THE ROLE OF CONTROL IN DESIGN: FROM FIXING PROBLEMS TO THE DESIGN OF DYNAMICS

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Abstract: We will advocate the need to change the role of dynamics and control community from fixing problems related to the detrimental dynamics using active control to the design for beneficial dynamics early in the design cycle. We will summarize lessons learned in industrial research on mitigation of flow and structure oscillations in jet engines. We will show how the decisions on the control system architecture (sensor and actuator location) impact the achievable level of suppression of oscillations (fundamental limitations of performance). Attempts to introduce control late in the design process and without proper attention to control architecture often fail because of high cost to modify the design to add on active control. We will also show how certain aspects of design (symmetry) contribute to the origin of detrimental oscillations and point out how the dynamical systems and control theory methods can guide the design to prevent the oscillations.

Keywords: Fundamental limits, nonlinear control, describing functions, combustion

1. INTRODUCTION

In this paper we will review the typical role of dynamics and control communities in the design cycle of new products and advocate the need to change this role from mainly *reactive* to strongly *proactive*. Case studies in active and passive control of oscillations in jet engines will be used to illustrate both the current and the proposed use of dynamics and control methods as well as the role of dynamics and control experts in developing technologies applicable to jet engines. While author's experience was restricted to jet engines, we hypothesize that the assessment of the current role of the control and dynamics communities and merits of the proposed new role apply broadly across multiple industries.

Dynamical phenomena such as transients and oscillations strongly affects operation of most devices. Active control is often used to modify the dynamics late in the design cycle. However, the dynamics and control communities play relatively insignificant role in the early design cycle for new products. Since not enough attention is paid in early design to the dynamic characteristics of the product, it is often discovered late in the design cycle, when the first prototypes are built and tested, that the dynamic properties of the product are not acceptable. To fix the dynamics problem a costly recovery process is launched. Often only at this point the dynamics and control experts are invited to participate. However, at this stage the design modifications required to modify the dynamics with control are extremely constrained by the hardware already built, cost, and schedule. As a result an active control solution is rarely accepted and a more practical passive control solution is sought for. Independently of whether an active or passive control solution is selected, its implementation cost at this late stage is much higher than a cost of a similar solution if it were implemented early in the cycle. In this paper we advocate the need for a *Design of Dynamics*, which amounts to an early introduction of dynamics and control methods in the design process to properly address the dynamic characteristics of products.

The paper is organized as follows. We begin with an overview of the current role of dynamics and control communities in the design. We argue that the dynamics and control communities are typically *reactive*, *excluded* from the process of selection of control architecture and a model, biased towards active control solution that is *external* to the product (extra hardware), and with *narrow* focus on the design of control algorithm. We provide case studies in control of flutter and thermoacoustic instabilities that point out the consequences of such behavior. We will show how lack of participation of the control experts in the process of selection of control architecture can lead to an intractable control design problem because of fundamental limitation of performance. We will also show how *engaging* in the process of selecting a control architecture and a model leads to an improved control system with clear understanding of the physical factors that fundamentally limit the control performance. Next, we will show on an example of analysis of wave phenomena in jet engines how manipulation of the natural physical feedback loops in the product can lead to a solution with minimal modifications to the product. We will argues that the dynamics and control experts need to be more *proactive*, *engaged* in the design process, and considering *broad* range of solutions with a preference toward these *internal* to the product. We will conclude by indicating some *technical* and *social* barriers that need to be overcome before the Design of Dynamics is introduced into industrial practice.

2. CURRENT ROLE OF DYNAMICS AND CONTROL IN DESIGN: FIXING PROBLEMS LATE IN DESIGN CYCLE

Despite decades of extensive research detrimental oscillatory wave phenomena still drive jet engines development and maintenance costs and limit their operability. Jet engines are designed for high performance, survivability, operability, and affordability. However, design for increased performance and low observability often leads to excitation of detrimental wave phenomena such as flutter, rotating stall, and thermoacoustic instabilities that reduce engine parts life and limit its operability. In particular, compressor and fan blade failure is still a common problem for all major engine manufacturers in spite of decades of extensive research in the area of design for flutter and High

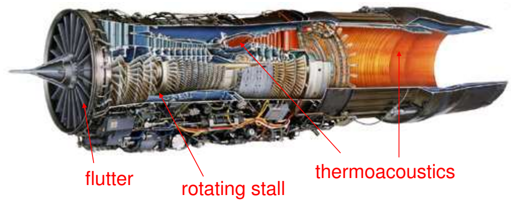

Fig. 1. Detrimental wave phenomena affecting operations of jet engines

Cycle Fatigue mitigation. While first discovered in 1950-ties, augmentor screech and rumble as of today are still common problems for military engines.

Mitigation of detrimental wave phenomena in jet engines is difficult since it involves controlling sensitive and complex dynamics in presence of model uncertainty and severe design constraints. Lightly damped structural, acoustic, and fluid dynamic modes easily become under-damped or unstable because of positive feedback coupling with other flow phenomena and excitation by broad band, and tonal flow disturbances. Physics that causes detrimental oscillations involves complex high Mach and Reynolds number flows, which cannot be reliably and accurately computed with current methods. State of the art computational methods based on CFD are too computationally expensive to be applied. Hence, model analysis is not utilized to exploit the design space and find innovative damping solutions in early design stage. The accuracy of reduced order models utilized is doubtful especially when chemical reaction, flow separation, or shocks are involved. Because of high uncertainty, models are not utilized for a design of robust oscillation mitigation solutions and robustness of chosen solutions to unexpected off-design conditions is not guaranteed. Avoiding oscillations by operating engines at regimes with large stability margins results in unacceptable performance loss.

Passive dampers can be used to control oscillations, but they undesirable additions, since they increase engine weight and complexity. Moreover, the positive effects of the passive damping devices are only utilized at a small portion of flight envelope when instabilities occur, but the negative aspects (like weight) impact engine performance at all operating conditions. Hence, active control is often considered as an alternative to passive dampers. Both passive and active control solutions are typically introduced late in the design cycle as a reaction to dynamics problem discovered when first prototypes are build and tested. The active control community typically becomes engaged late in the design process when the design process owners recognize possibility that active control can solve the dynamics problems. Very often the control experts do not participate in creating the model used for control

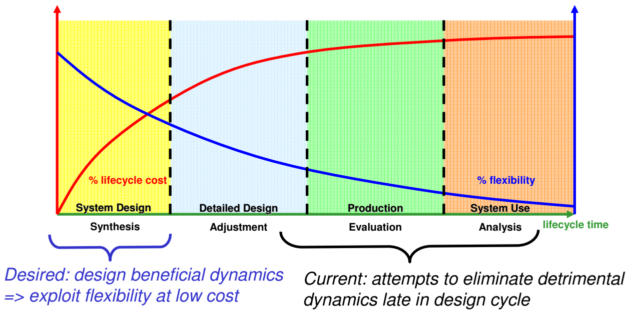

Fig. 2. Impact of dynamics and control methods on the design cycle

design and in the design of the control architecture, limiting its role to designing the control algorithms. In

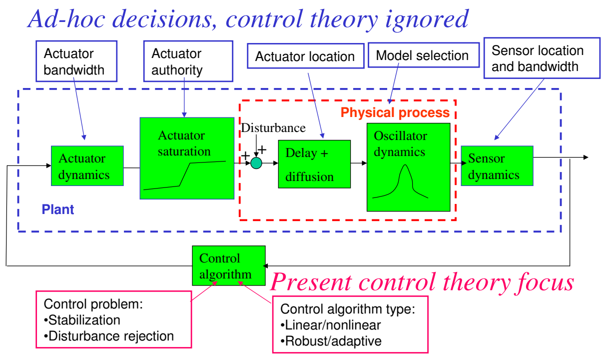

Fig. 3. Decisions that impact control performance

this paper we will show in examples how a selection of a control architecture that ignores the principles of control theory can lead to an intractable control problem in which the desired control performance cannot be achieved because the achievable control performance is *fundamentally limited* by the control architecture and physics of the problem.

