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# Decentralized Optimization for Energy Saving of HVAC Systems*

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*Abstract*—**Improving the control strategy of building HVAC (heating, ventilation, and air-conditioning) systems can lead to significant energy savings while preserving human comfort requirements. This paper formulates the optimization problem to obtain the setpoints of control variables with the lowest HVAC system energy consumption. The optimization problem is nonlinear, strongly coupled, and has both discrete and continuous control variables. This paper proposes a decentralized optimization method to divide the HVAC system into multiple subsystems and then solve the subproblems based on the local information. Numerical results demonstrate that with limited knowledge of its neighbouring subsystems, each subsystem can achieve the local/global optimal solution of the original optimization problem. The method proposed in this paper has a broad application potential since it does not demand the global system knowledge.**

## I. Introduction

The energy consumed by heating, ventilation, and air-conditioning (HVAC) systems accounts for approximately 40% of the domestic energy consumption in the US and 15% in China[1]. It is well known that the improvement to control strategies of HVAC systems can significantly reduce building energy consumption. However, in most cases, current strategies are not optimal. For example, the control strategies of HVAC systems in most of the 430,000 commercial and institutional buildings in Canada are inefficiently designed, resulting in energy losses of 15% to 30%[2]. Therefore, the potential to reduce these losses provide compelling reason to improve the control strategies of building HVAC systems.

The control of HVAC systems consists of 1) determining the setpoints of various HVAC system components and associated equipment, e.g. the setpoint of fan frequency, and 2) determining the parameters of local controllers, e.g. parameters of Proportion Integration Differentiation (PID) controller. This paper studies the first type of control. It aims to find the optimal setpoints of HVAC system components and associated equipment, so that the indoor air temperature requirement can be guaranteed with minimum energy consumption. The control process of controllers is not considered here. Therefore, the static models are used to find the optimal setpoints of the HVAC system.

To the best of our knowledge, most of the HVAC system optimization methods are centralized ones. However, all the knowledge of the HVAC system and building (e.g. the parameters of the system, the temperature of indoor air, and the cooling load of rooms) are required for centralized methods. Moreover, the indoor air condition usually change with time. All the change should be known by the centralized optimizer, and the optimal setpoints of the system should be recalculated. There will be difficulties to implement centralized methods when the scale of system becomes large.

Different from centralized methods, decentralized methods are promising to overcome these difficulties. The main idea of a decentralized method is to first decompose the system into multiple subsystems and then let each subsystem make the optimal decision or optimize sub-optimization problem based on the local system information. For the optimization problem in this paper, each room can be viewed as a subsystem. There is one agent for each subsystem. The agent makes a decision based on subsystem's local information and communicates with its neighbors iteratively and eventually to achieve the optimal solution of the original optimization problem. Decentralized methods decompose large scaled problem into small scaled ones. And only local information is required for optimization. Hence, the decentralized method has better scalability and can easily adapt to any change of subsystems.

The research on decentralized optimization methods dates back to the 1960s[3]. Bertsekas and Tsitsiklis developed several iterative methods to solve decentralized problems and presented the convergence conditions for these methods[4]. These decentralized methods require that each subsystem knows all the information associated with the decision variables of its sub-optimization problem and are not suitable to solve the optimization problem of HVAC systems in a decentralized way. The study on decentralized optimization has developed rapidly in the last decade. The decentralized methods are used to solve both multi-objective and single-objective optimization problems. Inalhan et al. and Semsar–Kazerooni and Khorasani developed decentralized optimization methods for multi-objective systems, and obtained the Pareto solutions of these problems [4], [5]. In this paper, the single-objective optimization for effective energy management of the HVAC system is studied, i.e. the subsystems of the HVAC system should cooperate with each other to minimize the summation of the subsystems' energy consumption. In [6]-[8], decentralized subgradient methods are applied to solve the single-objective optimization problem. Le and Hossain proposed a method based on a dual method to decompose the single-objective optimization problem, and solved the problem in a decentralized manner[9]. Dual methods are also used in [10]-[14] for decentralized optimization. However, all these methods require that the optimization problem is convex.

