OPERABILITY OF MULTIVARIABLE NON-SQUARE SYSTEMS

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Abstract: Non-square process control systems, with fewer inputs than the controlled outputs, are quite common in chemical processes. In these systems, it is impossible to control all measured variables at specific set points and many of the outputs are controlled within an interval. The objective of this paper is to introduce a multivariable Operability methodology for such non-square systems to be used in the design of non-square constrained controllers. In order to motivate the new concepts, we examine some simple non-square systems obtained from the control system of a Steam Methane Reformer process.

Keywords: Operability, Intervals, Output Variables, Polygons, MIMO, Model Based Control

1. INTRODUCTION

In this section, the problem definition is described and a brief introduction to prior work, concerning non-square systems and operability issues, is presented.

1.1 *Problem Definition*

In recent years, in the face of increasing complexity of chemical processes due to the integration of units, process optimization and strict environmental regulations the use of tools to evaluate the performance of a control structure has become very important. This has to be done in a more systematic manner than by trial and error closed-loop simulations and before the final controller step. Georgakis et al (2003) mentioned that it has been since the last decade that the integration of process and control design has received considerable attention. Skogestad (2004) emphasized that the field of control structure design in plant-wide control problems, which includes the selection of manipulated and control variables, is underdeveloped. Moreover the majority of the controllability methods developed address the design of multiple input – multiple output (MIMO) systems with respect to interactions and loop pairings, and often apply only to unconstrained systems. Few methods take into account the limited range available for the control inputs during the design phase. The Operability framework developed by Vinson and Georgakis (2000) was a contribution in this direction. The Operability methodology is an effort to integrate the process design and control objectives, helping to cope with the complexity of chemical processes. Essentially, the Operability measure can quantify the ability of a process to change from one steady-state to another and reject expected disturbances utilizing a limited control action available. This measure is important because once the design is fixed, no control methodology can overcome limitations on operability. It is only with a tool to evaluate the operability of a chemical process that one could analyze appropriately the economic aspects of the process. A review of the literature on integration of process and control design related to square systems can be found in Vinson (2000). A survey concerning mathematical and process-oriented approaches in plant-wide control was presented by Larsson and Skogestad (2000).

Based on the linear and steady-state Operability framework initially developed by Vinson and Georgakis (2000), the objective of this paper is the development of a multivariable *non-square* Operability methodology for linear systems. This would help in the design of non-square Model Predictive Controllers (MPC), with more outputs than inputs, commonly encountered in industrial chemical processes. Basically, MPC controllers are model-based controllers which account for process constraints. Based on the input constraints, generally specified a priori due to physical limitations of the process, an important task is to define the output ranges or constraints within which we want to control the process. The problem is that very tight constraints make the control design difficult, with the possibility of not being able to find the appropriate input variables to achieve the control objective. On the other hand, if the constraints are not tight enough, output specifications, such as desired product quality or environmental regulations, cannot be achieved. Therefore the Operability methodology can serve an important role in solving this problem. Through this framework it is possible to verify achievability of control objectives before implementing the MPC controller. In addition, according to Vinson (2000) some of the main features of the MPC strategy, such as being predominantly linear and using constraints for each manipulated and controlled variable, are directly associated with the developed operability framework. This problem functions as motivation for the current effort. The outline of this paper is the following: first a summary of the prior work concerning non-square systems and operability issues is given. Then, the basic theory used in the development of this paper is explained. The results from the analysis of some simple systems are presented next, closing with conclusions.

1.2 *Summary of prior work*

Non-square systems with more outputs than inputs are quite common in chemical processes. Apart from the common non-square nature of some chemical processes, a system with more outputs than inputs may occur if one of the actuators of an original square system is operating at constantly saturated levels. Several studies that analyze aspects of non-square systems can be found in the literature. Reeves and Arkun (1989) developed a block relative gain array (RGA) measure for non-square linear systems as a tool to analyze and evaluate control structures in steady-state before the controller design in order to specify the appropriate control structure. Similarly, Chang and Yu (1990) extended RGA for non-square multivariable systems, defining the non-square relative gain array (NRG). Both studies suggested that for non-square systems with more outputs than inputs, the outputs have to be controlled in the least square sense, minimizing offsets. One very important contribution that examines the design of non-square systems is the concept of Partial Control introduced by Shinnar (1981), and mathematically analyzed by Kothare et al. (2000). This methodology helps the control engineer to choose the appropriate set of measured variables to be controlled at the set-point, in a system having limited degrees of freedom. This choice must be made so that the other outputs can still be controlled at specified ranges while satisfying all the input and performance variable constraints and rejecting all the expected disturbances. This methodology would be useful in selecting the variables in the control design stage after the process operability quantification proposed here has been evaluated.

