paper provides an estimate in one step only. This is achieved using the transformation Jacobian and therefore it does not require to compute the inverse transformation. This approach is quite similar to that adopted by Kazantzis and Kravaris for their nonlinear observer (Kazantzis and Kravaris, 1998).

Example

Consider the following system

$$\dot{x}_1 = -x_1^2 - x_2 \tag{29}$$

$$\dot{x}_2 = 2x_1 x_2 - x_1^2 \tag{30}$$

$$y = x_1 \tag{31}$$

The system can be transformed into a pseudo linear one with the following coordinates change:

$$z_1 = x_1 \tag{32}$$

$$z_2 = -x_1^2 - x_2 \tag{33}$$

It can then be written as system (3) where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} 0 \\ 2y^3 + y^2 \end{pmatrix}$$

Two arbitrary independent high gain observers can be synthetised by taking

$$H_i = \begin{pmatrix} \alpha_1^i/\omega_i \\ \alpha_2^i/\omega_i^2 \end{pmatrix} \quad i = 1, 2$$

with ω_i positive and so that both following polynomials are Hurwitz:

$$s^2 + \alpha_1^i s + \alpha_2^i \quad i = 1, 2$$

The performance of the finite time observer are illustrated by a numerical simulation on Figure 1. The different parameters values used for the simulation are listed in Table 1. It can be seen that the estimate reaches the state exactly after the pre-defined delay D as expected.

The finite time convergence of the estimate provided by the above observer is guaranteed by the linearity of the reconstruction errors dynamics. However in practice, it is not always possible to find a suitable change of variable allowing to build

Table 1. Parameters for the numerical simulation

Variable	Value	Variable	Value
$x_1(0)$	0.5	$x_2(0)$	1
$\hat{x}_1(0)$	0	$\hat{x}_2(0)$	0
Parameter	Value	Parameter	Value
α_1^1	3	α_1^2	8
α_2^1	2	α_2^2	3
ω_1	0.1	ω_2	0.1

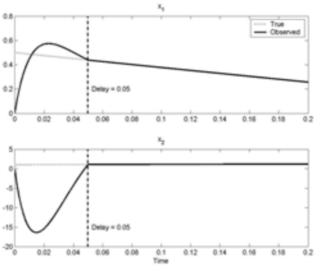

Fig. 1. Simulation results for exemple 1

observers with linear error dynamics. In particular, if the β function of Equation (3) depends on the state, the error dynamics are not linear. In this case, the use of high gain observers allows to converge within a finite time into a neighborhood of the state (Menold, 2004).

3. GENERALIZED FINITE TIME OBSERVER

In the following, we present a general approach of the finite time observation for nonlinear systems.

Assume that the matrices A_1, A_2 are Hurwitz and that the functions β_1, β_2 are such that $[A_i,\ \beta_i]$ form controllable pairs. Furthermore, let Ψ_1 and Ψ_2 be the solutions of the following systems of partial derivative equations :

$$\frac{\partial \Psi_1}{\partial x} f(x) = A_1 \Psi_1 + \beta_1(y) \tag{34}$$
$$\frac{\partial \Psi_2}{\partial x} f(x) = A_2 \Psi_2 + \beta_2(y) \tag{35}$$

Then the following systems are observers for $\Psi_1(x)$ ans $\Psi_2(x)$ respectively:

$$\dot{\hat{z}}_1 = A_1 \hat{z}_1 + \beta_1(y) \tag{36}$$
$$\dot{\hat{z}}_2 = A_2 \hat{z}_2 + \beta_2(y) \tag{37}$$

and we have the following linear dynamics :

$$\frac{d}{dt}\left(\hat{z}_1 - \Psi_1(x)\right) = A_1\left(\hat{z}_1 - \Psi_1(x)\right) \tag{38}$$
$$\frac{d}{dt}\left(\hat{z}_2 - \Psi_2(x)\right) = A_2\left(\hat{z}_2 - \Psi_2(x)\right) \tag{39}$$

Using the following notations :

$$z_1 = \Psi_1(x) \tag{40}$$
$$z_2 = \Psi_2(x) \tag{41}$$

and defining the reconstruction errors as:

$$\epsilon_1 = \hat{z}_1 - z_1 \tag{42}$$
$$\epsilon_2 = \hat{z}_2 - z_2 \tag{43}$$