The optimization problem of an HVAC system is nonconvex and includes both discrete and continuous decision variables. Therefore, the methods above are not suitable to

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solve it. This paper proposes an iterative method to solve the optimization problem of HVAC systems in a decentralized manner. In this method, central coordination is removed, and subsystems with limited knowledge of the optimization problem can obtain the optimal solution by communicating with their neighbors. Numerical results show that the iterative process can converge to the optimal solution of the original problem.

## II. Problem Formulation

A typical HVAC system is shown in Fig. 1. The HVAC system consists of terminals, pumps, and chillers. Terminals consist of air handling units (AHUs) and valves, where AHUs are located at rooms. When indoor air temperature is higher than a setpoint, it will be decreased by supplying cooling air from AHUs to rooms. At the same time, hot air returning to AHUs will be cooled by chilled water supplied by chillers. Therefore, extra heat of room air is taken away by chilled water through the AHU and the chilled water temperature is increased. Eventually chillers will be used to decrease the temperature of the chilled water. Pumps provide the power for the chilled water, and valves determine the flow rates of the chilled water through terminals. Fig. 1 also shows the control variables and energy consumptions ($E$) of components, where the superscript $w$ means the chilled water, and the subscript $ahu$, $ter$, $v$, $b$, $p$, and $ch$ mean AHU, terminal, valve, bypass, pump, and chiller respectively. In the figure, the control variables include the on-off status of component $A$ ($A$=1 means the component is working, otherwise $A$=0), frequency of the pump and AHU fan ($f_p$, $f_{ahu}$), opening of the valve ($K_v$), and outlet temperature of the chilled water through the chiller ($\mathcal{T}^w_{ch}$).

As shown in Fig. 1, the chilled water flows through each component. The inlet and outlet of each chilled water pipe are also shown in this figure, e.g. N111 is the outlet of the pipe where terminal 51 is located. In this paper, the components connected by a section of chilled water pipe directly are defined as neighbors. A neighborhood of a component $i$ is defined as $N_i$. For example, in Fig. 1, $N_{Terminal41}$ = {Terminal 31, Terminal 51}, and $N_{Terminal1}$ = {Terminal 11, Pump1, Pump2, Pump3}. With the decentralized optimization method in this paper, each component can determine the setpoints of its control variables by communicating with its neighbors. In the following, the models of components are shown. In order to distinguish the inlet and outlet variables, $\mathcal{T}$ and $\mathcal{P}$ are used to describe the outlet temperature and pressure. The static mathematical model of component $i$ is described by $F_i$. The details of $F_i$ is not given in this paper due to page limit.

### A. Terminal

As shown in Fig. 1, a terminal consists of an AHU and a valve, and the AHU is connected with rooms.

#### 1) Valve

Valve aims to control the flow rate of chilled water through a terminal. Denote the model of valve as $F_v$, then

$$s_{ter} = F_v\left(K_v\right) \tag{1}$$

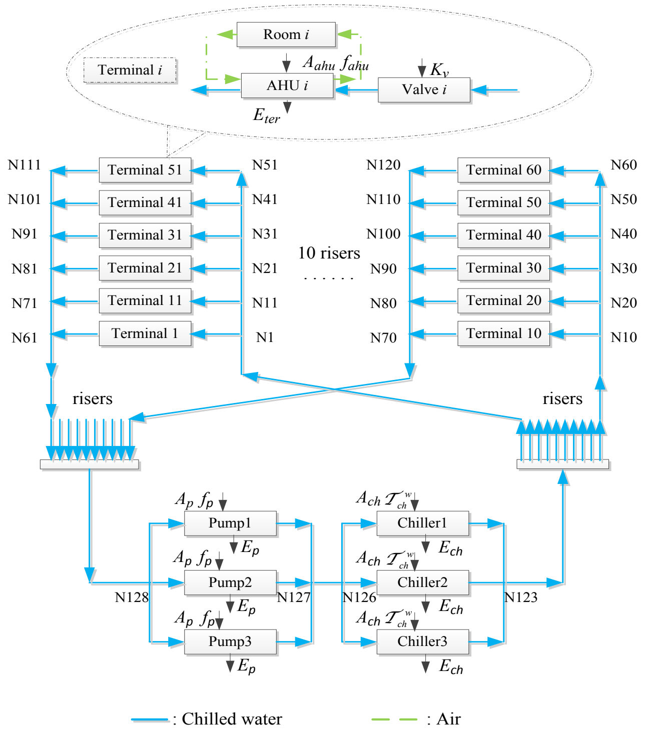


Figure 1 Example of HVAC system

The resistance of terminal ($s_{ter}$) is determined by the opening of valve ($K_v$). The terminal's resistance ($s_{ter}$) and flow rate ($G^w_{ter}$) satisfy the following equation

