MULTI MODEL APPROACH TO MULTIVARIABLE LOW ORDER STRUCTURED-CONTROLLER DESIGN

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Abstract: The method presented here offers an effective and time saving tool for robust low order multivariable controller design. The relation between controller complexity and closed loop performance can easily be evaluated. The method consists of five steps: 1. A desired behavior of the closed loop system is specified. Considering the nonminimum phase part of the process model the closed loop attainable performance is determined. 2. The process model and the attainable performance are scaled by the RPN-scaling procedure. 3. This defines an “ideal” scaled controller, which is usually too complex to be realized. 4. The frequency response of the ideal scaled compensator is approximated by a simpler one with structure and order chosen by the user. 5. Since the approximation in frequency response is performed with the scaled system, it is necessary to return to the original system’s units. This procedure can be implemented using a multi-model approach, what increase the robustness of synthesized controller.

Keywords: multivariable project design, frequency domain, multi models, low order controllers

1. INTRODUCTION

The increase of the complexity of the modern plants promoted an increase of the interaction among the variables of the process increasing the number of necessary control loops to maintain the conditions of desired operations and the quality of the obtained products.

Restrictions on the feedback compensator structure are often encountered in chemical plants, when several control stations are provided only with local measurements. Such decentralized information structures result in block-diagonal compensator matrices. Decentralized controllers are also attractive because the information about the feedback is concentrated in the diagonal blocks. This means they are easier to understand and to put into operation and more easily made failure tolerant than general multivariable control systems.

Even for plants with strong interaction, a decentralized controller can be attractive from a performance viewpoint, since depending on the disturbance direction and the model uncertainty can exhibit a better performance to disturbance rejection than a centralized one. Usually to improve the performance to set-point change is interesting to include some degree of decoupling between the main interacting loops. All these situations imply and require a structured controller.

The controller order is another point to be considered, since it is strongly related to implementation easiness. Low order controllers (e.g. PID) are much simple and easy to implement and maintain in industrial control systems (DCS) than a high order state space centralized controller

Due the uncertainties associated to the model and the need of working at different operating points (OPs) with different dynamic behaviours, it is required that the controller must exhibit certain robustness degree. Usually, it is common to design a controller for each OP separately, or to tune for the worst case and to test it to the other OPs, which in general does not produce the best achievable result.

The design of robust decentralized controllers remains a demanding problem; standard methods for robust design cannot be used for structured compensators. The standard techniques for robust full controller design (e.g., Hinf,μ) cannot be directly applied to design a robust structured low order controller. In this paper it is proposed a new methodology to solve this problem, which conciliates design simplicity with DCS implementation easiness.

The proposed approach is based on the multi-model system representation and on the frequency domain approximation. The basic idea of this approach is to approximate the high order full controller that achieves the desired attainable closed loop response by a low order structured controller.

2. METODOLOGY

Consider that the block diagram in figure 1 requires the closed-loop behavior to be a predetermined transfer function chosen $T_0(s)$. Given the model G, mathematically the requirement to make the process closed-loop exactly equal to $T_0(s)$ is satisfied if, and only if

$$C(s) = G^{-1}(s)\left(T_0^{-1}(s) - I\right)^{-1} \tag{1}$$

$C(s)$ is the "ideal" controller that can be a high order controller, since no restriction is used in (1). Although the ideal controller is usually not realizable, it provides the designer with the necessary information about the desired controller frequency response. The basic idea is to approximate in frequency domain the ideal controller (C_0) by a low order structured controller. Since we want that the approximated controller performs so close as possible to the ideal one, it is better to approximated the closed-loop frequency response, i.e. $\Delta T = T - T_0$, instead of approximating $\Delta C = C - C_0$ directly.

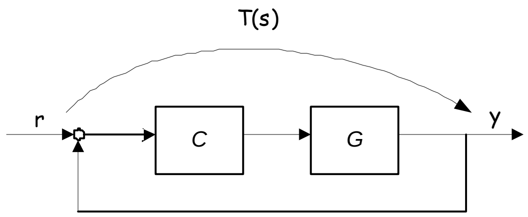

Fig. 1: Standard Feedback Configuration.