The dynamical systems academic community has created tools to *analyze* the dynamics if low dimensional models are available, but not to *design* the beneficial dynamics. This community is typically not engaged with industrial processes missing a significant opportunity for impact. In this paper we will show an example how, by using ideas from the theory of the *dynamical systems with symmetry*, one can identify the root causes of detrimental dynamics and how one can create *beneficial dynamic interactions* that eliminate the detrimental dynamic behavior.

Let us summarize here some attributes of the *current* role that the dynamics and control communities play in the jet engine design process. First, they act *reactively*. They will only act when called upon by the design process owners. This usually means a late entrance into the process, when the acceptable solutions are extremely constrained. Secondly, the control experts will rarely attempt to analyze the natural dynamics of the problem and solve the problem by manipulation of the natural dynamics. Instead, they will typically look for an active control solution which is *external* to the product, i.e., requires adding extra sensors and actuators. The solutions obtained in this way attempt to *override* the natural dynamics and require a nontrivial modifications of the design. In particular, it requires extra hardware, which means extra cost and complexity. This is always the least desirable solution. Third, they will often *accept assumptions* about the control problem definition, proper control architecture, and model of the process made by the process owners without questions. Given the model, the sensors, and the actuators defined by the design process owners, the control engineers will quickly proceed to the design of a control algorithm and its experimental verification. At best such behavior can result in expensive active control solution if the assumptions made by process owners are correct and the extra hardware addition is acceptable. However, since the assumptions that led to the definition of an active control concept were made without involving control experts, they often are incorrect and the active control solution does not satisfy the performance requirements. Even when these assumptions are corrected and the active control performance is acceptable, the active control approach often cannot meet the acceptable criteria in term of cost or complexity and is abandoned in favor of a cheaper and easier to implement passive control solution if the latter is found.

3. THE IMPORTANCE OF PROPER CHOICE OF CONTROL ARCHITECTURE

In this section we will describe an active control project in which initial lack of team play between the design process owners and the control engineers resulted in a failure of the project to achieve an acceptable control performance. After this initial failure, a close collaboration of the design process owners with control engineers was established, which resulted in a discovery of a superior control architecture and demonstration of an excellent control performance.

Despite advances in aeromechanical engineering, fan stall flutter (Forsching, 1984) remains a substantial constraint in jet engine designs. The motivation for the work described in this section was to investigate the extent to which active control of this aeromechanical instability can extend the operability of a given fan design. The control objective was damping augmentation of the flutter modes.

The details of modeling, control design, and experimental demonstration of active flutter control are summarized in the papers (Banaszuk *et al.*, 2002*a*; Banaszuk *et al.*, 2002*b*; Rey *et al.*, 2003).

The experimental setup shown in Figure 4 contains a 17 inch scale fan with flow characteristics and flutter margin comparable to of those found in high by-pass ratio commercial jet engines. The first attempts to provide flutter damping augmentation involved using

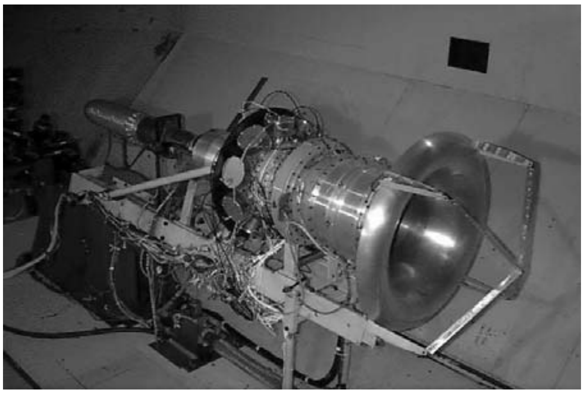

Fig. 4. 17” fan experimental rig

pressure sensors and five circumferentially located valves as actuators. This architecture was chosen by the turbomachinery experts responsible for the the experimental demonstration of a high performance flutter control system.

An active control algorithm was supposed to be designed using a model extracted from a frequency response of the pressure sensors. A typical frequency response with the pressure sensors is shown in Figure 5. Note that the lightly damped flutter pole represented by a spike in the magnitude response around 273Hz is accompanied by a zero only 1Hz apart. Such proxim-

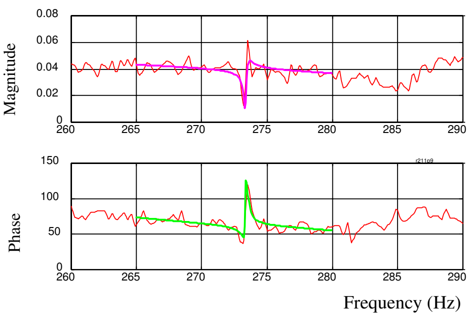

Fig. 5. Flutter frequency response using pressure sensors

ity of zero to the flutter pole resulted in a very difficult control problem. Four control engineers made four separate attempts using different control algorithm design techniques to provide damping augmentation for the 273Hz flutter pole. Since the close proximity of pole and zero indicates severe fundamental limitations of achievable control performance, the damping augmentation achieved was insignificant. Even though several pressure sensor configurations were tested, all sensor configurations resulted in a near pole/zero cancelation.

The failure of the attempts to control flutter using pressure sensors could be explained by an inadequate design of the control architecture (namely selection of sensors and actuators), which resulted in a pole/zero configuration that fundamentally limited achievable control performance. The turbomachinery experts who designed the architecture used their physical intuition, controllability and observability of flutter being the only control aspects that were analyzed properly. They were unaware of an importance of a proper design methodology leading to an architecture that avoids a near pole/zero cancelation. While the control experts understood the detrimental influence of a near pole/zero cancelation on the control performance, they were not involved in the design of the control architecture, simply because they did not insist on a participation in the control architecture design process.

The root cause of a near pole/zero cancelation was discovered during a discussion involving both turbomachinery and control experts that lasted only one and half hour. It was postulated that a strong direct feedthrough from the actuators to the pressure sensors dominating the pressure response is the root cause of a near poles/zero cancelation. When a large direct feedthrough term is added to a smaller transfer function representing flutter, it is easy to show (by combining the terms into one simple fraction) that a near pole/zero cancelation will occur. Linking the origin of a near pole/zero cancelation to the physics of the problem was the key development. The turbomachinery experts started to appreciate the value the control theory methods and became strong promoters of the active control methods.

Another discussion led to an identification of an easily implementable sensing approach that eliminated the direct feed-through. With the new eddy current sensors to measure the blade time arrival the direct feedthough was completely eliminated and zeros close to poles were removed (Rey *et al.*, 2003). Figure 6 shows the comparison of the frequency responses of flutter dynamics using the pressure and eddy current sensors.

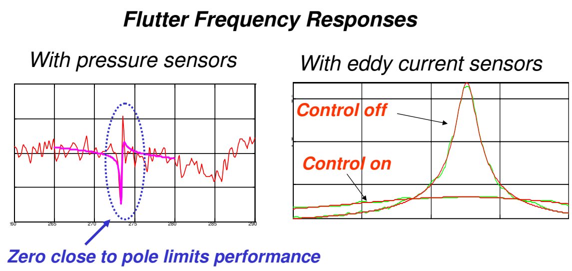

Fig. 6. Flutter magnitude response using pressure and eddy current sensors

The final active control hardware consisted of ten zero mean mass flow actuators equally spaced around the fan case between the blade row and the exit guide vanes. The actuators consisted of regular audio speakers enclosed in a pressure vessel. Flutter was sensed by means of eddy current sensors mounted on the fan casing. As blades speed past these sensors, the sensed signal is used to record the blade arrival time. Early and late arrivals are associated with combinations of forward and backward bending and twisting of the blades from which the flutter modes amplitudes can be derived in real-time.