$$P^w_{ter} - \mathcal{P}^w_{ter} = s_{ter} \cdot \left(G^w_{ter}\right)^2 \tag{2}$$

Therefore, when the differential pressure is fixed, the flow rate of the terminal can be determined by the opening of the valve.

#### 2) AHU

AHU is a heat exchanger. The room air is chilled by the chilled water in the AHU. Denote the model of AHU as $F_{ahu}$, then if the AHU is working, i.e. $A_{ahu}$=1, the equality constraints function of AHU $\mathbf{h}_{ahu}$ is

$$\begin{aligned}&\mathbf{h}_{ahu}\left(\mathbf{x}_{ahu}\right)\\&=\left[\mathcal{T}^w_{ter}\ \mathcal{T}^a_{ahu}\ Q_{ter}\ E_{ter}\right]^T - F_{ahu}\left(T^w_{ter}, G^w_{ter}, T^a_{ahu}, f_{ahu}\right) \quad (3)\\&=\mathbf{0}\end{aligned}$$

where $\mathbf{x}_{ahu} = [\mathcal{T}^w_{ter}\ \mathcal{T}^a_{ahu}\ Q_{ter}\ E_{ter}\ T^w_{ter}\ G^w_{ter}\ T^a_{ahu}\ f_{ahu}]$, $T^w_{ter}$ and $T^a_{ahu}$ are the inlet temperature of the chilled water and air through the AHU, $G^w_{ter}$ is the flow rate of the chilled water, $f_{ahu}$ is the frequency of the AHU fan which determines the flow rate of air $G^a_{ter}$, $\mathcal{T}^w_{ter}$ and $\mathcal{T}^a_{ahu}$ are the outlet temperature of the chilled water and air through the AHU, $Q_{ter}$ is the heat amount transferred from the room air to the chilled water, and $E_{ter}$ is the energy consumption of the AHU.

When the indoor air temperature is higher than a setpoint, it will be decreased by supplying cooling air from AHUs to rooms. At the same time, hot air returning to AHUs will be cooled by chilled water supplied by chillers. And then, as shown in Fig. 1, this cooled air will be supplied to rooms. The model of AHU shows that when the chilled water and air's inlet temperature ( $T_{ter}^{w}$ and $T_{ahu}^{a}$ ) and flow rates ( $G_{ter}^{w}$ and $G_{ter}^{a}$ ) are given, the outlet temperature ( $\mathcal{T}_{ter}^{w}$ and $\mathcal{T}_{ahu}^{a}$ ), the transferred heat amount($Q_{ter}$), and energy consumption of AHU ( $E_{ter}$ ) can be obtained.

If the AHU does not work ($A_{ahu}$=0), then the temperature of the chilled water flowing in and out AHU will not change, and the fan frequency and energy consumption of the AHU is zero, i.e.

$$\mathbf{h}_{ahu}\left(\mathbf{x}_{ahu}\right)=\left[\mathcal{T}_{ter}^{w}-\mathcal{T}_{ter}^{w}, f_{ahu}, E_{ter}\right]^{T}=\mathbf{0} \tag{4}$$

*3) Room*

AHU supplies chilled air to rooms. Denote the model of the room as $F_r$, then

$$\mathcal{T}_{r}^{a}=F_{r}\left(f_{ahu}, \mathcal{T}_{ahu}^{a}\right) \tag{5}$$

where the frequency of the AHU fan ($f_{ahu}$) determines the flow rate of supply air $G_{ter}^{a}$, and the supply air temperature is the outlet air temperature of the AHU. With the model of the room, the indoor air temperature ( $\mathcal{T}_{r}^{a}$ ) can be obtained. The indoor air temperature should be equal to a setpoint $T_{set}$, i.e. $\mathcal{T}_{r}^{a}=T_{set}$ .

