VARIABILITY MATRIX: A NEW TOOL TO IMPROVE
THE PLANT PERFORMANCE

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Abstract: This work introduces a novel methodology to quantify the profit gain due to reduction in the product variability. The base of the proposed approach is the variability matrix (VM), which relates how the loop variance of main loops is changed when the variance of the other loops are changed. Based on the potential reduction on the main loop variance, it is possible to quantify the economic impact produced by improving the tuning of given control loop. Based on the VM, it is possible to select the control loops responsible for the major impact in the variability of the products and which should be the vocation of the loop: good performance of robustness. The VM concept is applied to a simple distillation process. This example shows how the plant profitability can be improved by utility reduction and by selling products more impure. *Copyright © 2006 IFAC*

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1 INTRODUCTION

Reduce process variability allows the process arrives near the restriction, increasing its performance. In the most of processes the operating point with the high efficiency is normally in a corner of the operating window. Achieve this point means reduce energy and increase production (Seborg et al., 1989).

To keep the process near the maximum efficiency operating point, the performance of control system should be ensured.

Usual refinery or petrochemical plants has hundreds or thousand loops and guarantee the performance of all loops is impossible without a systematic procedure, which would determine and sort the loops following their economic impact to the process profitability. Usually, several control loops can be improved reducing the loop variability. But the main question is which loop should be attack firstly. Many times, some loops can have a great potential to reduce its variance, but they have a very small impact in the plant profitability. On the other hand, some other loops can have a little potential to reduce its variance, since they are already well tuned. But if these loops could be a little better, they would

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contribute more significantly to the final economical result. Therefore, to select and order, which loops should be firstly improved, it is necessary to quantify the corresponding economic impact.

In this paper the novel concept of variability matrix is introduced as a tool to quantify the economic impact of improving the control loop performance.

The base of the proposed approach is the variability matrix, which relates how the loop variance of main loops is changed when the variance of the other loops are changed. Based on the potential reduction on the main loop variance, it is possible to quantify the economic impact produced by improving the tuning of given control loop.

The article is structured as follows: in the section 2 some definition is introduced using a distillation process. In section 3, a new methodology to relate the decrease of the variability of a given product in profit will be defined. In section 4, the concept of variability matrix is formally presented. In section 5 the new methodology is applied to the distillation introductory exempla. Finally, the conclusions concerning to this work are shown in section 6.

2 DEFINITIONS

To quantify the economic impact, it is interesting to classify the control loops into two following categories

1. **Main or Primary Loops** are the loops that directly control the products specification. Its performance improvement causes the reduction in product variability, which can be directly translated into profitability.

2. **Auxiliary or Secondary Loops**: Loops that do not directly control the product quality, but can indirectly affect the product variability.