Figure 7 shows a schematic of the control system. The "Inverse DFT" block in the diagram performs an inverse spatial Fourier Transform which converts each flutter control signal into an actuator command according to the position of the actuator along the circumference. The amplitude of the mass-flow corresponds to that of a sine-wave of the nodal diameter and phase speed of the traveling wave.

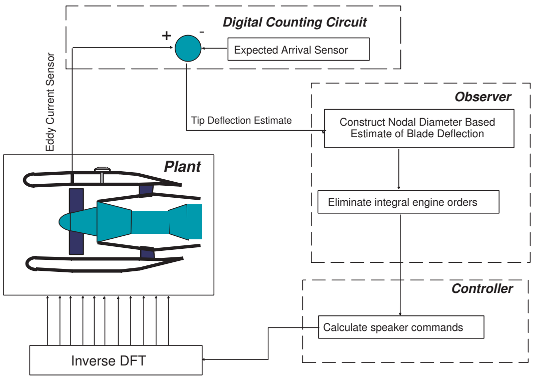

Fig. 7. Flutter control system schematics

The control system was able to add damping to the three critical flutter modes. The damping augmentation achieved was an order of magnitude larger than the intrinsic aeromechanical damping of the flutter modes at the design point. More details on the flutter control algorithms used in this work can be found in (Banaszuk *et al.*, 2002*b*).

Figure 8 shows a summary of the damping augmentation achieved for 0, 1 and 2 nodal diameter flutter of the blade first bending mode. Notice that the range of damping values for the open loop system between the design point (label "A") and the flutter boundary (label "B") is much smaller than the amount of damping added through active control.
Unfortunately, even though feasibility of active control of flutter with off-blade sensors and actuators was demonstrated in a rig, the technology did not make it to the product it was supposed to impact. A fan blade redesign resulted in an elimination of the dynamics problem on the product, and hence active control solutions was no longer needed.

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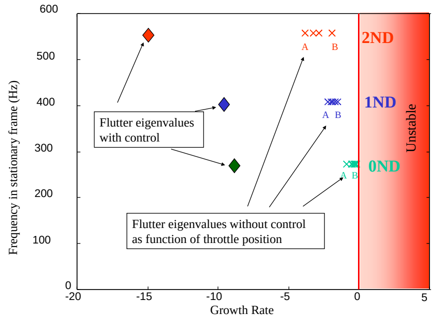

Fig. 8. Summary of flutter control experiments

of a superior control architecture and an experimental demonstration of an excellent control performance. The key factor of the success was an assumption of a *proactive* role by the control experts. Despite this technical success, the active control technology was abandoned in favor of a passive control solutions that was *internal* to the product.

4. THE IMPORTANCE OF CORRECT MODELING ASSUMPTION

In this section we will describe an active control project in which a *wrong modeling assumption* that the dynamics being controlled could be represented as a linearly unstable limit cycling system led to inadequate definition of control objective as a stabilization problem. These wrong assumptions resulted in a failure of the project to explain the poor performance achieved in some of the active control experiments. Eventually, a rigorous analysis revealed the faulty assumptions. A new assumption was stated that the dynamics should be modeled as a noise-driven system with a large control delay. The controlled system can be either stable or unstable depending on the particular values of the parameters. The presence of a large broad-band disturbance driving the system implies that a proper control objective is a disturbance attenuation, rather than stabilization. In addition, the presence of a large delay in the control path causes the achievable control performance to be fundamentally limited. This in turn explains a poor control performance observed in some control experiments. For more details we refer to papers (Banaszuk *et al.*, 1999*a*; Banaszuk *et al.*, 1999*b*; Cohen and Banaszuk, 2003; Mezic and Banaszuk, 2004).

Emphasis on reducing the level of pollutants created by gas turbine combustors has led to the development of premixed combustor designs, especially for industrial applications. Premixing large amounts of air with the fuel prior to its injection into the combustor greatly reduces peak temperatures within the combustor and leads to lower NOx emissions. However, premixed combustors are susceptible to the so-called thermoacoustic combustion instabilities. These instabilities arises due to a destabilizing feedback coupling between acoustics and combustion (unsteady heat release). It causes large pressure oscillation in the combustor that detrimentally affects the combustor durability and raises environmental noise pollution (Seume *et al.*, 1997).

Active Combustion Instability Control (ACIC) with fuel modulation has appeared an effective approach for reducing pressure oscillations in combustors. Promising experimental results have been reported by researchers at United Technologies Research Center (UTRC) (Cohen *et al.*, 1998; Hibshman *et al.*, 1999), Seimens kWU (Seume *et al.*, 1997; Hoffmann *et al.*, 1998), ABB/Alstom (Paschereit *et al.*, 1999), Honeywell Inc. (Anson *et al.*, 2002), Westinghouse/Georgia Institute of Technology (Sattinger *et al.*, 1998), and the U.S. Department of Energy (Richards *et al.*, 1995). However, the achieved reduction of pressure oscillation varies between these experiments from 6dB to 20dB. In many cases, the attenuation of the oscillation at primary frequency is accompanied by excitation of the oscillation in some other frequency band (Langhorne *et al.*, 1988; Fleifil *et al.*, 1997; Saunders *et al.*, 1999). This phenomenon is commonly referred to as *secondary peaking* or *peak splitting*.

A satisfactory explanation of the different attenuation levels and peak-splitting phenomena has not been presented in the literature. Much of the theoretical attention in the area of ACIC has focused on control design (Bloxsidge *et al.*, 1987; Bloxsidge *et al.*, 1988; Langhorne *et al.*, 1988; Chu *et al.*, 1998; Hathout *et al.*, 2000; Evesque *et al.*, 2000) – that is inherently dependent on the dynamics considered in the model or present in the experiment – and not so much on factors that actually limit the achievable performance. One of the reasons for this is that the thermoacoustic oscillations frequently arise as a limit cycle that requires nonlinear models of combustion dynamics. This limits the mathematical tools available for both control design as well as the analysis of resulting dynamics.

We investigated the factors that determined achievable reduction of the level of pressure oscillation in combustors using fuel control. Our studies have been motivated by experience with ACIC in the experiments conducted at UTRC (Cohen *et al.*, 1998; Hibshman *et al.*, 1999). These experiments were done in sub-scale single nozzle combustors.

An industrial engine is equipped with an annular combustor comprising of several premixing fuel nozzles arranged along the circumference. The ACIC experiments used sector embodiments of the annular combustor. Figure 9 depicts a four megawatt single-nozzle combustor and a three-nozzle sector combustor. In either setup, experiments were carried out at realistic operating conditions and between 10-17% of the net fuel was modulated for control using linear proportional or nonlinear on-off fuel valves. Pressure sensors inside the combustor were used for feedback. Additional details on the experiments appear in (Cohen *et al.*, 1998; Hibshman *et al.*, 1999).

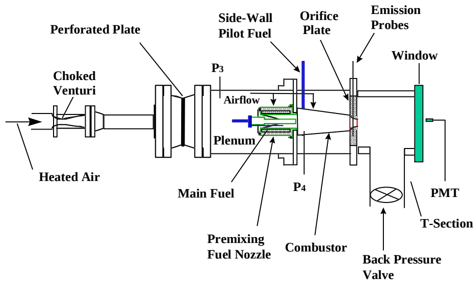

Fig. 9. UTRC single-nozzle 4MW combustor.

