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COORDINATED DECENTRALIZED MPC FOR PLANT-WIDE CONTROL OF A PULP MILL BENCHMARK PROBLEM

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Abstract: In large-scale model predictive control (MPC) applications, such as plant-wide control, the coordination of unit-based MPC controllers has been identified as both an opportunity and a challenge in enhancing the plant-wide control performance. This work discusses an efficient strategy for the coordination of decentralized MPC systems and illustrates the approach with an application to the pulp mill benchmark problem proposed by Castro and Doyle III (2004*a*). The decentralized unit-based MPC controllers are coordinated at the MPC steady-state target calculation stage by employing decentralized optimization techniques. The off-diagonal element abstraction technique and the price-driven coordination algorithm are used in the development of a coordination mechanism. The pulp mill case study shows that this coordinated, decentralized MPC framework is an effective approach to plant-wide MPC applications, which has high reliability, accuracy and efficiency.

Keywords: Decentralized MPC, Target calculation, Price-driven coordination, Decentralized optimization

1. INTRODUCTION

In many plant-wide control and optimization applications, a large-scale process model is decomposed into several smaller subsystems and a controller is developed for each subsystem. This may lead to a decentralized unit-based MPC framework. The coordination of the unit-based controllers has been identified as having significant potential benefit (Havlena and Lu, 2005).

The decomposition and coordination approaches to solving complex large-scale control problems attracted attention in 1970's and 1980's (Wismer, 1971; Titli, 1978; Jamshidi, 1983); but the interest diminished thereafter for a number of reasons (Havlena and Lu, 2005) including: limited implementation opportunities; inherent complexity and difficulty of the problem; and computational issues. The industrial success in applying control schemes with a decentralized architecture has stimulated increasing interest in coordination of decentralized MPC (Lu, 2003); however, the need for more research in this area has been well recognized (Havlena and Lu, 2005; Isaksson *et al.*, 2005).

Most commercial MPC products employ two-stages: a steady-state target calculation and a dynamic control calculation (Qin and Badgewell, 2003; Ying and Joseph, 1999; Rao and Rawlings, 1999). In the case of decentralized unit-based MPC, without plant-wide coordination,

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the optimum operations achieved by each unit-based MPC may provide significantly worse performance than the plant-wide optimum solution (Havlena and Lu, 2005).

The potential benefit of coordinating decentralized control schemes has garnered increasing interest by both researchers and practitioners. Camponogara *et al.* (2002) have proposed a distributed MPC scheme, where local control agents broadcast their states and optimization results to every other agent under pre-specified rules to help reach a plant-wide optimum. Decentralized optimization via Nash bargaining has been applied for solving multi-player coordination problem by Waslander *et al.* (2004). Venkat *et al.* (2004) have used augmented states to model interactions and their scheme involves iterative negotiations among decentralized MPC systems. One common feature of the above schemes is that the decentralized MPC controllers exchange information directly and thus stand at an equal status within their negotiation hierarchy.

In process industries, however, a wide-spread belief among practitioners is that the trend toward decentralization will continue until the control system consists of seamlessly collabrating autonomous and intelligent nodes with a supervisory coordinator overseeing the whole process (Scheiber, 2004). One approach to coordinating decentralized MPC is to employ a centralized optimization layer to perform a plant-wide target calculation (e.g., Honeywell's *ad hoc* technology); while an alternative approach is to take advantage of decentralized optimization with an additional coordination system (Havlena and Lu, 2005). Our previous work (Cheng *et al.*, 2004; Cheng *et al.*, 2005*b*) adopts this approach, where the Dantzig-Wolfe decomposition and price-driven coordination strategies are tailored to yield a coordination system for decentralized MPC.

This work discusses the development of a coordination system for decentralized MPC that employs the price-driven coordination algorithm and off-diagonal element abstraction technique (Cheng *et al.*, 2005*b*). The case study based on the pulp mill benchmark problem shows that the proposed coordinated, decentralized MPC framework can be a viable approach to solving plant-wide MPC problems.