Thus, the equalities involved with the terminal are

$$\mathbf{h}_{ter}\left(\mathbf{x}_{ter}\right)=\begin{bmatrix}\left[s_{ter}-F_{v}\left(K_{v}\right), P_{ter}^{w}-\mathcal{P}_{ter}^{w}-s_{ter}\cdot\left(G_{ter}^{w}\right)^{2}\right]^{T}\\ \mathbf{h}_{ahu}\left(\mathbf{x}_{ahu}\right)\\ \left[\mathcal{T}_{r}^{a}-F_{r}\left(f_{ahu}, \mathcal{T}_{ahu}^{a}\right), \mathcal{T}_{r}^{a}-T_{set}\right]^{T}\end{bmatrix}=\mathbf{0} \tag{6}$$

where $\mathbf{h}_{ter}$ is the equality constraints function of terminal, and $\mathbf{x}_{ter}=[s_{ter}\ K_{v}\ P_{ter}^{w}\ \mathcal{P}_{ter}^{w}\ G_{ter}^{w}\ \mathcal{T}_{r}^{a}\ \mathbf{x}_{ahu}^{T}]^{T}$ .

The frequency of the AHU fan ($f_{ahu}$), opening of the valve ($K_v$), and flow rate of the chilled water through the terminal ( $G_{ter}^{w}$ ) have lower and upper bounds. Denote the upper bound of any variable $x$ as $\bar{x}$ and lower bound as $\underline{x}$ , then these 'bounds' form the inequality constraints. The inequality constraints of terminals are as follows: if $A_{ter}$=1, then

$$\mathbf{g}_{ter}\left(\mathbf{x}_{ter}\right)=\left[f_{ahu}-\bar{f}_{ahu}, \underline{f}_{ahu}-f_{ahu}, K_{v}-\bar{K}_{v}, \underline{K}_{v}-K_{v}, -G_{ter}^{w}\right]^{T} \leq \mathbf{0} \tag{7}$$

where $\mathbf{g}_{ter}$ is the inequality constraints function of terminal.

## B. Pump

Pumps provide power for the chilled water. Denote the model of the pump as $F_p$, then if the pump is working, i.e. $A_p$=1, the model of the pump is

$$\left[\mathcal{P}_{p}^{w}\ E_{p}\right]^{T}=F_{p}\left(f_{p}, G_{p}^{w}, P_{p}^{w}\right) \tag{8}$$

where $f_p$ is the frequency of the pump. The frequency and inlet pressure of the pump ($f_p$ and $P_{p}^{w}$ ), and flow rate of the chilled water through the pump ( $G_{p}^{w}$) determine the outlet pressure ( $\mathcal{P}_{p}^{w}$ ) and energy consumption of the pump ( $E_p$ ).

In addition, the temperature of the chilled water flowing in the pump is equal to the temperature of the chilled water flowing out the pump, i.e. $\mathcal{T}_{p}^{w}=T_{p}^{w}$ .

If the pump is not working, i.e. $A_p$=0, then the frequency and energy consumption of the pump, and flow rate of the chilled water through the pump are zero, i.e. $E_p$=0, $G_{p}^{w}=0, f_{p}=0$ .

Thus, the equalities involved with the pump are

$$\mathbf{h}_{p}\left(\mathbf{x}_{p}\right)=\begin{cases}\begin{bmatrix}F_{p}\left(f_{p}, G_{p}^{w}, P_{p}^{w}\right)-\left[\mathcal{P}_{p}^{w}\ E_{p}\right]^{T}\\ T_{p}^{w}-\mathcal{T}_{p}^{w}\end{bmatrix}=\mathbf{0}, & \text{if } A_{p}=1;\\ \left[E_{p}\ \ G_{p}^{w}\ \ f_{p}\right]^{T}=\mathbf{0}, & \text{if } A_{p}=0\end{cases} \tag{9}$$

where $\mathbf{h}_p$ is the equality constraints function of pump, and $\mathbf{x}_{p}=\left[G_{p}^{w}\ T_{p}^{w}\ \mathcal{T}_{p}^{w}\ P_{p}^{w}\ \mathcal{P}_{p}^{w}\ A_{p}\ f_{p}\ E_{p}\right]^{T}$ .