The Operability Index (OI) was introduced by Vinson and Georgakis (2000) and Vinson (2000) as a measure to access the input-output open-loop controllability of a multivariable square chemical process. The concept of operability given by Vinson (2000) is the following: A process is operable if the available set of inputs is capable of satisfying the desired steady-state and dynamic performance requirements defined at the design stage, in the presence of the set of anticipated disturbances, without violating any process constraints. Vinson and Georgakis (2002) have demonstrated that the Operability Index is independent of the inventory control structure. This property allows one to compare the operability of competing designs before the process control structure is selected or implemented, i.e., during the process synthesis stage. Vinson and Georgakis (2000) have also shown that this measure can be applied to SISO and MIMO systems and is more appropriate than other design tools such as RGA or minimum singular values. Concerning non-square systems, Vinson (2000) analyzed the ability of the OI to enhance the performance of a non-square MPC controller, specifically DMCplus™ (AspenTech). Finally, an overview of all Operability definitions and concepts has been done by Georgakis et al (2003). In the next section, a brief explanation of the concepts and definitions of the Operability framework is given.

2. PROPOSED APPROACH FOR PROBLEM SOLUTION

The Operability Index (OI) was introduced by Vinson and Georgakis (2000) for analyzing square systems. It provides a quantitative result for multivariable systems and a graphical representation for systems less than 3-D, permitting the design to be modified in order to improve process operability before the control structure selection.

2.1 *Operability of Square Systems: Servo Operability Measure*

To make the idea of the Operability measure clear, it is necessary to define some useful spaces. The Available Input Space (*AIS*) is the set of values that the process input variables can take based on the design of process, limited by process constraints. Mathematically for an *n x n* square system:

$$AIS = \left\{ \underset{\sim}{u} \mid u_{A,i}^{\min} \le u_{A,i} \le u_{A,i}^{\max}; 1 \le i \le n \right\}.$$

Moreover, the Desired Output Space (*DOS*) is given by the desired values of the outputs of the process and is represented by:

$$DOS = \left\{ \underset{\sim}{y} \mid y_{D,i}^{\min} \le y_{D,i} \le y_{D,i}^{\max}; 1 \le i \le n \right\}.$$

Based on the steady-state model of the process, expressed by the process gain matrix (*G*), the Achievable Output Space (*AOS*) is defined by the output values that can be achieved using the inputs inside the *AIS*. We will use the notation $AOS_u(d^N)$ for the *AOS* calculated considering all points inside *AIS* (subscript u) when the disturbances lie in their nominal values (d^N), i.e., for the servo problem. For this problem, the outputs in the $AOS_u(d^N)$ are calculated by: *y=G(u)*, where $u \in AIS$. Based on those definitions, the Servo Operability Index with respect to the outputs is the following:

$$s-OI_y = \frac{\mu(AOS_u(d^N) \cap DOS)}{\mu(DOS)} \qquad (1)$$

Where μ represents a function that calculates the size of the space, for example for 3-D it is the volume and in 2-D the area. This index quantifies how much of the region of desired outputs can be achieved using the available inputs in the absence of disturbances and is useful in analyzing changes in the existing plant design to enlarge *AOS*. This Index has a value between 0 and 1. A process is considered completely operable if the index is equal to 1. If the OI is less than one, some regions in the *DOS* are not achievable. It is worth emphasizing that to calculate the OI, mathematical operations involving intersections of polytopes have to be performed. In this work, the MATLAB (Mathworks™, Inc) Geometric Boundary Toolbox (GBT) has been used. It was developed by Veres et al (1996) to evaluate those intersections. Details concerning the servo Operability Index with respect to the inputs and the Desired Input Space (*DIS*) can be found in Georgakis et al (2003), as well as in Vinson and Georgakis (2000).