2. PLANT-WIDE MPC

In the process industry, centralized or monolithic MPC schemes are considered to be not viable for complex process control and optimization problems (Lu, 2003; Havlena and Lu, 2005). Consequently, industrial practice has tended toward a decentralized MPC architecture.

Usually, any limited cooperation between decentralized MPC controllers is through an upper level optimization, such as real-time optimization (RTO), at a sampling time of hours; however, disturbances or setpoint changes in the interval between two RTO executions may drive the optimum operations away from the targets given by the RTO system; thus, it is necessary to perform re-optimization at a higher frequency to maintain optimum operations. This section focuses on the coordination strategies for decentralized MPC at the target calculation level, and as a result, at a sampling time comparable with that of the MPC control calculation.

2.1 Unit-based MPC

In this work, unit-based MPC refers to the decentralized MPC subsystems developed for individual operating units as shown in Figure 1.

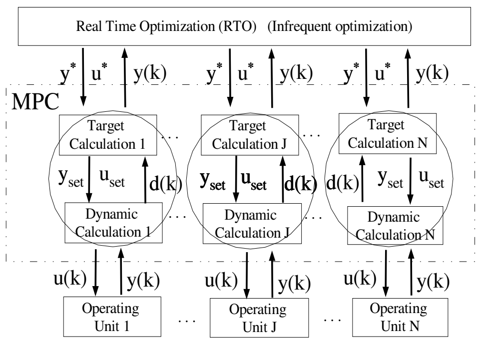

Fig. 1. Two-stage unit-based MPC system

Consider the following constrained quadratic programming (QP) formulation of MPC target calculation for an individual operating unit (Ying and Joseph, 1999):

$$\min_{\mathbf{y}_{set},\mathbf{u}_{set}} \; z = (\mathbf{y}_{set}(k) - \mathbf{y}^*)^T \mathbf{Q}_y (\mathbf{y}_{set}(k) - \mathbf{y}^*) + (\mathbf{u}_{set}(k) - \mathbf{u}^*)^T \mathbf{Q}_u (\mathbf{u}_{set}(k) - \mathbf{u}^*) + \mathbf{c}_y(\mathbf{y}_{set}(k) - \mathbf{y}^*) + \mathbf{c}_u(\mathbf{u}_{set}(k) - \mathbf{u}^*) + \epsilon^T \mathbf{c}_\epsilon^T \mathbf{c}_\epsilon^T \epsilon \quad (1)$$

s. t.

$$\begin{aligned} \mathbf{y}_{set}(k) &= \mathbf{K}\mathbf{u}_{set}(k) + \mathbf{d}(k) \\ \mathbf{d}(k) &= \mathbf{d}(k-1) + \delta(k) \\ \mathbf{y}_{min} - \epsilon \le \mathbf{y}_{set}(k) &\le \mathbf{y}_{max} + \epsilon \\ \mathbf{u}_{min} \le \mathbf{u}_{set}(k) &\le \mathbf{u}_{max} \\ \epsilon &\ge 0 \end{aligned} \quad (2)$$

where $\mathbf{y}^*$ and $\mathbf{u}^*$ are the optimal nominal "targets" computed by upper level optimizers, $\mathbf{y}_{set}(k)$ and $\mathbf{u}_{set}(k)$ are the achievable targets to be optimized, while $\mathbf{d}(k)$ is the estimated disturbance updated by:

$$\delta(k) = \mathbf{y}_m(k) - \mathbf{y}_{set}(k|k-1), \quad (3)$$

where $\mathbf{y}_m(k)$ are the measured outputs at time k and $\mathbf{y}_{set}(k|k-1)$ is the prediction of outputs in the previous control execution. ϵ may be defined as a violation tolerance of the output constraints that ensures a feasible solution to the QP. The steady-state gain matrix $\mathbf{K}$ can be calculated via linearization of the nonlinear model used in an upper optimizing layer or abstracted from the linear model used by lower level MPC dynamic control. Note that the above formulation considers only the local unit.