In this paper, the proposed methodology will use a two degree-of-freedom control loop configuration shown in Figure 2, where the controller C is separated into four blocks: C_{PI}, C_{PV}, C_{SP} and C_F.

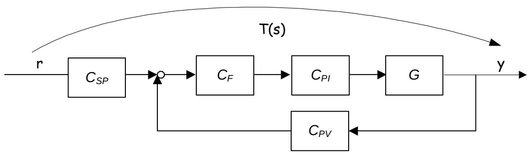

Fig.2: Two degree-of-freedom feedback control

The C_{PI} block is a PI controller whose structure is always fixed and always given by (2), whilst C_{SP} and C_{PV} are dependent on the PID controller parameterization (e.g., series, parallel, ISA-form).

$$C_{PI} = K_C\left(1 + \frac{1}{T_I s}\right) \tag{2}$$

As discussed by Faccin and Trierweiler (2004), the advantage to use the 2DOF control configuration is threefold: (a) It divides a typical nonconvex optimization problem (when the standard configuration is used) into two convex problems. (b) It consists in a common base, in which all possible industrial PID parameterization can be converted. In Faccin (2004) it is shown this conversion for several industrial PID parameterizations. (c) The controller order can be easily increased and implemented in modern DCS. For example, process filters for noise averting can be synthezed and incorporated into C_{PV}. The conversion of different PID or other control forms are very simple, since the algorithm relates the control action (? u) with the variable process (? y) and the variable of reference (? r), i.e., $\Delta u = M(s)\Delta y + N(s)\Delta r$

$$C_{PV}(s) = C_{PI}^{-1}(s)M(s) \ , C_{SP}(s) = -C_{PI}^{-1}(s)N(s) \tag{3}$$

When more than a PID is desired to control the system, it can be done using the block C_F. This block is also diagonal with elements given by the orthogonal serie:

$$\begin{gathered} C_F(s) = \sum_{k=1}^{n=ordem} \left(T_F\right)_k \varphi_k(s) \\ \varphi_k(s) = \varphi_1(s)\prod_{i=1}^{k}\frac{s-\lambda}{s+\lambda} \qquad \varphi_1(s) = \frac{\sqrt{2\lambda}}{s+\lambda} \end{gathered} \tag{4}$$

Where λ is the frequency point where the fit of the curve $\Delta T/s$ is more precarious and the coefficients T_F are the decision variables of the optimization problem.

2.1 Optimization Problem

After algebraic manipulation

$$\begin{aligned} &\Delta T(s) = T - T_0 = \\ &S(s)[G(s)C_{PI}(s)\left(C_{SP}(s) - C_{PV}(s)T_0(s)\right) - T_0(s)] \end{aligned} \tag{5}$$

If $S \cong S_0$ ($S_0 \cong I - T_0$), and C_{SP}, C_{PV}, C_F are diagonal blocks ,the problem can be seen as that the j-th column of ΔT is only influenced by the j-th column of ΔC, so the problem is independent in the column, and can be solved separately. The objective function (6) consists of the Euclidian norm of the step response of the transfer function ΔT on frequency domain for N frequency points.

$$FO = \min_{PID} \sum_{s=j\omega_1}^{s=j\omega_N} \left\| T(s) - T_0(s) \right\|^2 \tag{6}$$

The problem is solved in an iterative and sequential way. In the initiation, C_{SP}, C_{PV}, $C_F = I$, and the parameters from the PI block is determined. In agreement with the selected algorithm, the C_{SP} and C_{PV} blocks are determined fixing the PI. A new iteration starts always fixing the knowns parameters from then previous iteration. This procedure is executed until that the stop approach is satisfied.

When the PID is determined in this sequential and iterative method, it is fixed and the C_F block is solved, to determining T_F.

These problems can be formulated as a least squares problem for each model. If it is desired a control design using the multi model representation, each model generates the same kind of problem. So, the whole problem can be solved as a weighted least squares problem, and these weights are selected by the project designer.