2.2 *Operability of Non-Square Systems*

In order to quantify the operability of non-square linear systems, some modifications to the definitions initially proposed by Vinson and Georgakis (2000) are required. First, it is worth classifying the process outputs according to the way that they have to be controlled into two categories: *set-point controlled*: variables that are controlled in their exact set-point (for instance, production rates and product qualities); *set-interval controlled*: variables that are controlled at specified ranges (pressure, temperature and level); the operability in the latter case is defined as *interval operability*. The set-point and range variables can be chosen according to an economic objective and given by a supervisory strategy. The idea of the new definition of operability is to fix critical outputs at their set-points, allowing the others to vary within their maximum and minimum limits defined a priori. This definition should also recognize the necessity to control some outputs at ranges rather than require that all points of the *DOS* be reached. In interval operability, process outputs must have at least one feasible operating point within the desired interval for the process to be considered operable. Using the same *AOS* definition as in the square case will lead us to a poor definition of operability. Therefore, it is necessary to redefine the Achievable Output Space in order to analyze the non-square operability issue properly. At this point, it is necessary to define the Expected Disturbance Space (*EDS*). This space represents all the steady-state disturbances that affect the process which can also be used to reflect uncertainties in model parameters employed in the design. Finally, the goal is to formulate a multivariable steady-state methodology to obtain the *AOS,* given *EDS* and *AIS*, to quantify the operability of any non-square linear system. As a starting point in this development, we will examine 2 simple cases in the next section which involve sub-systems of the Steam Methane Reformer (SMR) process that has 4 manipulated, 1 disturbance and 9 controlled variables.

3. RESULTS

A 2 x 1 sub-system of the SMR model cited in the previous section will be used as a starting point to demonstrate the importance of the proposed problem. It is worth mentioning that the SMR model has only non-integrating outputs, and the process dynamics are neglected here since we are studying steady-state Operability. Using the same notation as above and considering the effect a disturbance has on the process, we write:

$$y = G u_1 + G_d w_1 \quad \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} u_1 + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} w_1 \qquad (2)$$

Where w_1 represents the Expected Disturbance Space ($EDS = \{w_1 \mid -1 \le w_1 \le 1\}$) and G_d is the disturbance gain matrix. $AIS = \{u_1 \mid -1 \le u_1 \le 1\}$ and $DOS = \{y \in \Re^2 \mid \|y\|_\infty \le 1\}$.

Rearranging equation (2); we have:

$$y_1 = a_{11}u_1 + d_1 w_1 \Rightarrow u_1 = \frac{y_1 - d_1 w_1}{a_{11}} \qquad (3)$$

$$y_2 = a_{21}u_1 + d_2 w_1 \Rightarrow y_2 = a_{21}\frac{y_1 - d_1 w_1}{a_{11}} + d_2 w_1 \qquad (4)$$

Based on the system of equations above, two cases result.

Case 1: Consider the following scaled steady-state gain matrices: $G = [1.41, 0.66]^T$; $G_d = [1, 0]^T$; In this particular case, since $d_2=0$, equation (4) can be rewritten as:

$$y_2 = a_{21}u_1 \Rightarrow y_2 = a_{21}\frac{y_1 - d_1 w_1}{a_{11}} \qquad (5)$$

Thus, the base case servo *AOS* (*AOS* when $w_1 = 0$) is given by a straight line. In this particular case, the disturbance gains shift the servo *AOS* horizontally. This can be observed in Figure 1, where we have also sketched the *DOS*.