2.2 Coordination of Decentralized MPC

Centralized Optimization A centralized optimization approach for coordinating decentralized MPC systems has been discussed in Havlena and Lu (2005). In that framework, as is shown in Figure 2, a monolithic optimization problem is formulated

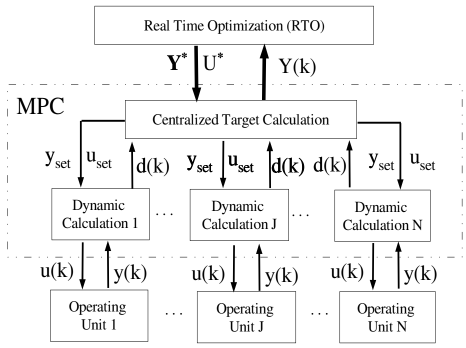

Fig. 2. Centralized MPC target calculation

at the target calculation stage for the entire plant. A plant-wide gain matrix is used, which relates the manipulated variables (MVs) and controlled variables (CVs) of all decentralized MPC subsystems. Although a centralized steady-state optimization approach may accurately track optimal plant operations, it can lack the reliability of the decentralized control structure.

Decentralized Optimization Depicted in Figure 3, a coordinator is designed to deal with the interactions among decentralized MPC controllers and makes use of the price-driven coordination method (Cheng *et al.*, 2005*a*). The task of the coordinator is to ensure that the coordinated system finds the optimal plant operations. Note that in the figure "S. I." denotes the term *sensitivity information*, which is the Lagrangian-like information flow used in the coordination mechanism.

A key step in coordinator design is to identify appropriate interactions for linking constraint formulation. The linking constraints contain process variables from multiple operating units (or unit-based MPC controllers). These linking constraints are used in the coordinator's optimizing scheme.

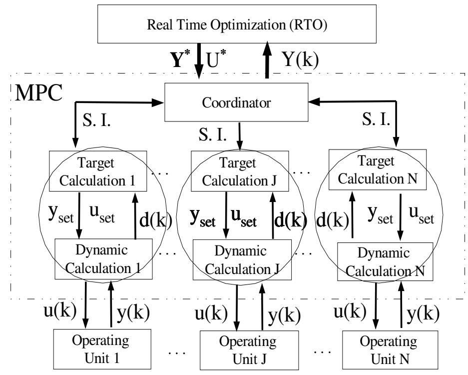

Fig. 3. coordinated, decentralized MPC target calculation

Several methods can be used to model the interactions, such as the interstream consistency (Cheng *et al.*, 2005*a*), off-diagonal elements abstraction, and ratio control constraint augmentation (Havlena and Lu, 2005).

Off-diagonal Elements Abstraction Here we briefly discuss the off-diagonal elements abstraction method for constructing linking constraints. Quite often, advanced control strategies are designed and implemented at different times for different operating units. In this situation, the CVs and MVs have been specified and grouped in a unit-based sense. Assume that we have a full gain matrix for a plant with N operating units:

$$\mathbb{A} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \dots & \mathbf{K}_{1N} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \dots & \mathbf{K}_{2N} \\ \dots & \dots & \dots & \dots \\ \mathbf{K}_{N1} & \mathbf{K}_{N2} & \dots & \mathbf{K}_{NN} \end{bmatrix} \tag{4}$$

A unit-based implementation of MPC in (2) uses the block-diagonal information $\mathbf{K}_{ii}$ of the plant model in their calculations, while the off-diagonal blocks may be treated as disturbances in their models. This way of dealing with the off-diagonal information can introduce undesirable uncertainty when the interactions are significant. Note that the plant-wide model:

$$Y(k) = \mathbb{A}U(k) + D(k) \tag{5}$$

where $Y(k)$ and $U(k)$ are vectors containing the CVs and MVs of N local operating units, respectively, and is equivalent to:

$$\mathbf{y}_i(k) = \mathbf{K}_{ii}\mathbf{u}_i(k) + \mathbf{e}_i(k) + \mathbf{d}_i(k) \tag{6}$$

$$\mathbf{e}_i(k) - \sum_{j=1}^{N} \mathbf{K}_{ij}\mathbf{u}_j(k) = \mathbf{0} \quad j \neq i \tag{7}$$