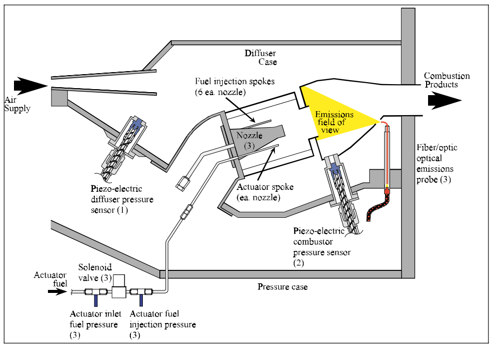

Fig. 10. UTRC three-nozzle sector combustor.

Combustion dynamics arise due to a feedback coupling between the acoustic modes of the combustor cavity and the unsteady heat released due to combustion of fuel-air mixture. The resulting feedback interconnection is typically referred to as a *thermoacoustic loop*. In the simplest setting considered here, the acoustics is modeled by the bulk Hemholtz mode of the combustor cavity. The precise physical mechanisms underlying the unsteady heat release are complex and reduced order models for the same are not well-understood. Here, the unsteady heat release was modeled as a fluctuation in the equivalence ratio (normalized fuel/air ratio) expressed as a nonlinear function of acoustic velocity input. Only the simplest two effects are considered to model the functional relationship. One is the bulk fluid convection effect that is modeled by a time delay and the other is the effect due to time-delay and nonlinearities in the burning rate where the latter that is modeled by a static saturation nonlinearity. The resulting thermoacoustic model equations arise as

$$\frac{d}{dt}\begin{bmatrix}\rho c u_i\\ \rho c u_e\\ p\end{bmatrix} = \begin{bmatrix}-M_i c_i/l_i & 0 & c_i/l_i\\ 0 & -M_e c_e/l_e & c_e/l_e\\ -A_i c/V & -A_e c/V & 0\end{bmatrix}\begin{bmatrix}\rho c u_i\\ \rho c u_e\\ p\end{bmatrix} + \begin{bmatrix}0\\ 0\\ H\left(u_i(t-\tau)+u_t(t), w(t-\tau)\right)\end{bmatrix}, \quad (1)$$

where p is the combustor chamber (modeled as a capacitance) pressure, $\rho c u_i$ is the upstream nozzle mass velocity, $\rho c u_e$ is the downstream exit mass velocity, $w(t)$ is the fuel mass flow input, and $u_t(t)$ is used to model the stochastic turbulent flow velocity in the nozzle – assumed to be a broad-band white noise. The heat release function $H\left(u_i(t-\tau)+u_t(t), w(t-\tau)\right)$ in (1) captures in a reduced order fashion the nonlinear effects due to combustion. The parameter τ represents the cumulative time delay – primary delay due to convection plus delay because of chemical reaction and fuel-air mixing. For additional details on the model and explicit characteristics of the forcing term, see (Peracchio and Proscia, 1998).

In the existing thermoacoustic literature a commonly accepted assumption was that a presence of peaks in the pressure spectra is an indication of a limit cycle. This assumption was adopted by the UTRC team that included combustion engineers, dynamical system experts, and control engineers. As a consequence of this modeling assumption the adopted control objective was a *stabilization of a linearly unstable limit-cycling plant*. A simple phase-shifting algorithm using pressure sensors was designed to control the fuel valves.

The amount of the pressure amplitude attenuation with control varied between the rigs and between various operating conditions. Figure 11 shows spectra of pressure without and with active fuel control. Note that 5.5x attenuation was achieved in the single-nozzle rig, while in the sector rig the attenuation was only 2x. The attenuation in the sector rig was limited by the peak-splitting phenomenon mentioned above. This result was puzzling, since the peak-splitting could not be easily explained using the limit-cycling plant assumptions.

Eventually a breaktrough was achieved when the limit-cycling model assumption was questioned. Methods presented in the paper (Mezic and Banaszuk, 2004) led to identification of regions of validity of linear and nonlinear models for thermoacoustic oscillations. An alternative hypothesis was formulated that the low amplitude pressure oscillations should be modeled using a *linearly stable, noise-driven model*. To verify this hypothesis, a control-oriented thermoacoustic models were identified by fitting the experimentally obtained frequency response from fuel valve input to the pressure sensor output. The frequency

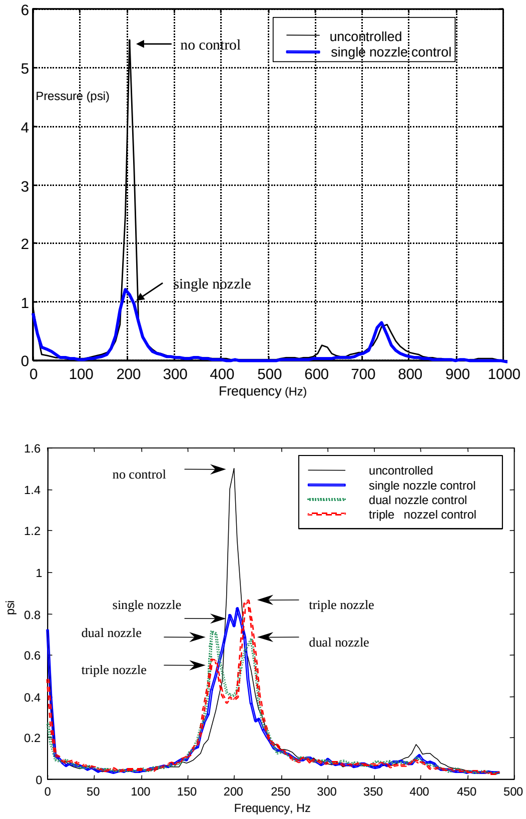

Fig. 11. Open and closed-loop pressure spectra for the single nozzle and sector combustors.

response experiments were carried out a in a range of operating conditions with both the single-nozzle and sector (three-nozzle) combustors. For the single nozzle combustor operating at the high equivalence ratio condition, a linear model consisting of a lightly damped second order system together with a (large) delay was found to fit the data well. In the following, we describe the identification and validation of the linearity hypothesis with this model.

At the high equivalence ratio condition, the pressure oscillations observed in the single nozzle combustor are relatively small and the proportional actuator used for control operates in its linear range. Therefore, it was hypothesized that a linear plant and controller model may be used to analyze the behavior of the controlled system. Figure 12 depicts the structure of the feedback control system. Figure 13 compares the experimentally obtained frequency response to it's model fit. The identified model arises as a second order lightly damped oscillator with a delay of $\tau = 4.4$ ms chosen to match the phase roll-off in the $300 - 400$ Hz frequency range. As the identified model is linear and stable, a model of external noise is needed to account for the pressure oscillations observed in the experiments. We use the model structure for the noise in Eq. (1) together with the identified model to estimate a noise model. In particular, a white noise model is built at the plant input (see Figure 12) to match

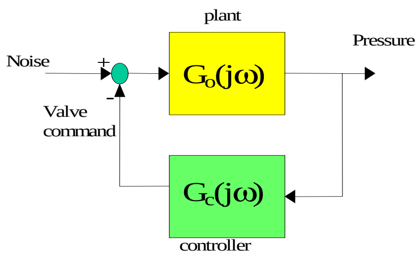

Fig. 12. Feedback control of thermoacoustic plant in the presence of noise: a pressure measurement is used to obtain the fuel valve control input.