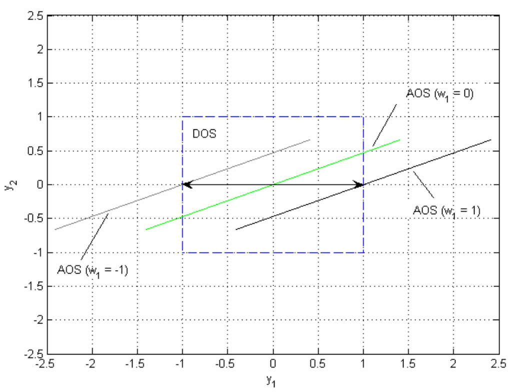

Figure 1: servo *AOS*, shifted *AOS* and *DOS*

The movement of the *AOS* with different disturbance values depends on the values of G_d. If $G_d = [0, 1]^T$, the servo *AOS* would be moving vertically rather than horizontally as in the previous case. For every value of the disturbance, a different straight line is obtained. The union of all the Achievable Output Spaces that correspond to all the expected disturbance values will be called *AOS(d)*. This space and the *DOS* are plotted in Figure 2, along with the Achievable Output Interval Space (*AOIS*). This new space, *AOIS* represents the rectangle that touches, but does not cross, the lines associated with the minimum and maximum disturbance values of *AOS(d)*. This means that if we control the two outputs within some constraints that are not larger than the *AOIS*, the process will not be interval operable with the available input ranges and in the presence of the expected disturbances. In other words, the system will be interval operable if *DOS* covers *AOIS* completely. Therefore, the Interval Operability Index with respect to the outputs (IOI_y) can be now defined as:

$$IOI_y = \frac{\mu(DOS \cap AOIS)}{\mu(AOIS)} \qquad (6)$$

Where μ is the area of the corresponding polygon in this example, and the volume for the 3-D case. It is worth mentioning that we are assuming a rectangular

DOS. If the *DOS* is not rectangular, the solution has to be modified appropriately.

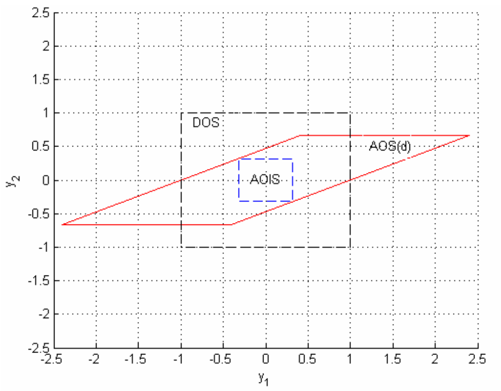

Figure 2: *AOS(d)*, *DOS* and *AOIS* for case 1

Case 2: Disturbance inserted in both output variables: Assume the same *AIS*, *DOS* and *EDS* from the previous case and the following gain matrices: $G = [1.41, 0.66]^T$; $G_d = [-0.6,\ 0.4]^T$;

In this case, the system of equations (3) and (4) holds. The servo *AOS* is now shifted along a diagonal, as shown in Figure 3. In this figure we have also plotted an example of a *DOS* and *AOIS* calculated for this case.

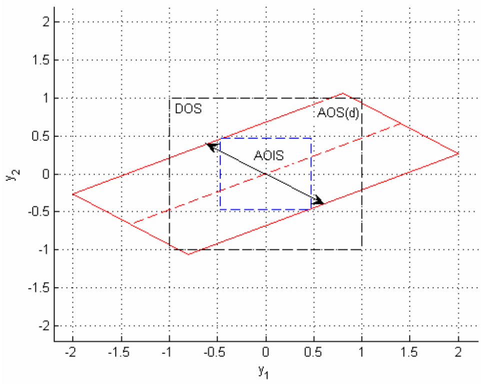

Figure 3: *AOS(d)*, *DOS* and *AOIS* for case 2

For both 2x1 cases above, the system is fully interval operable, since *DOS* is large enough to cover *AOIS*. We can actually see that if we make the *DOS* smaller than the *AOIS*, the process will not be interval operable.

An inoperable case would happen if the disturbances affecting the process were increased in absolute value: $EDS = \{w_1 \mid -3 \le w_1 \le 3\}$. Thus, the *AOIS* would also be enlarged. Figure 4 shows *AOS(d)*, *DOS* and *AOIS* for this scenario. In this case, we notice that the IOI_y would be smaller than 1. This means that the system is only interval operable for some disturbance values considered. In order to make it fully operable, the process constraints should be relaxed, which means the *DOS* should be enlarged to cover the *AOIS* entirely.