In addition, the flow rate of the chilled water through the pump ( $G_{p}^{w}$), and frequency and differential pressure of the pump ($f_p$ and $\Delta P_p$) have lower and upper bounds, i.e. if $A_p$=1, then

$$\mathbf{g}_{p}\left(\mathbf{x}_{p}\right)=\left[-G_{p}^{w}, f_{p}-\bar{f}_{p}, \underline{f}_{p}-f_{p}, \Delta P_{p}-\overline{\Delta P}_{p}, \underline{\Delta P}_{p}-\Delta P_{p}\right]^{T} \leq \mathbf{0} \tag{10}$$

where $\mathbf{g}_p$ is the inequality constraints function of pump.

## C. Chiller

Chillers reduce the temperature of the chilled water. High temperature chilled water flows in the chiller, and turns into low temperature chilled water. Denote the model of the chiller as $F_{ch}$, then if the chiller is working, i.e. $A_{ch}$=1,

$$\left[T_{ch}^{w}\ E_{ch}\right]^{T}=F_{ch}\left(\mathcal{T}_{ch}^{w}, G_{ch}^{w}, Q_{ch}\right) \tag{11}$$

where $\mathcal{T}_{ch}^{w}, G_{ch}^{w}$ are the outlet temperature and flow rate of the chilled water through the chiller, $Q_{ch}$ is the heat amount that the chiller takes away from the chilled water, and $T_{ch}^{w}$ is the inlet temperature of the chilled water.

The heat amount that chillers take away from the chilled water ($Q_{ch}$) is equal to the total heat amount that transferred from the indoor air to the chilled water in terminals ($Q_{ter}$), i.e. $\sum Q_{ch}=\sum Q_{ter}$ . Therefore, by using AHUs, the extra heat of indoor air is transferred to the chilled water, and then with the chiller, this heat is taken away from the chilled water. With the model of the chiller, the inlet temperature of the chilled water ( $T_{ch}^{w}$ ), and energy consumption of the chiller ($E_{ch}$) can be obtained when the outlet temperature and flow rate of the chilled water ($\mathcal{T}_{ch}^{w}, G_{ch}^{w}$), and heat amount of the chiller ($Q_{ch}$) are given.

The pressure of the chilled water through the chiller decreases because of the resistance of the chiller, i.e.

$$P_{ch}^{w} - \mathcal{P}_{ch}^{w} = s_{ch} \cdot \left(G_{ch}^{w}\right)^{2} \tag{12}$$

where $P_{ch}^{w}, \mathcal{P}_{ch}^{w}$ are the inlet and outlet pressure of the chilled water through the chiller, and $s_{ch}$ is a constant meaning the resistance of the chiller.

If the chiller is not working ($A_{ch}$=0), then the flow rate of the chilled water through the chiller and energy consumption of the chiller are zero, i.e. $E_{ch} = 0$ , $G_{ch}^{w} = 0$.

Thus, if $A_{ch}$=1, the equalities involved with the chiller are

$$\mathbf{h}_{ch}\left(\mathbf{x}_{ch}\right) = \begin{cases} \begin{bmatrix} \left[T_{ch}^{w}\ E_{ch}\right]^{T} - F_{ch}\left(\mathcal{T}_{ch}^{w}, G_{ch}^{w}, Q_{ch}\right) \\ \sum Q_{ch} - \sum Q_{ter} \\ P_{ch}^{w} - \mathcal{P}_{ch}^{w} - s_{ch} \cdot \left(G_{ch}^{w}\right)^{2} \end{bmatrix} = \mathbf{0}; \\ \left[E_{ch}\ G_{ch}^{w}\right]^{T} = \mathbf{0}, \text{if } A_{ch} = 0 \end{cases} \tag{13}$$

where $\mathbf{h}_{ch}$ is the equality constraints function of chiller, and $\mathbf{x}_{ch} = \left[G_{ch}^{w}\ T_{ch}^{w}\ \mathcal{T}_{ch}^{w}\ P_{ch}^{w}\ \mathcal{P}_{ch}^{w}\ A_{ch}\ Q_{ch}\ E_{ch}\right]^{T}$ .