All this procedure is performed in a very fast way, but it is just an approximation because if difference ΔC is not sufficiently small, S deviates from S_0, and the computation error of the column-by-column optimization may be large. The controller can be improved by a non-linear optimization, which considers the closed-loop resulting directly.

The cost function in the non-linear optimization is

$$FO_{Global} = \sum_{k=1}^{ni}\sum_{i=1}^{no}\sum_{l=1}^{N}\left\| \frac{\Delta T_{ik}(j.\omega_l)}{j.\omega_l} \right\|^2 \qquad (7)$$

Where *no* and *ni* are the number of outputs and inputs of the system respectively and ***N*** is the number of frequencies in the frequency vector. The controller from the column-by-column optimization is used as a starting point for the non-linear optimization. The following equation can be formulated

$$\begin{aligned} &\min \gamma \\ &\gamma, x \in \Re \\ &subject \quad to: \quad FO_i(x) - w_i\gamma \le 0 \end{aligned} \qquad (8)$$

Where γ is an auxiliary variable and w_i is the weight for each FO_i calculated for the model i.

2.2 General procedure

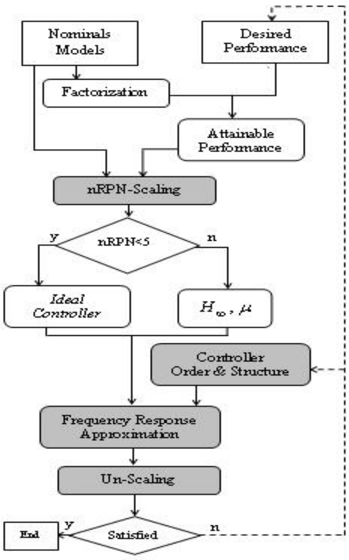

Fig.3: General Procedure.

Fig. 3 shows the general procedure. The desired performance is established to each output through specifications in the time domain (rise time and maximum % of overshoot) that are mapped into a second order transfer function. The models must be factorized to insert some restrictions in the performance like RHP - zeros and -poles and time delay to maintain the internal stability of the feedback system (Trierweiler *et al.*, 2000).

The RPN (robust performance number) (Trierweiler and Engell, 1997) and nRPN (Farenzena and Trierweiler, 2004) when it is working with multi-model are calculated. Small values indicate a good performance using this method. Diagonal matrices that minimize the condition number of the system at the frequency that RPN assumes its maximal value, are used to scale the models. With the controller structure and order, the frequency response approximation is used to calculate the blocks (C_{PI}, C_{PV}, C_{SP} and C_F). The controller is returned to the original units and if the simulation shows a poor performance, the desired performance or its structure and order can be modified.

3. CASE STUDY

The case study consists of a six spherical tank plant. The unit is composed by six level tanks interacting to each other, two control valves, one recycle tank and one pump. The objective is to control the levels h_3 and h_6, manipulating the two valves defining the flow rates F_1 and F_2.

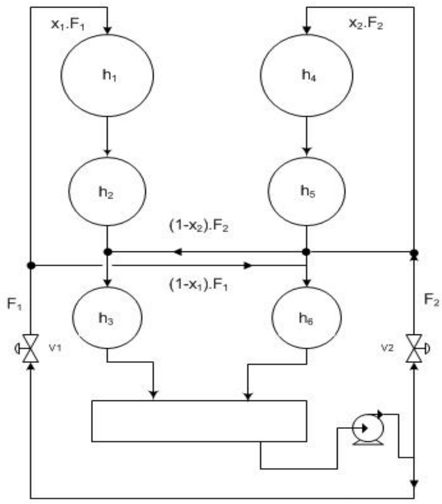

Fig. 4: The six spherical tanks process.

For this process the simplified model expressed by (9) was developed. Where g is the constant of gravity, a_i is the section area of the discharge pipe from the tank i, and D_i is the diameter of the tank i.