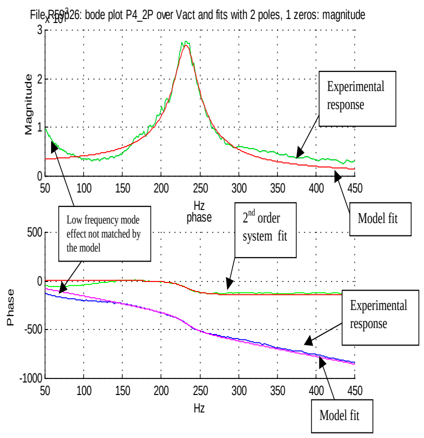

Fig. 13. Results of the linear model identification of the frequency response obtained in the single nozzle rig experiment.

the experimentally obtained PSD of the uncontrolled pressure. Figure 14 shows that the identified noise

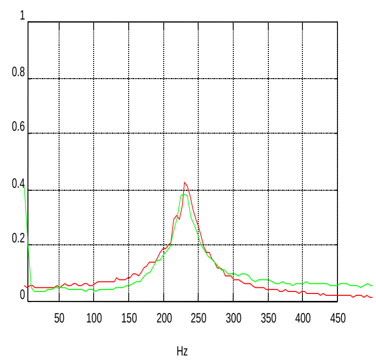

Fig. 14. Square root of the PSD of pressure from experiment and from model simulation.

model allows us to match the experimentally obtained pressure PSD with the results of the model simulations using SIMULINK. In the combustion experiment, a wide band turbulent air velocity fluctuation in the nozzle is one of the sources for the presence of noise. The identified plant model includes the fuel valve actuator dynamics together with the thermoacoustic model dynamics of Eq. (1). The frequency response of the actuator is effectively flat over a wide frequency band about the resonant thermoacoustic frequency ω_r. As a result, additional states are not needed and a second order model with delay consistent with Eq. (1) is sufficient.

Finally, feedback control experiments were used to validate the implicit linearity hypothesis and the noise model. An observer-based phase-shifting controller was used both in the experiment and in the model simulations. Figure 15 compares the experimentally obtained pressure PSD with the PSD obtained from simulations with various phase-shifting controllers. The fact that the two PSDs are nearly identical implies that out plant and noise models are valid and suitable for the control design at the high equivalence ratio condition in the single-nozzle combustor.

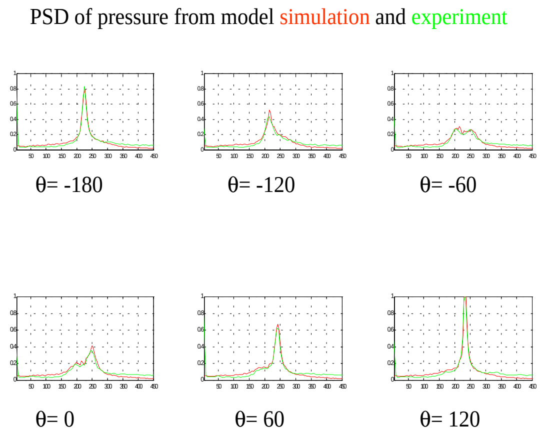

Fig. 15. Results of validation using feedback control: effect of a phase-shifting controller on pressure PSD in experiment and simulation.

The correction of the model assumptions led to an explanation of the peak splitting phenomenon and eventually to understanding of the fundamental limitations of the achievable control performance. This key technical contribution showed the value of the control theory methods to the design process owners (in this case the combustion engineers) and increased *credibility* of the control engineers among the combustion engineers. The process owners became open to learning the basic principles of the feedback control theory. Translation of the control theory principles to the language of physics was most important in breaking the *language barrier* between the dynamics and control group and the process owners.

5. FUNDAMENTAL LIMITATIONS OF PERFORMANCE

In this section we will discuss a relationship between the physics of a problem, the control architecture selection, and the achievable *control performance* using the thermoacoustic problem introduced in the previous section as an example. The performance limitations of the achievable suppression of oscillations will be described in terms of controller-independent lower bounds on the sensitivity function gain. The lower bounds will depend on the physics of the problem and the selection of control architecture. Since these factors cannot be analyzed independently, it will become mandatory that a high performance control architecture can only be designed by a team including the experts in control and in the physics of the problem. For more details we refer to papers (Banaszuk *et al.*, 1999*a*; Banaszuk *et al.*, 1999*b*; Cohen and Banaszuk, 2003; Mehta *et al.*, 2004).

In this section, the fundamental limitations associated with the feedback control of combustion instabilities are discussed. The theory is applied to obtain bounds on achievable performance in the high equivalence ratio experiments where linear plant and control models are adequate. In particular, the analysis helps explain the peak splitting phenomenon observed in UTRC and other ACIC experiments. In frequency domain, the closed-loop transfer function from the noise model to the pressure measurement is given by

$$\frac{p(j\omega)}{n(j\omega)} = G_0(j\omega)S(j\omega), \tag{2}$$

where

$$S(j\omega) = \frac{1}{(1 + G_0(j\omega)G_c(j\omega))} \tag{3}$$

denotes the sensitivity function. The control objective is to stabilize the closed loop system and shape the sensitivity function with the objective of reducing the noise driven pressure oscillation. In particular, the controller attenuates the noise at frequencies where $|S(j\omega)| < 1$ and amplifies the noise otherwise. Figure 16 depicts the experimentally obtained Nyquist diagram for the controlled single nozzle combustor. The attenuation and excitation frequency bands are also shown. The effect of the phase-shifting controller is to rotate the diagram so that the attenuation is maximized at the resonant frequency ω_r. The presence of a large delay in the loop makes it difficult to achieve broadband attenuation of pressure oscillations – the sidelobes in the diagram are the regions of secondary peaks. This observation in our closed-loop combustion experiments (Cohen *et al.*, 1998; Hibshman *et al.*, 1999) together with a wide range of performance results in the ACIC literature (Seume *et al.*, 1997; Hoffmann *et al.*, 1998; Paschereit *et al.*, 1999; Anson *et al.*, 2002; Sattinger *et al.*, 1998; Richards *et al.*, 1995) motivated us to study the fundamental limitations of ACIC. Our objective was to better understand – in a controller independent fashion – the effect of delay, limited actuator bandwidth and authority and plant dynamics (unstable poles) on the achievable performance and study the resulting trade-offs.

Fundamental limitations in obtainable performance (and robustness) are determined by certain conservation laws that govern the balance of negative and positive areas under the sensitivity (and complementary sensitivity) frequency response (Seron *et al.*, 1997; Freudenberg and Looze, 1987). These laws are used to obtain controller-independent bounds on performance and robustness with *any* LTI controller. For the sensitivity function, obtainable performance bounds can be derived from the celebrated Bode integral formula

$$\int_0^\infty log \mid S(j\omega) \mid d\omega = 2\pi\sigma_r, \tag{4}$$

where σ_r is the real part of the resonant unstable pole-pair; right-hand-side is zero for open-loop stable plant. The integral formula shows that noise attenuation (which requires $\mid S(j\omega) \mid < 1$) over a certain frequency band is always accompanied by noise amplification $\mid S(j\omega) \mid > 1$ over some other frequency band. (This is sometimes referred to as the waterbed effect.) In the presence of unstable poles, a larger penalty is paid in terms of sensitivity amplification. Figure 17 provides a graphical representation of the area formula: sensitivity reduction (negative area in the integral) is always accompanied by sensitivity amplification (positive area).

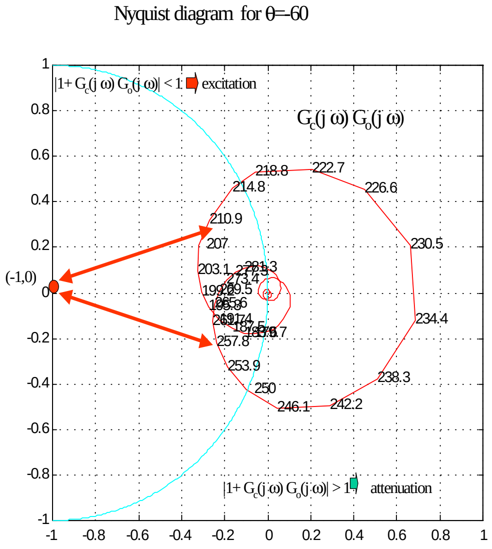

Fig. 16. Nyquist diagram for the phase-shifting controller with optimal phase shift.