In addition, the flow rate of the chilled water through the chiller has lower and upper bounds, i.e. if $A_{ch}$=1, then

$$\mathbf{g}_{ch}\left(\mathbf{x}_{ch}\right) = \left[G_{ch}^{w} - \bar{G}_{ch}^{w}\ \ \underline{G}_{ch}^{w} - G_{ch}^{w}\right]^{T} \leq \mathbf{0} \tag{14}$$

where $\mathbf{g}_{ch}$ is the inequality constraints function of chiller.

## D. Coupling of components

Fig. 1 shows that the components are located at the chilled water pipes. For each conjunction node (denoted as N in Fig. 1) of the chilled water pipes, the chilled water flowing in and out the node should satisfy mass balance and heat balance. Suppose that node $k$ is a conjunction node. Denote

$$\begin{aligned} I &= \{i \mid \text{node } k \text{ is the outlet of componnent } i\} \\ J &= \{j \mid \text{node } k \text{ is the inlet of componnent } j\} \end{aligned} \tag{15}$$

For component $i$, denote $T_i$ and $P_i$ as the inlet temperature and pressure of the chilled water through component $i$, and $\mathcal{T}_i$ and $\mathcal{P}_i$ as the outlet temperature and pressure. Then,

$$\begin{aligned} &\sum_{i \in I} G_{i}^{w} = \sum_{j \in J} G_{j}^{w} \\ &\sum_{i \in I} G_{i} \cdot c\left(\mathcal{T}_{i}^{node\_k} - T^{node\_k}\right) = 0 \\ &T_{j}^{node\_k} = T^{node\_k}, \forall j \in J \end{aligned} \tag{16}$$

where $\mathcal{T}_{i}^{node\_k}$ is the outlet temperature of the chilled water through component $i$, $T^{node\_k}$ is the inlet temperature of chilled water through component $j$, $c$ is the specific heat.

In addition, the outlet pressure of the chilled water through component $i$ is equal to the inlet pressure of the chilled water through component $j$, i.e. $\mathcal{P}_{i}^{node\_k} = P_{j}^{node\_k}$ .

Denote $\mathbf{h}_{node\_k}$ as the equality constraints function involved with node $k$, then

$$\mathbf{h}_{node\_k}\left(\mathbf{x}_{node\_k}\right) = \begin{bmatrix} \sum_{i \in I} G_{i}^{w} - \sum_{j \in J} G_{j}^{w} \\ \sum_{i \in I} G_{i} \cdot c\left(\mathcal{T}_{i}^{node\_k} - T^{node\_k}\right) \\ T_{j}^{node\_k} - T^{node\_k}, \forall j \in J \\ \mathcal{P}_{i}^{node\_k} - P_{j}^{node\_k}, \forall i \in I, j \in J \end{bmatrix} = \mathbf{0} \tag{17}$$

where $\mathbf{x}_{node\_k}$ is the vector consisting of $G_{i}^{w}, G_{j}^{w}, \mathcal{T}_{i}^{node\_k}$ , $\mathcal{P}_{i}^{node\_k}, P_{j}^{node\_k}, T^{node\_k}$ and $T_{j}^{node\_k}$ .