The error dynamics linearity allows to write the relations between the reconstruction errors at different times instances as follows:

$$\epsilon_1(t) = e^{A_1 D} \epsilon_1(t-D) \tag{44}$$
$$\epsilon_2(t) = e^{A_2 D} \epsilon_2(t-D) \tag{45}$$

This allows to write the same set of four equations as in Section 2, which becomes, after differentiation :

$$\dot{\hat{z}}_1(t) = \nabla\Psi_1 \dot{x}(t) + \dot{\epsilon}_1(t) \tag{46}$$
$$\dot{\hat{z}}_2(t) = \nabla\Psi_2 \dot{x}(t) + \dot{\epsilon}_2(t) \tag{47}$$
$$\dot{\hat{z}}_1(t-D) = \nabla^D\Psi_1 \dot{x}(t-D) + e^{-A_1 D}\dot{\epsilon}_1(t) \tag{48}$$
$$\dot{\hat{z}}_2(t-D) = \nabla^D\Psi_2 \dot{x}(t-D) + e^{-A_2 D}\dot{\epsilon}_2(t) \tag{49}$$

where the following notations are used ($i = 1, 2$):

$$\nabla\Psi_i = \left(\frac{\partial \Psi_i}{\partial x}\right)_{x=\hat{x}(t)} \tag{50}$$
$$\nabla^D\Psi_i = \left(\frac{\partial \Psi_i}{\partial x}\right)_{x=\hat{x}(t-D)} \tag{51}$$

The formulation of the observer proposed in the following theorem arises from solving this set of equations for $\dot{x}(t)$.

Theorem 2. Assume that the matrices A_1, A_2 are Hurwitz and that the functions β_1, β_2 are chosen so that the following systems are controllable:

$$\dot{\hat{z}}_1 = A_1 \hat{z}_1 + \beta_1(y) \tag{52}$$
$$\dot{\hat{z}}_2 = A_2 \hat{z}_2 + \beta_2(y) \tag{53}$$

and let Ψ_1, Ψ_2 be the solutions of the PDE systems (34) and (35). Furthermore, assume that the matrices A_i are chosen so that, for any positive constant D, the following term is invertible:

$$e^{-A_1 D}\nabla\Psi_1 - \nabla^D\Psi_1 \left(\nabla^D\Psi_2\right)^{-1} e^{-A_2 D}\nabla\Psi_2 \tag{54}$$

Then if the initial conditions are set as follows:

$$\hat{z}_1(t_0) = \Psi_1(\hat{x}(t_0)) \tag{55}$$
$$\hat{z}_2(t_0) = \Psi_2(\hat{x}(t_0)) \tag{56}$$

and satisfy the following condition:

$$\hat{z}_1(t_0) = \nabla^0\Psi_1 \left(\nabla^0\Psi_2\right)^{-1} \hat{z}_2(t_0) \tag{57}$$

the following dynamical system:

$$\dot{\hat{x}}(t) = \Omega^{-1}\left(\Delta(\dot{\hat{z}}_1) - \underline{\nabla}\Delta(\dot{\hat{z}}_2)\right) \quad (58)$$

where

$$\Delta(z) = e^{-A_1 D} z(t) - z(t-D) \quad (59)$$

and

$$\underline{\nabla} = \nabla^D \Psi_1 \left(\nabla^D \Psi_2\right)^{-1} \quad (60)$$
$$\Omega = e^{-A_1 D}\nabla\Psi_1 - \underline{\nabla} e^{-A_2 D}\nabla\Psi_2 \quad (61)$$

is a finite time observer for (1) in the sense that the estimate reaches the states after the predefinite time delay D.