The auxiliary variable $\mathbf{e}_i$, which is an abstraction of the off-diagonal elements, represents the influence of the inputs of other operating units on the local system. Without the equations in (7), the auxiliary vector $\mathbf{e}_i$ can be regarded as local variables only involved with the i^{th} operating unit, and they can play a role as decision variables in the optimization.

The abstracted equality constraints in (7) are the linking constraints to be incorporated into the coordinator's optimization problem. Different decentralized optimization strategies have different usage of the linking constraints, but all of them aim to find a set of $[\mathbf{y}_i, \mathbf{u}_i, \mathbf{e}_i]$ so that the optimum plant operations are achieved.

Price-driven Coordination Method In our previous work (Cheng *et al.*, 2005*a*), the price-driven coordination method was developed to efficiently solve large-scale QP problems with equality linking constraints in a decentralized optimization manner. The work was based on ideas presented in Jose and Ungar (1998*a*) and (1998*b*).

Using the price-driven coordination method, the MPC target calcultion for a local operating unit can be modified as:

$$\min_{\mathbf{y}_{set},\mathbf{u}_{set}} \quad z = (\mathbf{y}_{set}(k) - \mathbf{y}^*)^T \mathbf{Q}_y (\mathbf{y}_{set}(k) - \mathbf{y}^*)$$
$$+ (\mathbf{u}_{set}(k) - \mathbf{u}^*)^T \mathbf{Q}_u (\mathbf{u}_{set}(k) - \mathbf{u}^*) + \mathbf{c}_y (\mathbf{y}_{set}(k) - \mathbf{y}^*)$$
$$+ \mathbf{c}_u (\mathbf{u}_{set}(k) - \mathbf{u}^*) + \epsilon^T \mathbf{c}_\epsilon^T \mathbf{c}_\epsilon^T \epsilon - \mathbf{p}^T \mathbf{e}(k) \quad (8)$$

s. t.

$$\begin{aligned} \mathbf{y}_{set}(k) - \mathbf{K}\mathbf{u}_{set}(k) &= \mathbf{e}(k) + \mathbf{d}(k) \\ \mathbf{d}(k) &= \mathbf{d}(k-1) + \delta(k) \\ \mathbf{y}_{min} - \epsilon \le \mathbf{y}_{set}(k) &\le \mathbf{y}_{max} + \epsilon \\ \mathbf{u}_{min} \le \mathbf{u}_{set}(k) &\le \mathbf{u}_{max} \\ \epsilon &\ge 0 \end{aligned} \quad (9)$$

where we omitted the subscript i for simplicity. Note that there is a minor modification to the objective function and unit model.

Note that a price vector $\mathbf{p}$ is introduced in (8). It has been proved that there exists an equilibrium price vector $\mathbf{p}^*$ that optimally coordinates the independently solved subproblems (unit-based optimization problems). To find the equilibrium price vector, the generalized Newton's method with stepsize determination is used to solve the following system of equations:

$$\mathbf{p}^T \Delta(\mathbf{p}) = 0 \quad (10)$$

$$\Delta(\mathbf{p}) = \mathbf{e}_i - \sum_{j=1}^{N} \mathbf{K}_{ij}\mathbf{u}_j \quad i = 1...N \quad j \ne i \quad (11)$$

for updating the price vector $\mathbf{p}$, and the equilibrium price vector $\mathbf{p}^*$ satisfies the above two equations. One may also notice that Δ in (11) is an implicit function of the price vector $\mathbf{p}$. When the price vector is appropriately updated, the composition of unit-based MPC solutions will converge to the plant-wide optimum.