To exemplify these definitions, Figure 1 shows a typical distillation process with a vessel to smooth the feed of the second column (V1). The objective is obtaining all products with high purity.
The primary loops are the three cascades that control the composition of both products (AC1, AC-3, and AC-4). The other loops are called secondary loops.
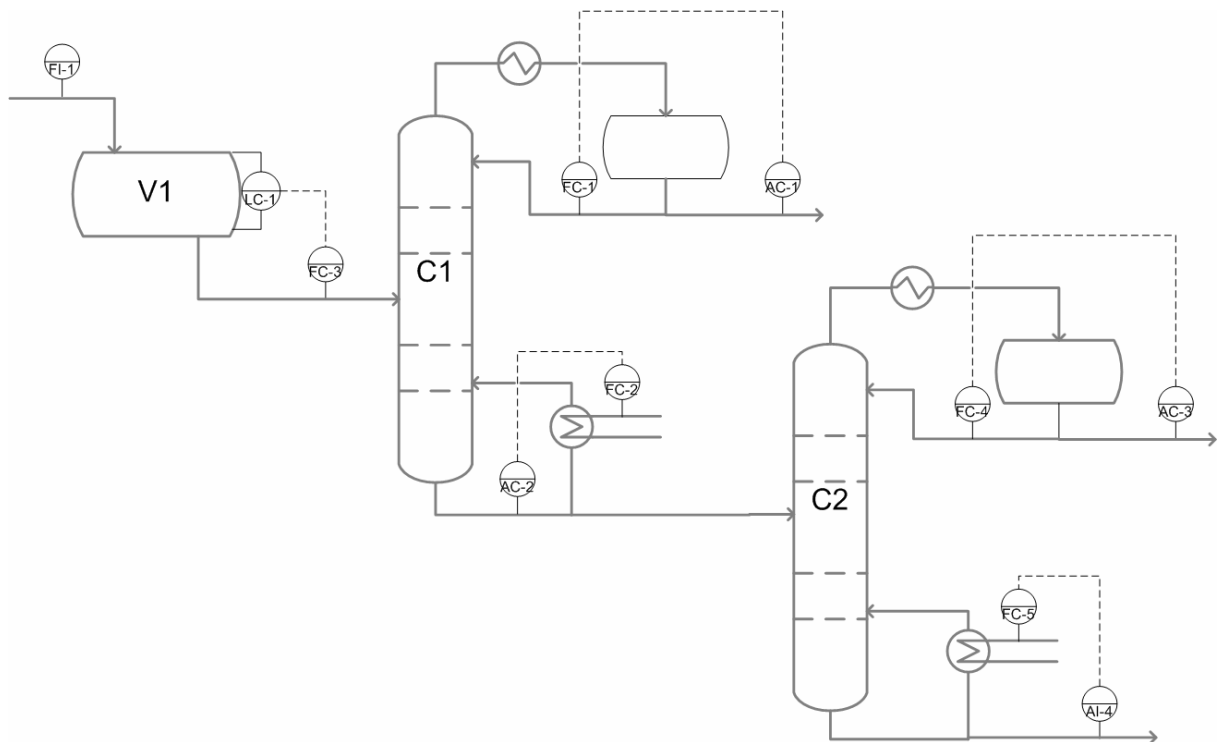
Figure 1: Schematic representation for a typical distillation process

The impact of each loop and its magnitude in the variability of the products will be shown in the section 4, where the concept of the matrix of variability is introduced.

Some of control loops should have a good performance to ensure the products specification (e.g. AC-2) and others should only smooth de disturbances, to stabilize the feed of the columns (e.g. the level of V1).

3 ASSESSING THE PROFIT OF THE PRIMARY LOOPS

In the literature there are some methodologies to estimate the profit as function of the operating point. The most famous is called Taguchi (Taguchi, et al., 1989) and relates the profit as a quadratic function of product purity where the vertex is in the specification and the profit decreases with a quadratic constant. The form of the curve that relates the profit as function of process performance is purely heuristics. Besides, the parameters obtaining is also a difficult task, and does not use explicitly the energy reduction or production increase.

In this article a simple methodology based on first principles (mass balance) is introduced to quantify the profit of the reduction of the primary loops. In the next section, this methodology is applied to translate the how can a reduction of variability of the secondary loops into final product variability.

When the process arrives closer to the specification, there are three ways of increasing the profitability:

1. Reduction of energy consume: when the specification of the products is lower, the energy spent is also lower, in the most of process.
2. Sell impurity as product: with the product near the specification a part of impurities can be sold with the same price of expensive product.
3. Increasing of the unity production: If the equipment restricts the increase of the plant production, the variability reduction allows the production to increase. This analysis is relevant but complex, and will not be considered in this article.
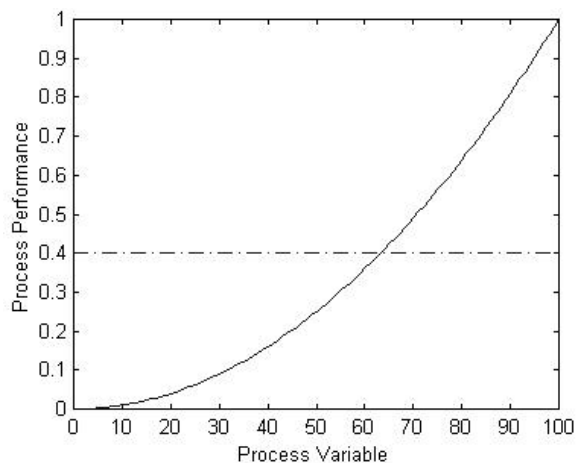
3.1 Process Performance

The key factor to quantify the profit due to variability reduction is measure the process performance. The performance is specific for each process (e.g. product composition, conversion, profit).