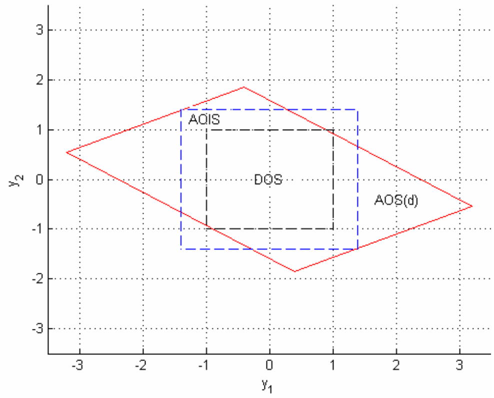

Figure 4: *AOS(d)*, *DOS* and *AOIS* for inoperable case

Now, the problem of controlling 3 outputs at ranges, when having only 2 inputs, will be presented here. The system of equations considered is:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} w_1 \qquad (7)$$

The gain matrices from the SMR process are the following:

$$G = \begin{bmatrix} 1.41 & 0.27 \\ -0.39 & -0.20 \\ 0.66 & 0.49 \end{bmatrix}; \quad G_d = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix};$$

Also: $DOS = \{y \in \Re^3 \mid \|y\|_\infty \le 1\}$, $AIS = \{u \in \Re^2 \mid \|u\|_\infty \le 1\}$ and *EDS* is assumed the same as before.

The *AOS(d)* is now the union of all planes, instead of straight lines, corresponding to different values of disturbances affecting the process. Thus, *AOS(d)* will be an oblique parallelepiped, and *DOS* and *AOIS* are, in general, orthogonal parallelepipeds and, in this case, cubes. *AOS(d)* and *DOS* are displayed on Figure 5. Figure 6 shows *AOS(d)* and *AOIS*, and Figure 7 shows *DOS* and *AOIS*.

Observing Figure 6, one notices that *AOIS* touches both extreme planes of *AOS(d)*. As drawn in Figure 7, the *DOS* is quite larger than the calculated *AOIS*. This implies that the *DOS* can be reduced in size and the process will continue to be output operable as long as *AOIS* continues to be a subset of *DOS*.

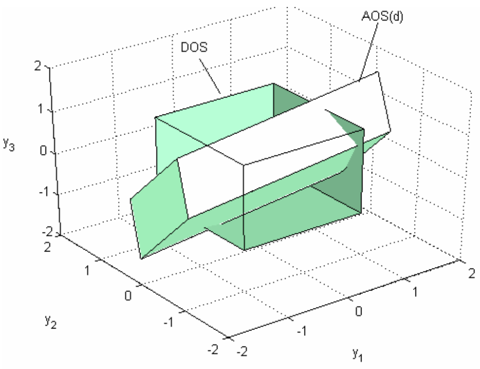

Figure 5: *AOS(d)* and *DOS* - 3x2 problem

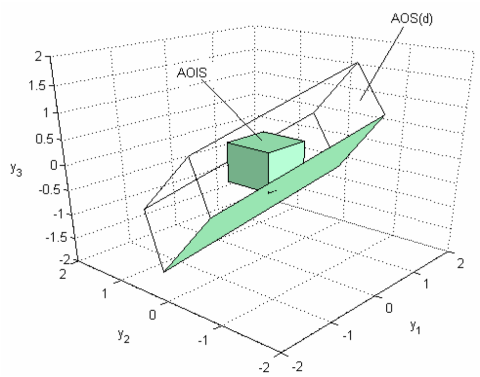

Figure 6: *AOS(d)* and *AOIS* - 3x2 problem

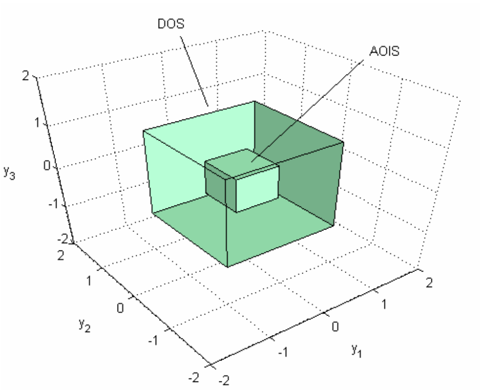

Figure 7: *DOS* and *AOIS* - 3x2 problem

4. CONCLUSIONS

In this paper we presented an extension of the previously defined concept of operability to the case of non-square systems, where some of the output variables need to be controlled within intervals rather than a set-point. Through the detailed examination of 2 case studies we have demonstrated the motivation for calculation of the Achievable Output Interval Space (*AOIS*) as the smallest possible interval constraints for the outputs that can be achieved with the available range of the manipulated variables and when the disturbances remain within their expected values.

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