After linearizing the model and transforming into Laplace domain at the operating point (h_{1s},h_{2s},h_{3s},h_{4s}) the corresponding transfer matrix is given by (11).

$$
\begin{aligned}
A_1 \frac{dh_1}{dt} &= f_1 = x_1 \cdot F_1 - R_1\sqrt{h_1} \\
A_2 \frac{dh_2}{dt} &= f_2 = R_1\sqrt{h_1} - R_2\sqrt{h_2} \\
A_3 \frac{dh_3}{dt} &= f_3 = (1-x_2)\cdot F_2 + R_2\sqrt{h_2} - R_3\sqrt{h_3} \\
A_4 \frac{dh_4}{dt} &= f_4 = x_2 \cdot F_2 - R_4\sqrt{h_4} \\
A_5 \frac{dh_5}{dt} &= f_5 = R_4\sqrt{h_4} - R_5\sqrt{h_5} \\
A_6 \frac{dh_6}{dt} &= f_6 = (1-x_1)\cdot F_1 + R_5\sqrt{h_5} - R_6\sqrt{h_6}
\end{aligned} \tag{9}
$$

where

$$
A_i = \pi\left(D_i h_i - h_i^2\right) \quad \text{and} \quad R_i = a_i\sqrt{2g} \tag{10}
$$

$$
\begin{bmatrix} h_3(s) \\ h_6(s) \end{bmatrix} = \begin{bmatrix} \dfrac{x_1 c_1 e^{-0.9s}}{\prod_{i=1}^{3}(\tau_i s+1)} & \dfrac{(1-x_2)c_1 e^{-0.3s}}{(\tau_3 s+1)} \\ \dfrac{(1-x_1)c_2 e^{-0.3s}}{(\tau_6 s+1)} & \dfrac{x_2 c_2 e^{-0.9s}}{\prod_{i=4}^{6}(\tau_i s+1)} \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \tag{11}
$$

with

$$
c_1 = \frac{2\sqrt{h_{3s}}}{R_3}, \quad c_2 = \frac{2\sqrt{h_{6s}}}{R_6} \tag{12}
$$

$$
\tau_i = \frac{2A_i\sqrt{h_{is}}}{R_i} \tag{13}
$$

When the sum of x_1 and x_2 is greater than one, the system has a RHP-zero. If $x_1+x_2=1$, the system has a zero located at the origin and as greater goes this sum, the zero is moved away of the origin along the positive axis.

Table 1: Process Parameters.

Parameters	Value
D_1 D_4 [cm]	35
D_2 D_5 [cm]	30
D_3 D_6 [cm]	25
R_1 R_4 [$cm^{2.5}min^{-1}$]	1690
R_2 R_5 [$cm^{2.5}min^{-1}$]	1830
R_3 R_6 [$cm^{2.5}min^{-1}$]	2000

Table 2: Operating Points.

Variables	OP 1	OP 2	OP 3	OP 4
h_3 [cm]	4.8400	17.0156	8.4100	11.7306
h_6 [cm]	3.2400	11.3906	8.1225	5.4056
F_1[L/min]	4	7.5	4	7.5
F_2[L/min]	4	7.5	7.5	4
x_1,x_2	0.7,0.6	0.7,0.6	0.7,0.6	0.7,0.6
RHP-zero	1.0246	0.1915	0.3818	0.3158
y_z (output zero direction)	$\begin{bmatrix} 0.68 \\ -0.79 \end{bmatrix}$	$\begin{bmatrix} 0.58 \\ -0.81 \end{bmatrix}$	$\begin{bmatrix} 0.52 \\ -0.85 \end{bmatrix}$	$\begin{bmatrix} 0.69 \\ -0.72 \end{bmatrix}$
u_z (input zero direction)	$\begin{bmatrix} -0.58 \\ 0.59 \end{bmatrix}$	$\begin{bmatrix} -0.77 \\ 0.63 \end{bmatrix}$	$\begin{bmatrix} -0.65 \\ 0.75 \end{bmatrix}$	$\begin{bmatrix} -0.85 \\ 0.51 \end{bmatrix}$

Table 1 shows the parameters used in the model, while Table 2 summarizes the steady-state and operating conditions of the studied OPs.