When the performance curve has a high slope, increase controller performance will improve the plant performance. On the other hand, if the curve is flat, a low variability controller will achieve almost the same process performance of a controller with poor performance.

With a process histogram, the mean process performance can be easily calculated. The process performance is multiplied by the frequency that the process assumes that process variable interval

(Marlin, 1991). Figure 2 shows two different performance curves.

Depending on the performance curve, the process variable can have a narrow or broad distribution: when the performance is the same in all operating point (curve 2) the simple reduction of variability of the loop does not represent increase profit. On the other hand, when the performance has abrupt slope, small changes in variability will cause significant increase in the performance (values near 100 of curve 1).

Figure 2: Schematic representation of performance curves

3.2 Reduction of Energy Consume

When the process achieves an operating point of higher efficiency, the energy consumed usually decreases, because the higher profit operating point is in a corner of the operating window (Marlin, 1995). To quantify the profit given by energy reduction (P_{ER}), the difference in the utilities streams (ΔU) are calculated:

$$\Delta U = G(0)^{-1}\Delta Y \qquad (1)$$

Where $G(0)$ is the static gain matrix for the system and ΔY is the difference in the operating points specification for the controlled variables variability reduction.

The profit given by energy reduction (P_{ER}) can be estimated multiplying the difference in the utility streams (ΔU) by the energy cost (E_C).

$$P_{ER} = \left(G^{-1}\Delta Y\right)^{'} E_C \qquad (2)$$

Suppose a system with two manipulated variables, with the following static gain matrix:

$$G(0) = \begin{bmatrix} -0.01 & 0.005 \\ 0.005 & -0.01 \end{bmatrix} \qquad (3)$$

After the process variability reduction, the difference in the specification of products is 0.005 and 0.01. The energy cost are (US$) 300 and 200 per energy unity. The profit due to energy reduction can be calculated as follows:

$$P_{ER} = \left(\begin{bmatrix} -0.01 & 0.005 \\ 0.005 & -0.01 \end{bmatrix}^{-1} \begin{bmatrix} 0.005 \\ 0.01 \end{bmatrix} \right)' \begin{bmatrix} 300 \\ 200 \end{bmatrix} = 733 \tag{4}$$

3.3 Sell a product more impure

When the variability of a given product becomes smaller, the mean composition can be reduced to a new value closer the specification, allowing mix in the final product a greater part of impurities, less valuable than the product.

To estimate the profit (P_{IP}), we need to determine the amount of impurities (ΔF) can be mixed in the product. A simple mass balance can be done, based on the actual flow (F), the final and initial contamination of a given stream (y_F and y_I) and the increased flow (ΔF), considering that the amount of product is constant (Fy_I).

$$Fy_I = (F + \Delta F)y_F$$
$$\Delta F = F\left(\frac{y_I}{y_F} - 1\right) \tag{5}$$

Multiplying the increased flow (ΔF) by the price difference between the main product of the stream and the contaminant (P_E - P_C), the increased profit can be estimated.

$$P_{IP} = F\left(\frac{y_I}{y_F} - 1\right)(P_E - P_C) \tag{6}$$

4 ASSESSING THE PROFIT OF THE SECONDARY LOOPS – THE VARIABILITY MATRIX

4.1 Definition

To determine the influence of each loop in the final product we need to translate the variance reduction of all control loops into the primary loops. The variability matrix is a matrix where the elements (i,j) quantify how the change in the variance of the control loop (j) produces a change in the variance of the main loop (i), i.e.,

$$VM(i, j) = \frac{\Delta \text{variance}(cv_i)}{\Delta \text{variance}(cv_j)} \tag{7}$$

The structure of the variability matrix is the following:

- Rows: The rows show the influence of each loop in the same final product. The number of rows is the same as the products or main loops.
- Columns: Shows the influence of a given loop in each final product. The number of columns is the same as the number of control loops implemented in the plant. The first columns correspond to the main loops.