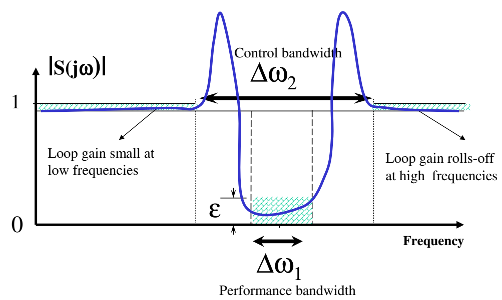

Fig. 17. A typical sensitivity function for control of oscillations

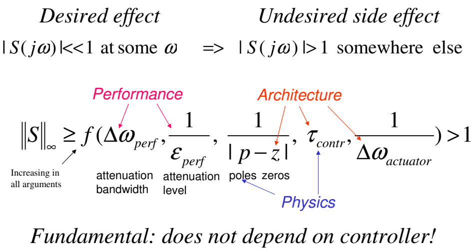

Fig. 18. Fundamental limitations of control performance showed as a lower bound on the sensitivity function gain

The performance objective for ACIC is to shape the sensitivity function so that it is small at and near the resonant frequency ω_r, i.e.,

$$|S(j\omega)| < \epsilon \tag{5}$$

for $\omega \in \Delta\omega_1$ where $\Delta\omega_1$ is the performance bandwith centered at ω_r. Meeting this performance objective creates negative area in the integral and this leads to noise amplification at some other frequencies. If the control bandwidth were infinite, the positive area may be distributed over a wide frequency band so amplification at any given frequency may be designed to be arbitrarily small. However, if the control bandwidth is finite (so the loop rolls off beyond certain low and high frequencies), the positive area would have to be accommodated in a smaller frequency band (where loop gain is high) and this would necessarily result in peaking of the sensitivity function.

In the industrial ACIC settings at UTRC, the linearity hypothesis and subsequent control-oriented analysis of the preceding section applies only to a limited set of operating conditions. For most operating conditions of practical interest, the linearity hypothesis is not applicable because of in the industrial settings, the high power requirements of fuel modulation due to control means that the actuator essentially operates in its saturated nonlinear range. Next, On-Off actuator is a popular and cheap fuel actuator that is widely used for ACIC. ACIC experiments in sector combustor (Hibshman *et al.*, 1999) used On-Off actuators. The resulting closed-loop feedback system was thus nonlinear. Experimental results obtained with a linear controller showed peak splitting for a range of operating conditions. Figure 19 depicts the PSD of the pressure oscillations with one, two, and three fuel nozzles operating.

In order to understand the nonlinear effects because of On-Off actuators, the operating conditions for the uncontrolled case are specifically chosen to verify the linearity hypothesis for the thermoacoustic model. A linear thermoacoustic and noise model are identified from experiments using the procedure described in the previous section The thermoacoustic model now includes a larger time delay of $\tau = 7$ ms and a second order linear system with resonant frequency $f_r = 208.9$ Hz.

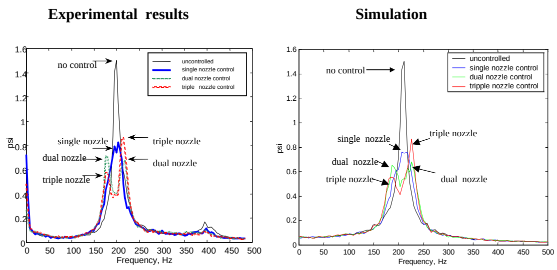

Fig. 19. PSD of pressure signal with on-off control of one, two, or three liquid fuel nozzles showing the peak splitting phenomenon observed in experiment and simulation.

In the models of the ACIC experiments with On-Off actuators, Gaussian balance using Random-Input Describing Functions (Gelb and Velde, 1968) yields an approximation of the feedback loop with respect to the Gaussian noise balance. The analysis in (Banaszuk *et al.*, 1999*b*; Cohen and Banaszuk, 2003) shows that the loop $G_1(j\omega)N_R(A(\sigma),\sigma)$ (where $N_R(A(\sigma),\sigma)$ denotes the Gaussian input describing function) yields a well-posed closed-loop system for all $\sigma \neq \sigma_0$. Note that this is the case independently of the dynamics of the open loop $G_1(j\omega)$, the amplitude of the limit cycle A, and the values of Gaussian process standard deviation σ. The resulting sensitivity function is stable and one can formally write down an area formula which gives peak splitting for the approximation. Under the assumption that the approximation yields a good representation of the nonlinear model, this explains the peak splitting seen in the PSD of the ACIC experiments with On-Off actuators.

The above considerations give a formal framework for extending the fundamental limitations analysis for control of thermoacoustic loops. One considers the *modified sensitivity function* with respect to the noise balance. Peak splitting is a consequence of the area formula as applied to the modified sensitivity function. For the case of On-Off nonlinearity with G_1 stable, we showed that the *modified sensitivity function* is stable and well-posed independent of the dynamics of G_1 and the noise (variance). We expect this to be true for a larger class of nonlinearities.

Analysis provided indicates that the peaking phenomenon observed in ACIC experiments is to a large extent inevitable for combustion systems with large delay controlled with actuators of limited bandwidth. This is reflected in the fact that the sensitivity with the linear actuator case or the modified sensitivity function with the nonlinear On-Off actuator achieves values exceeding 1.

We also used the analysis to explain the difference between the experimental results obtained in single-nozzle and sector combustors (see Fig. 11). Compared to the sector combustor, the single nozzle combustor shows higher open-loop oscillations but in a narrower band of frequencies. Such is the case because of lower damping of the thermoacoustics in the single nozzle combustor. Next, the plant delay identified from the frequency responses is higher in the sector combustor than in the single-nozzle combustor. As a result, limitations and peaking in the sector combustor – with its broadband performance objective for a thermoacoustic plant with large delay – is more severe than in the single nozzle combustor.

In this section we discussed a relationship between the physics of a problem, the control architecture selection, and the achievable *control performance*. Since these factors cannot be analyzed independently, a high performance control architecture can only be designed by a *team* including the experts in control and the physics of the problem. Such teamwork helps articulate the value of analytic methods of control theory and greatly facilitates breaking the *language* barrier between the control theory expert and the design process owners. The best possible outcome of this process is convincing the design process owners that the analysis methods of dynamical systems and control theory are an efficient way of exploiting the physics of the problem.

6. DESIGN OF BENEFICIAL DYNAMIC INTERACTIONS

In this section we will discuss how using methods of dynamics and control to analyze the natural dynamics of the product can lead to a solution of a dynamics problem that is *internal* to the product, and hence easily implementable without necessity of adding extra hardware and complexity. Such solution can only be found if the experts in the physics of the problem and the dynamics and control experts work together as a *team*. Since such team work is not a natural act, it requires prior establishment of credibility and breaking of the language barrier. Working on the active control projects such as flutter and thermoacoustics described in the previous sections can greatly facilitate creation of such a team, even if the active control technology is not implemented on a product.

In (Hagen and Banaszuk, 2004) we examined how spatial variations of the system parameters can affect the system stability properties. Recent work has focussed on analysis of heterogeneous distributed systems (Dullerud and D'Andrea, 1999; Hagen, 2004; Jovanovic *et al.*, 2003). Symmetry-breaking is commonly referred to as mistuning in the literature regarding the dynamics of arrays of turbine blades on a disk. Studies of stability properties of turbine blade flutter through the introduction of spatial nonuniformities has appeared in (Bendiksen, 2000; Rivas-Guerra and Mignolet, 2003). Optimal mistuning in arrays of bladed disks has appeared in (Petrov *et al.*, 2000; Shapiro, 1998). A study of the effects of asymmetry on compressor stall inception has appeared in (Graf *et al.*, 1998).