In conclusion, the optimization problem can be stated as follows

$$\begin{aligned} &\underset{\mathbf{x}_{all}}{minimize}\ \sum_{i=1}^{n_{ch}} E_{ch,i} + \sum_{i=1}^{n_{p}} E_{p,i} + \sum_{i=1}^{n_{ter}} E_{ter,i} \\ &subject\ to\ \ \mathbf{h}_{all}(\mathbf{x}_{all}) = \mathbf{0};\quad \mathbf{g}_{all}(\mathbf{x}_{all}) \leq \mathbf{0}; \\ &\qquad\qquad A_{i} = 0 \text{ or } 1, \\ &\qquad\qquad \forall i \text{ is a component of the system} \end{aligned} \tag{18}$$

where $\mathbf{x}_{all} = \left[\mathbf{x}_{ter}^{T}\ \mathbf{x}_{p}^{T}\ \mathbf{x}_{ch}^{T}\right]^{T}$,

$\mathbf{g}_{all}(\mathbf{x}_{all}) = \left[\mathbf{g}_{ter}^{T}(\mathbf{x}_{ter})\ \mathbf{g}_{p}^{T}(\mathbf{x}_{p})\ \mathbf{g}_{ch}^{T}(\mathbf{x}_{ch})\right]^{T}$,

$\mathbf{h}_{all}(\mathbf{x}_{all}) = \left[\mathbf{h}_{ter}^{T}(\mathbf{x}_{ter})\ \mathbf{h}_{p}^{T}(\mathbf{x}_{p})\ \mathbf{h}_{ch}^{T}(\mathbf{x}_{ch})\ \mathbf{h}_{node}^{T}\left(\mathbf{x}_{node}\right)\right]^{T}$ , $\mathbf{h}_{node}\left(\mathbf{x}_{node}\right)$ are the equality constraints function of all the conjunction nodes of the chilled water pipes.

$\mathbf{h}_{all}(\mathbf{x}_{all})$ are nonlinear and nonconvex since the models of the working components ($F$) are nonlinear and nonconvex. Both the equality and inequality constraints are discrete since the component in the system can be working or not. And the optimization problem is strongly coupled since the mass and heat balance shown in subsection D.

# III. Decentralized Optimization Method

Denote the vector of decision variables involved with component $i$ as $\mathbf{z}_i$. Thus, $\mathbf{z}_i$ is formed by the elements of $\mathbf{x}_i$ and $\mathbf{x}_{node\_k}$ (denote node $k$ as the inlet or outlet node of component $i$). Denote $\mathbf{z}_{ij}$ as the vector of decision variables both involved with component $i$ and component $j$. From subsection $D$ , it can be seen that if component $i$ and component $j$ are neighbors, and node $k$ is their conjunction node, then the variables both involved with component $i$ and $j$ are $G_i$, $G_j$, $\mathcal{T}_{i}^{node\_k}$ , $\mathcal{P}_{i}^{node\_k}$ (or $T_{i}^{node\_k}$ and $P_{i}^{node\_k}$ , if node $k$ is the inet of component $i$), and $T_{j}^{node\_k}$ , $P_{j}^{node\_k}$ (or $\mathcal{T}_{j}^{node\_k}$ , $\mathcal{P}_{j}^{node\_k}$ , if node $k$ is the outlet of component $j$). Denote the set of elements of vector $\mathbf{z}$ as $Y$. And the sets of elements of vector $\mathbf{z}_i$ and $\mathbf{z}_{ij}$ are $Y_i$ and $Y_{ij}$. The variable separation method is shown in Fig. 2. In $Y_i$ and $Y_j$, the common variable set $Y_{ij}$ is replaced by $Y_{ij,i}$ and $Y_{ij,j}$ respectively. Denote the new decision variable set involved with component $i$ and $j$ as $\hat{Y}_i = \{Y_i / Y_{ij}, Y_{ij,i}\}$ , $\hat{Y}_j$ = $\{Y_j / Y_{ij}, Y_{ij,j}\}$ , and the corresponding decision variable vectors are $\hat{\mathbf{z}}_i$ and $\hat{\mathbf{z}}_j$ . Then there is no common variable between $\hat{\mathbf{z}}_i$ and $\hat{\mathbf{z}}_j$ . Also, the decision variable vectors corresponding to $Y_{ij,i}$ and $Y_{ij,j}$ are $\mathbf{z}_{ij,j}$ and $\mathbf{z}_{ij,j}$. It can be seen that $\mathbf{z}_{ij,j} = \mathbf{z}_{ij,j}$.