Proof

Let us introduce the reconstruction errors defined by Equations (42) (43). Equation (58) becomes:

$$\dot{\hat{x}}(t) = \Omega^{-1}\left(\Delta(\dot{z}_1) - \underline{\nabla}\Delta(\dot{z}_2) - \Delta(\dot{\epsilon}_1) + \underline{\nabla}\Delta(\dot{\epsilon}_2)\right) \quad (62)$$

By definition of z_1 and z_2 (Equations (40) and (41)), it can be seen that :

$$\Delta(\dot{z}_1) - \underline{\nabla}\Delta(\dot{z}_2) = \Omega\dot{x} \quad (63)$$

Therefore, Equation (62) can be rewritten as follows :

$$\dot{\hat{x}}(t) = \dot{x}(t) - \Omega^{-1}\left(\Delta(\dot{\epsilon}_1) - \underline{\nabla}\Delta(\dot{\epsilon}_2)\right) \quad (64)$$

As for any time greater than $t_0 + D$, Equations (44) (45) hold, both $\Delta(\dot{\epsilon}_1)$, $\Delta(\dot{\epsilon}_2)$ vanish and we have

$$\forall t \geq t_0 + D \ : \ \dot{\hat{x}}(t) = \dot{x}(t) \quad (65)$$

This implies that the estimate dynamics and the state dynamics are the same after the convergence time interval. It remains to show that the estimate reaches the value of the state variables at time $t = t_0 + D$

Let us focus on the time interval $[t_0 \quad t_0 + D]$. During this time interval, the delayed values for the different variables remain constant and equal to their initial values :

$$\hat{z}_1(t-D) = \hat{z}_1(t_0) \quad (66)$$
$$\hat{z}_2(t-D) = \hat{z}_2(t_0) \quad (67)$$
$$\underline{\nabla} = \underline{\nabla}^0 \quad (68)$$

The observer expression becomes:

$$\dot{\hat{x}}(t) = \Omega^{-1}\left(e^{-A_1 D}\dot{\hat{z}}_1 - \underline{\nabla}^0 e^{-A_1 D}\dot{\hat{z}}_2\right) \quad (69)$$

A simple expression for the integral of the above expression is difficult to compute since in particular Ω is not constant. However, it can be written as follows:

$$\Omega\dot{\hat{x}}(t) = e^{-A_1 D}\dot{\hat{z}}_1 - \underline{\nabla}^0 e^{-A_1 D}\dot{\hat{z}}_2 \quad (70)$$

and using the definition of Ω leads to the following equation:

$$e^{-A_1 D}\dot{\Psi}_1(\hat{x}(t)) - \underline{\nabla}^0 e^{-A_2 D}\dot{\Psi}_2(\hat{x}(t)) = \quad (71)$$
$$e^{-A_1 D}\dot{\hat{z}}_1 - \underline{\nabla}^0 e^{-A_2 D}\dot{\hat{z}}_2 \quad (72)$$

As the initial conditions are set as follows :

$$\Psi_1(\hat{x}(t_0)) = \hat{z}_1(t_0) \quad (73)$$
$$\Psi_2(\hat{x}(t_0)) = \hat{z}_2(t_0) \quad (74)$$

the above equation can be integrated between t_0 and t to give :

$$e^{-A_1 D}\Psi_1(\hat{x}(t)) - \underline{\nabla}^0 e^{-A_2 D}\Psi_2(\hat{x}(t)) = e^{-A_1 D}\hat{z}_1(t) - \underline{\nabla}^0 e^{-A_2 D}\hat{z}_2(t) \quad (75)$$

This equation can be rewritten as follows using the reconstruction errors expressions:

$$e^{-A_1 D}\Psi_1(\hat{x}(t)) - \underline{\nabla}^0 e^{-A_2 D}\Psi_2(\hat{x}(t)) = e^{-A_1 D}\Psi_1(x(t)) - \underline{\nabla}^0 e^{-A_2 D}\Psi_2(x(t)) - e^{-A_1 D}\epsilon_1(t) + \underline{\nabla}^0 e^{-A_2 D}\epsilon_2(t) \quad (76)$$

The evaluation of the above expression at time $t = t_0 + D$ leads to the following expression :

$$e^{-A_1 D}\Psi_1(\hat{x}(t_0+D)) - \underline{\nabla}^0 e^{-A_2 D}\Psi_2(\hat{x}(t_0+D)) = e^{-A_1 D}\Psi_1(x(t_0+D)) - \underline{\nabla}^0 e^{-A_2 D}\Psi_2(x(t_0+D)) - \epsilon_1(t_0) + \underline{\nabla}^0\epsilon_2(t_0) \quad (77)$$