As in the case of mistuning in arrays of bladed disks in turbines, this form of passive control is often more feasible than implementing an active control scheme. This may also be true for the case in combustion chambers, where high temperatures prohibit adequate sensing and may damage the actuators required for active control. Furthermore, symmetry-breaking can be a more cost-effective means of stability enhancement.

Within recent years at UTRC the analysis of the role of jet engine design symmetry in the dynamics of detrimental rotating waves led to explanation of the origin of the waves and practical means of their passive control demonstrated in an engine test. It is worth pointing out that these developments were inspired by Igor Mezic analysis of the impact of the symmetry structure of DNA molecules on DNA dynamic behavior (Mezic, 2005) that provided an inspiration to the authors of the current paper that led to a discovery of the beneficial and detrimental symmetry patterns in jet engines. This key inspiration ultimately led to the concept of the Design of Dynamics for the wave phenomena in jet engines described in this paper.

Oscillatory phenomena such as thermoacoustic instabilities and turbomachinery fan blade flutter could be modeled using wave equation with a nonlinear dynamic feedback representing coupling of lightly damped acoustic or structural waves with flow or combustion. An elegant explanation of the role of jet engine design symmetry in the inception and suppression of instabilities such as thermoacoustics and flutter was provided. The explanation does not require any particular physics-based model for combustion and flow phenomena, because it only utilizes its symmetry properties. In particular, it was shown that the so-called skew-symmetric feedback is always detrimental, while breaking the symmetry of the circumferential wave speed pattern is always beneficial. The research led to a methodology for designing engines with greater dynamic stability margins that was transitioned to an engine company. The effectiveness of symmetry breaking in quenching detrimental rotating wave oscillations was demonstrated in a full-scale engine test.

To derive results on stability, it was shown that under the assumption of identical feedback elements (identical combustion flameholders, identical fan blades, etc.), any feedback model can be decomposed as a sum of symmetric and a skew-symmetric feedback. Conceptually, the symmetric feedback corresponds to dynamics that have reflection (about centerline) symmetry while the skew-symmetry is a result of local asymmetry in feedback. The symmetric feedback causes the two eigenvalues to move as a pair in the same directions. It can either stabilize or de-stabilize depending upon the feedback model. The skew-symmetric feedback, on the other hand, is always detrimental regardless of the feedback model. It splits the eigenvalues, causing one rotating mode to gain damping while causing the other rotating mode to lose the same amount of damping. Using only the time-series data from experiments, the instability such as flutter and screech seen in experiments was explained as a consequence of the skew-symmetric feedback. The presence of a skew-symmetric feedback also explains why rotating wave instabilities in jet engines have preferential direction of rotation.

The second idea was to modify the structural aspects of the model in order to control the instability. This was accomplished by introducing precise spatial variations (mistuning) in the "mean properties" such as wave speed of the wave equation. While the skew-symmetric feedback causes the two eigenvalues to move apart, mistuning causes the eigenvalues to move closer. In either case, the net amount of damping in the system remains the same. This net damping depends upon the net symmetric feedback due to the presence of liner etc. and is not affected by spatial variation in mean. In effect, the mistuning utilizes the more heavily damped system modes to augment the damping of the lightly damped modes.

For a given skew-symmetric feedback (split of eigenvalues), there is an optimal amount of mean variations that reverses the detrimental effect of skew-symmetric feedback. This optimal amount corresponds to the eigenvalue diagram where the nominally double eigenvalues are the closest. Decreasing the amount of mistuning from the optimal amount causes one of the modes to become more damped at the expense of the other mode, which becomes less damped. On the other hand, increasing the mistuning beyond the optimal amount causes the frequencies of the two counter-rotating modes to shift without any additional damping augmentation. Figures 20 and 21 illustrate the concepts described above.

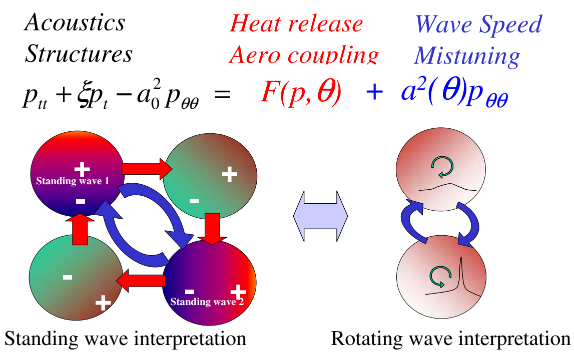

Fig. 20. Harmful and beneficial energy exchange between the rotating waves in engines: the effect of skew-symmetric feedback and symmetry breaking (wave-speed mistuning)

Finally, we comment on the robustness of the method that makes the symmetry breaking feasible for practical applications. The method exploits the dynamics of the problem for the purpose of creating beneficial energy exchange between traveling waves and thus enjoys several advantages. In particular, the method works by using the heavily damped rotating wave to provide damping augmentation for the lightly damped (or unstable) one, resulting in an overall decrease in the oscillation amplitude. In case of thermoacoustic waves, the method is applicable to general combustion schemes including swirl and bluff-body stabilized combustors. The approach does not require very accurate physics-based dynamic models for unsteady combustion or aero coupling and is robust to many un-modeled physical effects, such as changes in frequency, as long as the modal structure of the problem is approximately preserved.

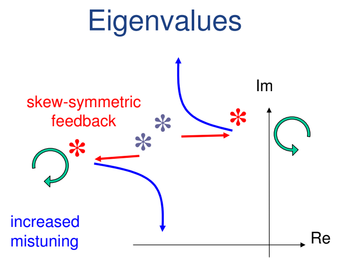

Fig. 21. Harmful and beneficial energy exchange between the rotating waves in engines: the effect of skew-symmetric feedback and symmetry breaking (wave-speed mistuning) on the model eigenvalues and the amplitudes of the rotating waves

Let us summarize what the Design of Dynamics means. If natural dynamics of a product needs modification, active or passive control using *external* devices is just one possible solution and often the least desirable one. However, the principles of dynamics and control can be utilized to find a solution that is *internal* to the product. The idea is to find a decomposition of a model of the dynamics into a *system of interacting components* and use the dynamics and control methods to create *beneficial dynamic interactions* between the component. For instance, the control of oscillations using external devices can be realized by interconnection of the lightly damped or unstable mode of the system with a heavily damped external device by choosing appropriate gain and phase of the closed-loop system. The same principle can be used to interconnect a lightly damped or unstable mode of the system with a heavily damped natural mode of the system. For instance, the wave speed mistuning interconnects underdamped traveling waves with heavily damped waves traveling in the opposite direction using the wave-speed perturbation as an interconnecting feedback. Figure 22 illustrates this idea.

7. BARRIERS IN DESIGN OF DYNAMICS

Several barriers are present in a way of introduction of Design of Dynamics into the industrial practice.

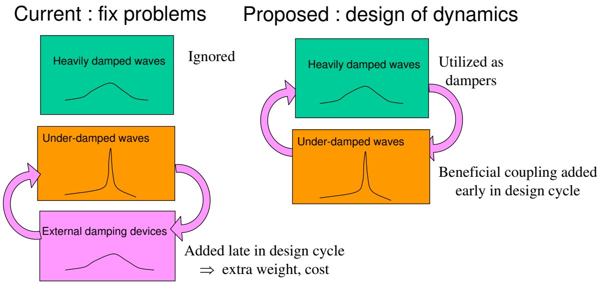

Fig. 22. The idea behind the Design of Dynamics

For the purpose of this paper we will group them into two areas: the first one technical related to an intrinsic difficulty of analysis of the dynamic phenomena and the other mostly social related to the perception of the role of the dynamics and control community.