Figure 3 shows a schematic representation of variability matrix.

Figure 3: Schematic representation of variability matrix

4.2 Similarities to a gain matrix

The variability matrix has similar properties to static gain matrix. Its values can be positive or negative. A positive value of the element VM(i,j) means that increasing the variance of the auxiliary loop j will increase the variance of the main loop i. For the control loop j "faster is better".

On the otherwise, if VM(i,j) is negative, it will decrease the variance of the main loop i by increasing the variance of the loop j. This situation is typical for buffer tanks, which typically should reduce the propagation of the disturbance by a variation on the tank level. Here, for the control loop j, "slower is better" is true. These loops should have a poor performance, being responsible for smooth variations in the process. The limit in this case is the safety of the unity. In this case, usual PID tuning methodologies are not adequate. We suggest in this case a methodology to tuning level controllers for tanks shown in Smith (2002).

The procedure to construct the variability matrix is analogous to build the static gain matrix. Any identification technique can be applied to identify it. For that, it should be used as inputs the variance of the primary and secondary loops and as outputs the variance of the main loops. As by standard identification procedure, it is necessary to make a perturbation in the inputs. In this case the perturbation can be produced by a change in controller parameters or through the addition of known perturbation in the control loop, which will change the control loop variance.

5 EXAMPLE

This section will show the use of these new concepts to reduce the variability of a given plant, showing the profit before and after the tests.

5.1 The plant

Consider the example shown in the Figure 1. This hypothetical plant is feed by three different products (A, B, and C).

The plant has two distillation columns with internal trays. Before the columns, the system has a vessel to smooth the variations in the unity feed, which are the main disturbance. Both columns have total condenser.

The total feed of the unity is 6000 ton/day. In the top of the first column (C1) the less valuable component (A) is removed from the unity. In the bottom of the column, the products B and C are removed. This stream feeds the second column (C2). The products B and C are removed in the top and bottom of C2, respectively.

The main disturbance of the unity is the feed flow of V1 that has a periodical oscillation. The columns have on-line analyzers that can provide their composition to the control system.

We assume that the inventory control is properly tuned. The control structure is build as shown in Table 1:

Table 1: Control structure for the distillation process

Loop	CV	MV
AC-1	x_B in top stream of C1	FC-1
AC-2	x_A in bottom stream of C1	FC-2
LC-1	Level of V1	FC-3
AC-3	$(x_A + x_C)$ in top stream of C2	FC-4
AC-4	x_B in bottom stream of C2	FC-5

Where x_i is the mass fraction of the component *i*. All these variables are controlled using a proportional-integral controller (PI).

The objective of this unity is split all the three components. Table 2 shows the value of each product (US$/ton), their specification and price.

Table 2 shows that the product A is the less valuable while the product C is the more valuable. The specification is the same for all the three products.
To ensure the products specification, the mean of each controlled specification must be 3 standard deviation from the restriction.

Table 2: Values and feed of each product

Prd	Feed (ton/day)	Specification	Price (US$/ton)
A	1000	$x_B < 0,05$	400
B	2000	$x_{A+C} < 0,05$	900
C	3000	$x_B < 0,05$	1200

Table 3 shows the utility consumed in the unity and their costs US$/ton:

Table 3: Utility consumed and price

Utility	Consume (ton/day)	Cost (US$/ton)
C1 - Condenser	2500	20
C1-Reboiler	2000	80
C2 - Condenser	1300	200
C2-Reboiler	1000	150

The transfer matrix G1 for the column C1 is given by

$$G_1 = \begin{bmatrix} \dfrac{-0.0001}{8s+1}e^{-2s} & \dfrac{0.000085}{12s+1}e^{-3s} \\ \dfrac{0.00007}{15s+1}e^{-3s} & \dfrac{-0.0001}{10s+1}e^{-20s} \end{bmatrix}. \quad (8)$$