The assumption on the initial conditions (Equation (57)) is such that

$$\hat{x}(t_0+D) = x(t_0+D) \quad (78)$$

This shows that the estimate reaches the state at time $t = t_0 + D$. This completes the proof. □

The above procedure is more general than the previous one. It does not require to tune and compute two observers with linear error dynamics. It only requires to compute two functions by solving a set of partial derivative equations. Solving this system can be done by expanding the different functions in Taylor series as proposed in (Kazantzis and Kravaris, 1998). It is worth noting that both functions Ψ_1 and Ψ_2 may be the same provided the matrices A_1, A_2 are such that $\left(e^{-A_1 D} - e^{-A_2 D}\right)$ is invertible. In this case the situation is exactly the same as the one presented in Section 2.

Example

Consider the following Van der Pol oscillator :

Table 2. Parameters for the numerical simulation

Variable	Value	Variable	Value
$x_1(0)$	1	$x_2(0)$	1
$\hat{x}_1(0)$	0.5	$\hat{x}_2(0)$	0.2
Parameter	Value	Parameter	Value
b_1	-7	b_3	14
b_2	-10	b_4	69

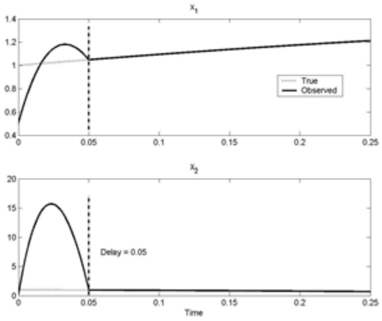

Fig. 2. Simulation results for exemple 2

$$\dot{x}_1 = x_2 \quad (79)$$

$$\dot{x}_2 = -x_1 + x_2 - x_1^2 x_2 \quad (80)$$

$$y = x_1 \quad (81)$$

and the following matrices and functions:

$$A_1 = \begin{pmatrix} b_1 & 1 \\ b_2 - 1 & 1 \end{pmatrix} \beta_1 = \begin{pmatrix} b_1 y + \frac{y^3}{3} \\ b_2 y + \frac{y^3}{3} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -1 - b_3 & 1 \\ -3 - b_4 & 2 \end{pmatrix} \beta_2 = \begin{pmatrix} b_3 y - \frac{y^3}{3} \\ b_4 y - 2\frac{y^3}{3} \end{pmatrix}$$

where b_1, b_2, b_3 and b_4 are constants to be chosen so that the matrices A_1, A_2 are Hurwitz. Solving the PDE systems (34) and (35) leads to the following expressions

$$\Psi_1 = \begin{pmatrix} x_1 \\ x_2 + \frac{x_1^3}{3} \end{pmatrix}, \Psi_2 = \begin{pmatrix} x_1 \\ x_2 + x_1 + \frac{x_1^3}{3} \end{pmatrix}$$

The implementation of the nonlinear finite time observer (58) requires to define both following systems ($i = 1, 2$)

$$\dot{z}_i = A_i z_i + \beta_i(y) \quad (82)$$

$$y = [1\ 0]\, z_i \quad (83)$$

and to compute the Jacobian of Ψ_1 and Ψ_2 which are respectively given by:

$$\nabla\Psi_1 = \begin{pmatrix} 1 & 0 \\ x_1^2 & 1 \end{pmatrix}, \nabla\Psi_2 = \begin{pmatrix} 1 & 0 \\ 1 + x_1^2 & 1 \end{pmatrix}$$

Simulation results are shown on Figure 2 where it can be seen that the estimate reaches exactly the state after the predefinite time delay D as expected.

4. CONCLUSION

In this paper we have presented a finite time observer for nonlinear systems that proceeds in one step and does not require to compute any inverse coordinates transformation. The estimation only requires to compute the Jacobian of the change of coordinates that transforms the system into a pseudo linear one allowing to build observers with linear error dynamics.

As the change of coordinates that transforms the system into a linear one is not always trivial, a more general approach has been envisaged. It consists in defining two pseudo linear systems allowing to compute two functions by solving a set of partial derivative equations. The estimate is then computed from the integration of a dynamical system using the Jacobian of each of the computed functions.

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