The four OPs have different dynamics and RHP-zero. The model 2 (M_2) is considered as the nominal model and it has the slowest dynamic. The model 1 (M_1) is the critical point OP, since the dynamic differs on most.

This process is difficult to control due to the time delay and the RHP-zero (which limit the achievable closed loop performance making the response slower). Figure 5 shows the step response to the models. The RGA (*Relative Gain Array*) in the channel (1,1) from all models is equal to 1.4 indicating some interaction and the correct choice to decentralized project designs.

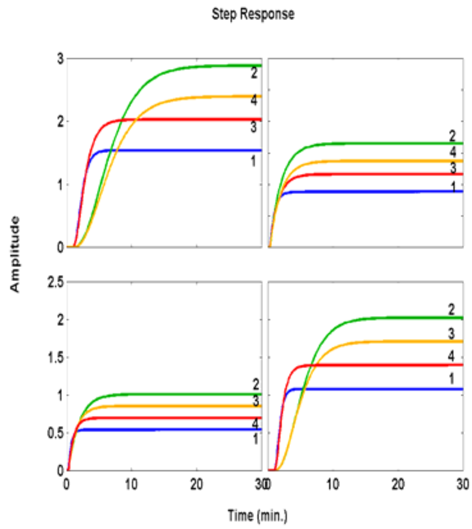

Fig. 5: Step Response of the Models.

4. RESULTS

Table 3 shows the desired performance applied to design the controllers. In the frequency domain, the RHP-zeros (and pure time delays) constraint the bandwidth up to which effective disturbance attenuation is possible. The largest bandwidth is determined by the RHP-zero closest to the origin (0.1915 in this case). The performance was established considering the limitation imposed by this zero, making the performance as fast as it is possible.

Table 3: Desired Performance (T_d).

Characteristic	Td
Rise time [y1,y2] (min.)	10,7
Overshoot %	10,10

The model must be factorized since it has an RHP-zero and time delay. The zero with the same output direction and the factorable time delay must be present in the closed loop transfer function to keep the internal stability of the feedback system. The time delay that cannot be factored out is approximated by Padé. For a second order Padé approximation, the zero is moved to 0.1731 indicating that the nonfactorable time delay have a unfavorable, but not significant, influence on the system controllability.

To analyse the controller's performance it was used a set point change (servo problem) in opposite directions, which is the worst situation than the controller can face according with the output zero direction as shown in Table 2. Similarly it was used as regulatory problem the unitary change to a at u_1 and -a at u_2 according the input zero direction.

It was designed three full controllers to the nominal model with the desired performance from Table 3. The simulation is presented in Figure 6. PID +F2 is used to indicate a PID with a second order filter.

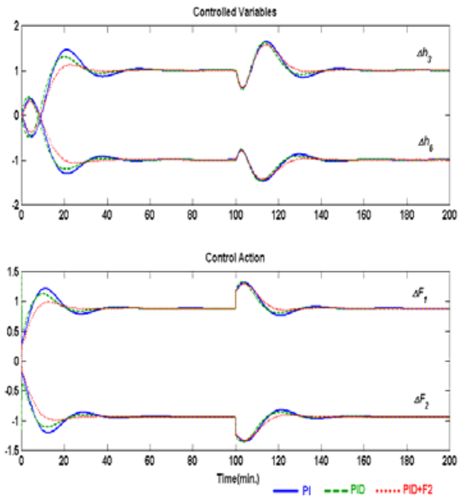

Fig. 6 Servo (r=[1 -1]) and regulatory (d=0.4.[1 -1]) response to the centralized controllers PI, PID,PID+F2 for the nominal model using the desired performance Td.