Whereas G2 is the transfer matrix of the column C2 and is given by

$$G_2 = \begin{bmatrix} \dfrac{-0.00015}{8s+1}e^{-6s} & \dfrac{0.0001}{12s+1}e^{-3s} \\ \dfrac{0.00009}{15s+1}e^{-3s} & \dfrac{-0.00021}{10s+1}e^{-5s} \end{bmatrix}. \quad (9)$$

The transfer function G3 is the vessel V1

$$G_3 = \frac{0.5}{40s}. \quad (10)$$

All the five controllers are tuned using the methodology based in frequency domain (Engell and Müller, 1993) using a desired performance 2 times faster than open loop.

Table 4 shows the mean and the standard deviation for each product, with the mentioned tuning.

Table 4: Mean and standard deviation for each stream

Product	Mean	Standard deviation
A (C1 – Top)	0.957	0.0022
B (C2 – Top)	0.974	0.0081
C (C2 – Bottom)	0.963	0.0042

We consider that the mean composition of A in bottom of C1 is maintained constant during all tests. The variability matrix (VM) for the system is given by

$$VM = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccccc} A & B & C & V1-LC & C1-BOT \\ \begin{bmatrix} 1 & 0 & 0 & -0.0001 & -0.0155 \\ -0.41 & 1 & -0.08 & -0.0006 & 1.25 \\ -0.29 & -0.05 & 1 & -0.0002 & 0.5 \end{bmatrix} \end{array} \qquad (11)$$

The variability matrix shows that the increase of the variability of the product A will decrease the variability of the others products (B and C). The same can be said of the product B and C, if the variability of one product decrease, the variability of the other will increase. But nothing will occur with the variability of A. The control of the V1 level could reduce the variability of all products, if a robust tuning is applied.

Based on VM, we can see that the V1 level causes increase of variability in all products. A new set of parameters is used, based on the methodology shown in Smith (2002) to retune the level control. The impact of new tuning is shown in Table 5.

Table 5: Mean and standard deviation for each stream with the level of V1 more robust

Product loop	Mean	Standard deviation
C1 – Top	0.955	0.0018
C2 – Top	0.965	0.0048
C2 – Bottom	0.959	0.0029

Table 5 shows that the variability of all products are reduced, allowing the system arrive nearer the restriction.

The loop that control the composition of the product A also cause impact in all other loops, as shown by (11). A new adjust more robust is made (3 times slower than open loop). Table 6 shows the new operating point, with the new parameters:

Table 6: Mean and standard deviation for each stream with the C1-Top more robust

Product loop	Mean	Standard deviation
C1 – Top	0.966	0.0053
C2 – Top	0.956	0.0019
C2 – Bottom	0.956	0.0019

Table 6 shows that the variability of product A becomes higher, however the B and C standard deviation becomes lower, allowing the system arrives closer to the restrictions.

Now, the indexes to quantify the profit (eqs. 2 and 6) of the new tuning will be applied. Table 7 shows the profit for each controller that is retuned for the reduction in the utilities and sell products more impure (US$/day).

Table 7 shows a visible increase in the profit, with energy reduction and product more impure. This example shows that key loops are responsible for high variations in the unity and the VM is a powerful tool to detect these loops, guiding the control engineer to achieve a more profitable operating point for his plant.

Table 7: Profit increase for energy reduction and product more impure

Increase profit	Energy reduction	Product more impure	Total (US$/day)
Level Retuning	33800	12200	46000
x_A loop retuning	7560	16800	24360
Total	41300	29000	**70360**

6 CONCLUSIONS

In this article a new tool called Variability Matrix (VM) is introduced. The VM show the influence of a given loop in the variability of all products of the plant. This information allows identifying the loops that cause the variability in each product, determining what would be the best performance of each controller (fast or robust).

Besides, this article shows also a methodology to quantify the gain caused by the variability reduction, due to the utility reduction and sell a product more impure (closer to the specification).

This new methodology is applied to a hypothetical distillation process, showing the potentialities of the VM. Based on VM, two loops are retuned, showing a visible increase in the plant profitability.

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