The technical barriers in mitigation of dynamic problems in products affected by unsteady flow phenomena (such as aerospace and chemical industry) become clearly visible when one realizes that the technical problem amounts to controlling sensitive and complex dynamics in presence of model uncertainty and severe design constraints. Lightly damped structural, acoustic, and fluid dynamic modes easily become underdamped or unstable because of positive feedback coupling with other flow phenomena and excitation by broad band, and tonal flow disturbances. Physics that causes detrimental oscillations involves complex high Mach and Reynolds number flows, which cannot be reliably and accurately computed with current methods. State of the art computational methods based on CFD are too expensive to be applied. Hence model analysis is not utilized to exploit the design space and find innovative dynamics mitigation solutions in early design stage. Accuracy of reduced order models utilized is doubtful especially when chemical reaction, flow separation, and shocks are involved. Because of high uncertainty, models are not utilized for design of robust oscillation mitigation solutions and robustness of chosen solutions to unexpected off-design conditions is not guaranteed. In this paper we partially addressed mitigation of the technical difficulties in modeling and analysis of dynamics by utilizing the symmetry properties of the product.

The social barriers that prevent the Design of Dynamics from becoming industrial practice are related to an underestimation of the full potential of the dynamics and control methods by the design process owners and the dynamics and control experts alike. In this paper we presented three case studies of dynamics and control analysis applied to jet engines technologies and discussed how the social barriers influenced the impact of the technologies on the product.

Let's reverse the negative aspects of the current role of the dynamics and control community outlined in Section 2 in the design process and postulate a new role for these communities. First, they should act *proactively* and play a critical role in early design process when a design flexibility is high and a cost of introduction of solutions to mitigate dynamics problems is the lowest. Second, they should focus on finding a solution that is *internal* to the product, i.e., does not involve extra hardware. This can be accomplished by analysis of the natural dynamics of the problem. The solution of the problem should *exploit* the natural dynamics by creation of beneficial dynamic interactions within the products with minimum external intervention. Third, rather than accepting assumptions from the design process owners, the dynamics and control community should *accept responsibility* for defining the best control architecture in terms of performance and cost. This direction will involve control and dynamics experts learning more of the physics of the problem than they typically accept as necessary. In particular, they will have to drive the modeling and experimental activities rather than be just the users of the results.

We hope that the three case studies presented in this paper will serve its intended purpose of convincing the control and dynamics experts about the benefits of the Design of Dynamics and the attributes they need to have to succeed. However, a much harder problem is how to articulate the benefits of the Design of Dynamics to the current design process owners so they will embrace the expanded role of the dynamics control methods and experts in the early design stages. The social barriers that need to be overcome here should not be underestimated. In fact, in author's experience, these are the most difficult barriers to overcome.

First, the design process owners have a *perception of limited applicability* of dynamics and control methods. Active control solutions are perceived as external to the product, involving extra cost and complexity, and are rarely implementable. As such they are treated as last resources. Moreover, because of the perception that active control methods are only useful for design of an active control system, the idea to use the dynamics and control methods in passive control design does not naturally occur to the design process owners.

Second, a *language barrier* between the dynamics and control community and the design process owners limits ability of the process owners to fully appreciate the potential of the dynamics and control methods to be used in a non-standard way postulated in this paper. The design process owners use the language of the most relevant discipline to the process (physics, chemistry, biology, etc.) and they require that all potential dynamics mitigation technologies be explained in this language. On the other hand, the dynamics and control experts use the language of dynamical systems or control theory often biased towards mathematical rigor and formalism. Inability to adequately articulate the control concepts in the native tongue of the design process owners will often result in the control concept being rejected.

Third, consider a *lack of credibility* of dynamics and control experts among the early design process owners combined with *territorial behavior* of the latter group. This combination will likely result in a control solution proposed by the dynamics and control experts to be treated with suspicion and possibly rejected. After all, the dynamics and control experts typically are not experts in any of the disciplines considered the most relevant to the product design. To contrary, the design process owners typically are experts in the disciplines most relevant to the product being designed. Why would they even consider solutions proposed by non-experts, when the experts struggle to exploit as much of the domain expertise to solve their problems? Use of a different type of models by the process owners and dynamics and control experts exacerbates the situation further. Low order models typically required by the dynamics and control experts are considered simplistic and inadequate by the design process owners. Using a suspect model as a basis of proposed design modification is a likely reason for a rejection of the proposed modification.

The last issue is a danger of a *competition* between the established design process owners and the dynamics and control experts. When the established process owners insist on solving the problem themselves without external help, they are likely to treat any attempt to bring outside expertise as an unnecessary distraction and a competition for limited resources available to solve the problem. This type of competition or perception of such is never healthy and should be avoided at all cost.

The social barriers mentioned above can be overcome, but the process of doing so is lengthy, difficult, frustrating, and fragile. In fact, it often fails. Here are some necessary conditions for increasing the role of dynamics and design communities in the early design process.

First, it is necessary to reach the state when a *perception of inadequacy* of the current design process is widely accepted among the process owners and their management. Such a perception is typically a result of a major *crisis* in a product design process. When the current design process is widely acknowledged to be faulty, the technical design process owners and even more so their managers become more open to a control solution. This is a best point of entry for the dynamics and control experts to get involved.

At this point it is important that the control and dynamics experts learn as much as possible about the scientific disciplines most relevant to the problem and their relationship with the internal dynamics of the product. They also have to show *commitment to work towards solving the problem* using the simplest possible means, including passive methods and exploitation of the natural dynamics, and avoid the trap of pushing for an active control solution at all cost. In this way they will present themselves as *team players*.

Along the way the dynamics and control experts need to show some partial *successes*. This can be accomplished utilizing traditional strengths that the dynamics and control communities exhibit, such as an ability to extract a low order model of a process directly form experimental data or assess the validity of a given physics-based model. Utilizing the rigor of mathematics is a great value, but it has to be balanced with translation to the language of physics tom be convincing.

Last but not least, just demonstrating the technical progress is not sufficient. It is extremely important that the dynamics and control experts use every opportunity to *educate* the process owners on all levels about the basic principles of dynamics and control theories and show why these principles are relevant to solving the problem at hand. Ability to translate the physics of the problem the the language of dynamics, finding solution in the dynamics domain, and translating the dynamic solution back to the language of physics will go particularly long ways towards eliminating the language barrier. The best possible outcome of this educational process is convincing the design process owners that the analysis methods of dynamical systems and control theory are just alternative efficient ways of exploiting the physics of the problem. This behavior will help establish the *credibility* with the design process owners and their management.

8. CONCLUSION

We advocated the need to change the role of dynamics and control community from fixing problems related to the detrimental dynamics using active control to the design for beneficial dynamics early in the design cycle. The paper summarized lessons learned in industrial research on mitigation of flow and structure oscillations in jet engines (thermoacoustic instabilities and turbomachinery flutter). We showed how the decisions on the control system architecture (sensor and actuator location) impacted the achievable level of suppression of oscillations (fundamental limitations of performance). Attempts to introduce control late in the design process and without proper attention to control architecture often fail because of high cost to modify the design to add on active control. We also showed how certain aspects of design (symmetry) contribute to the origin of detrimental oscillations and point out how the dynamical systems and control theory methods can guide the design to prevent the oscillations. The control and dynamics methods used early in design allow one to manipulate the physical feedback loops in the system to create beneficial dynamics and exploit design flexibility at low cost. To increase impact of experts in control and dynamics on the design process, the experts need to establish credibility in the technical community that owns the design process. In industrial environment this can be accomplished by playing a key role in a response to a crisis, and following up with teaching of basic principles of dynamics and control to the design community and their management.

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