The figure shows how the RHP-zero can limit the speed of the control loop. Even the PI controller can present less overshoot making the controller slower. On the other hand, the increase of the order has a stabilizing effect on the performance. It allows doing the controller faster without harming its performance

In figure 7, three decentralized controllers were designed to the nominal model using the same performance. In this case the order increase has less effect because the PI controller shows a slow, but satisfactory, performance. Comparing these results with figure 6, it can be concluded that due the interaction (as indicated by a RGA analysis) the decentralized controllers with the same order are slower and presents a larger interaction, even though present good results.

The best controllers (decentralized/full) designed to the nominal model was simulated using the model linearized at the OP_1 (the smallest gain) and it indicated a poor performance (very slow).

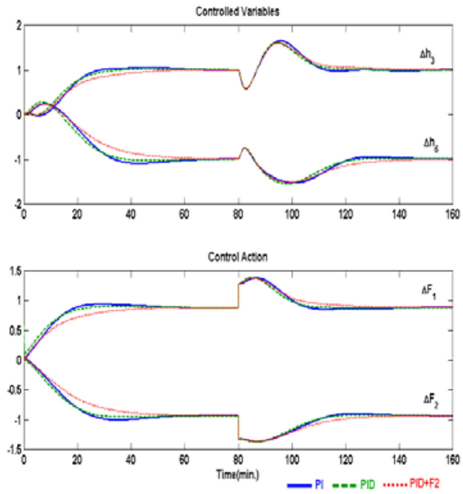

Fig. 7 Servo (r=[1 -1]) and regulatory (d=0.4.[1 -1]) response to the decentralized controllers PI, PID, PID+F2 to the nominal model using the desired performance Td.

Figure 8 shows the performance to the nominal model using all four models with the same weight to design three full controllers.

This is the problem choosing the critical case (high gain) to design only one controller to the whole process. The performance in the region of low gain can be made very slow, but not use the critical point can affect the stability making the process unstable in the high gain region.

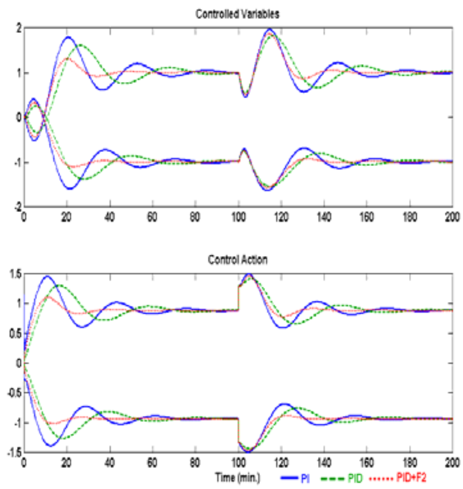

Fig. 8 Servo and regulatory response of the nominal model with the controller designed with all four models.

The results show that the increase of the order has more stabilizing effect when the controller is designed for multi-model case than the nominal model one. Also, it is important to realize that increasing the order of the controller using the polytope make the performance slow, but it is more significant in the region of high gain (M_2).

It was selected two full controllers that shown the best performance using the nominal model (C_n) and the polytope (C_p) respectively. This controller in both cases were a PID with a second order filter controllers.

Figure 9 compares the results to a servo response of C_n and C_p. The polytope controller is faster in all OPs, and it shows a performance as good as the nominal controller even in the nominal OP.

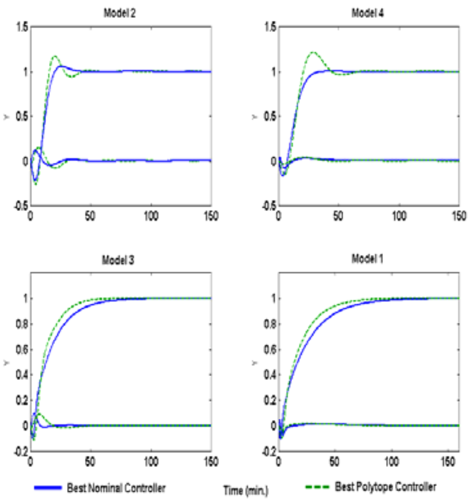

Fig. 9: Step response of C_n and Cp to all OPs.