3. PLANT-WIDE CONTROL OF A PULP MILL PROCESS

The pulp mill model given in Castro and Doyle III (2004*a*) is a newly published industrial benchmark problem, which may be suitable for the study of process modeling and estimation, process control and optimization, and fault detection and diagnosis. This pulp mill model includes the fiber-line and the chemical recovery loop. The primary goal of the pulp mill is to produce wood pulp of a given Kappa number or brightness while minimizing energy costs, utilities and chemical make-up streams. The control objectives, modes of operation, process constraints and measurements are all defined in Castro and Doyle III (2004*a*).

3.1 Existing Unit-based MPC Schemes

In Castro and Doyle III (2004*b*), a decentralized control system has been proposed. At the unit level, it involves two control layers: unit-based MPC and decentralized regulatory control loops. This work focuses on the MPC layer.

The existing MPC consists of four separate controllers, one each for the digester and oxygen reactor, the bleach plant, the evaporators, and the lime kiln/recaust areas, respectively. In their configuration, the MPC layer only contains the dynamic control calculation stage and involves totally 21 CVs and 20 MVs. The MPC is designed to track the set-point trajectories given by an upper level optimization.

3.2 Modeling for Target Calculation

Since we focus on MPC target calculation, the plant-wide linear steady-state model matrix $\mathbb{A}$ in (4), from the MVs to CVs, is obtained via step response tests to ensure that the steady-state gains are consistent with the dynamic simulation.

In this work, the effect of disturbances are compensated via the bias update strategy in Ying and Joseph (1999).

3.3 Unit-based MPC Target Calculation

This study uses the decentralized, two-stage MPC system discussed in Ying and Joseph (1999) and takes the formulation given by (1) and (2) for target calculation. The control calculations in this paper use the configuration of Castro and Doyle III (2004*b*).

In the unit-based MPC target calculation, the interactions between units were ignored. Thus, the gain matrix $\mathbf{K}$ in (2) is actually $\mathbf{K}_{ii}$, the block-diagonal elements of the overall-plant gain matrix $\mathbb{A}$. The effect of off-diagonal elements was treated as disturbances, through $\mathbf{d}(k)$. The bounds for variables are the same as in the dynamic control calculation, and the weightings $\mathbf{Q}_y$ and $\mathbf{c}_y$ are given in Table 1.

3.4 Closed-loop Performance

This section compares three control schemes: the centralized, the decentralized, and the coordinated, decentralized MPC target calculation. The centralized optimization scheme uses the entire plant-wide gain matrix and is used to define the performance benchmark for our study.

It is desired to closely track the setpoints given by an upper level optimization at the same time maximize production rate and minimize oxygen reactor coolant flow and kiln fuel flow. In this case study, the plant-wide objective function is defined as a combination of those objectives with weightings given in Table 1. The optimization problems in all of the schemes are formulated as minimization problems. The weightings for the MVs used in all MPC control schemes are adopted from the work by Castro and Doyle III (2004*b*).

Table 1. Important CV Weightings

Controlled variables	$\mathbf{Q}_y/100$	$\mathbf{c}_y$
production rate	1.5	-80
digester kappa No.	1.5	0
oxygen reactor kappa No.	1.0	0
oxygen reactor caustic flow	1.0	0
oxygen reactor steam flow	0.5	0
oxygen reactor coolant flow	0.75	30
E kappa No.	1.0	0
D_2 brightness	1.0	0
slaker temperature	1.0	0
kiln $\mathbf{O}_2$ excess %	1.0	0
kiln fuel flow	0.5	30

Using the coordination strategy given in Section 2.2, the plant operation can be driven to the optimum operation. This usually takes a few communication cycles between the coordinator and subsystems.