Both controllers were simulated with the nonlinear model. The simulation starts in the OP2 and the process is changed to the OP4, OP1, OP3 successively until the time of 400 minutes where a set point change in opposites directions (the worst situation that the controller can face it) and at the time 500 the values of x1 and x2 are inverted (x1=0.6 and x2=0.7) and so the process return to the OP2. The simulation results are shown in figure 10 demonstrated that the performance of the polytope controller is better than the nominal controller to the most changes because the first one consider all the OPs into the design. Also, choosing the weights allows improving the performance in a given region.

In table 4 are presented the parameters of C_n and C_p. The equation used to derivative action is given by

$$C_{PV}(s) = C_{SP}(s) = \frac{T_D s + 1}{\alpha T_D s + 1}(series) \qquad (13)$$

Table 4: Controller's parameters.

Parameter	Controller i \ j	C_n/C_p 1	2
K_P	1	0.178/0.022	-0.226/-0.194
	2	-0.159/-0.073	0.112/0.038
T_I	1	3.130/0.339	6.126/4.362
	2	6.164/2.502	1.5926/0.463
T_D	1,2	0.993/2.554	0.994/0.990
T_F	1,2	$\begin{bmatrix}0.93\\0.41\end{bmatrix} / \begin{bmatrix}0.93\\0.49\end{bmatrix}$	$\begin{bmatrix}0.82\\0.29\end{bmatrix} / \begin{bmatrix}0.97\\0.49\end{bmatrix}$
λ	1,2	1.054/0.873	

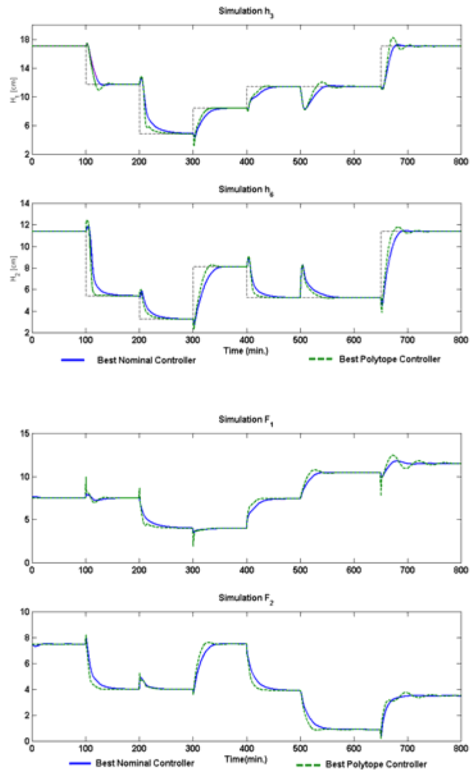

Fig. 10: Nonlinear simulation with C_n e C_p.

5. CONCLUSIONS

It was presented a fast and efficient method to design and to evaluate alternative multivariable control structures. The methodology is very flexible, allowing its use even in complex process with RHP-zero and time delay. Moreover, the controller designed with all OPs can provide a trade-off between the performance and robustness.

REFERENCES

Faccin, F. (2004). Uma abordagem inovadora para o projeto de controladores PID (In Portuguese). Master Thesis. Federal University of Rio Grande do Sul, Chemical Eng. Department, Brazil.

Faccin &Trierweiler (2004), A Novel Tool for multi-model PID Controller Design. . In: Dycops- (7th IFAC Symposium on Dynamics and Control of Process Systems), 2004, Boston. CD: Dycops 2004, 2004. v. 1. p. 176-186.

Farenzena, M.; Trierweiler, J.O. (2004), System Nonlinearity Measurement Based on the Rpn Concept. In: Dycops2004 (7th IFAC Symposium on Dynamics and Control of Process Systems), 2004, Boston. CD DYCOPS 2004, 2004. v. 1. p. 181-191

Trierweiler, J.O., R. Müller and S. Engell (2000), Multivariable Low Order Structured Controller design by frequency response approximation. In: *Brazilian Journal of Chemical Engineering* Vol.17, No 04-07, pp 793-807, Brazil.