Results based on a 8000-minute (about 140 hours) closed-loop simulation are reported. The disturbance set imposed on the process was adopted from Castro and Doyle III (2004*b*). Because the coordinated scheme provides identical performance to that of the centralized scheme, the following figures only show the closed-loop responses for the coordinated scheme and the original decentralized scheme.

The responses of some key process variables are reported in Figure 4. Note that, if the existing decentralized MPC behaves satisfactorily (i.e., is stabilizing and robust) under certain disturbances, the proposed coordination mechanism will not impact these characteristics. Moreover, the proposed control scheme can provide the optimum plant operations as given by the centralized scheme. In this study, the decentralized scheme exhibits significant offset from the optimum production rate and use of raw materials and energy.

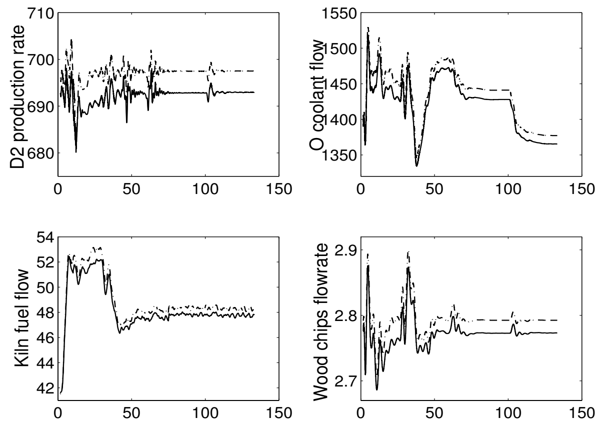

Fig. 4. Pulp mill closed-loop responses: solid line (coordinated); dash line (decentralized)

Table 2 reports the profit/cost function values and computational times for all three control schemes. Note that the accumulated value function is a

Table 2. Performance comparisons

control schemes	value function	optimization time* (per MPC execution)
centralized	1.22×10^5	0.06 s
unit-based	1.32×10^5	0.04 s
coordinated	1.22×10^5	0.14 s

* Simulations performed in Matlab 6.1, AMD Athlon 1.4G Hz, 1024M RAM machine.

time-integration of the objective function evaluated using the measured process variables. The optimization time is an average value based on the observed computational times. As we have defined the value function of the centralized scheme as a benchmark, we can see that the coordinated decentralized MPC provides the same plant-wide operations, which produces an 8.2% improvement on that of the decentralized scheme. In the case study, the optimization problems involve dozens of decision variables and hundreds of constraints. The coordinated MPC scheme provides solutions at a reasonable computational speed and as a result, exhibits a good trade-off between accuracy, reliability and computational load.

3.5 Remarks: Some Implementation Issues

In the case study, the sampling time for target calculation is chosen as 10 minutes, which is the least common multiple of the sampling times of MPC subsystems. Based on our observations from simulations, the selection of sampling time for coordination should also depend on the frequencies of the disturbances that the control system must deal with.

In general, good initial points can substantially enhance the efficiency of optimization. A practical target calculation formulation should not provide too aggressive changes in the reference trajectories, so the optimal solutions from two consecutive target calculation executions should not differ too much. Therefore, the equilibrium price vector from the previous execution works very well as an initial guess for the current target calculation.

4. CONCLUSIONS AND FUTURE WORK

In the MPC applications for plant-wide control, the coordination of unit-based MPC controllers can substantially improve plant operations. With the price-driven coordination algorithm and the off-diagonal element abstraction technique, a coordination mechanism was developed for a coordinated, decentralized MPC framework. The proposed control approach is applied to a pulp mill benchmark problem, and shows a significant improvement in performance in comparison to the existing decentralized MPC systems. Thus, the case study shows that this coordinated, decentralized MPC framework may be a viable technology for plant-wide MPC applications.

A number of issues still need to be further investigated. One of these is an understanding of the complexity and scaling behavior of the price-driven coordination algorithm. In addition, it is vital to understand the relationship between the structure of the decentralized MPC system and the performance of the coordinated system.

5. ACKNOWLEDGMENTS

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