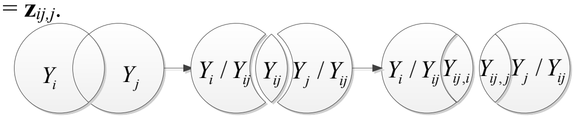


Figure 2 Variable decoupling method

Now it is aimed to obtain the sub-optimization problem of the decentralized optimization problem. And then components can obtain the optimal solution by solving their sub-optimization problems. The problem (18) can be transformed into (19) based on the variable decoupling method.

$$
\begin{aligned}
&\underset{\hat{\mathbf{z}}_{all} \in R^m}{minimize} \ \sum_{i=1}^{n_{all}} (E_i) \\
&subject\ to\ \mathbf{h}_i(\hat{\mathbf{z}}_i) = \mathbf{0}, i = 1,2,...n_{all}; \\
&\qquad \mathbf{g}_i(\hat{\mathbf{z}}_i) \le \mathbf{0}, i = 1,2,...n_{all}; \\
&\qquad \mathbf{h}_{node\_k}(\hat{\mathbf{z}}_{node\_k}) = \mathbf{0}, \qquad (19) \\
&\qquad \forall \text{node } k \text{ is conjuction of pipes}; \\
&\qquad \mathbf{z}_{ij,j} - \mathbf{z}_{ij,i} = \mathbf{0}, i = 1,2,...n_{all}, j \in N_i; \\
&\qquad A_i = 0 \text{ or } 1,\ \ i = 1,2,...n_{all}
\end{aligned}
$$

where $n_{all}$ is the total number of components. The equality constraint $\mathbf{z}_{ij,i} = \mathbf{z}_{ij,j}$ can be transformed into penalty function, as shown in (20).

$$
\begin{aligned}
&\underset{\hat{\mathbf{z}}_{all} \in R^m}{minimize} \ \sum_{i=1}^{n_{all}} \left( E_i + \sum_{j \in N_i} \alpha_i \left\| \mathbf{z}_{ij,j} - \mathbf{z}_{ij,i} \right\|^2 \right) \qquad (20) \\
&subject\ to: \hat{\mathbf{z}}_{all} \in \bigcup D_i, i = 1,2,...n_{all}
\end{aligned}
$$

where penalty factor $\alpha_i \in R^+$, whose initial value is finite and will approach to infinity as iteration moves on. With the penalty function transformation, $\mathbf{z}_{ij,I}$ and $\mathbf{z}_{ij,j}$ are not required to be equal when $\alpha_i$ is not big enough. $D_i$ in (20) is the feasible region of component $i$.

$$
D_i = \left\{ \begin{aligned} &\hat{\mathbf{z}}_i \mid \mathbf{h}_i(\hat{\mathbf{z}}_i) = \mathbf{0}, \mathbf{g}_i(\hat{\mathbf{z}}_i) \le \mathbf{0}, A_i = 0 \text{ or } 1, \mathbf{h}_{node\_k}(\hat{\mathbf{z}}_{node\_k}) \\ &= \mathbf{0},\ \forall \text{node } k \text{ is a conjunction node of component } i \end{aligned} \right\}
$$

It is obvious that $D_i$ only depends on the model of component $i$, and other components cannot influence it. Note that in (20), both the objective and feasible region can be decomposed into $n_{all}$ parts. Each part is a sub-optimization problem of (20). For component $i$, the sub-optimization problem is

$$
\begin{aligned}
&\underset{\hat{\mathbf{z}}_i}{minimize} \ \ E_i + \sum_{j \in N_i} \alpha_i \left\| \mathbf{z}_{ij,j} - \mathbf{z}_{ij,i} \right\|^2 \qquad (21) \\
&subject\ to\ \ \hat{\mathbf{z}}_i \in D_i
\end{aligned}
$$

The decentralized optimization algorithm for component $i$ is denoted by ***Algorithm I****:*

Step 1: Set penalty factor $\alpha_i$=$\alpha_{i,0}$, decision variable $\hat{\mathbf{z}}_i$ (0)= $\hat{\mathbf{z}}_{i0}$ , time of iteration $k$=0, step length $\beta > 1$, and a big enough value $M$, e.g. $M$=$10^5$;

Step 2: $k$=$k$+1;

Step 3: Send $\mathbf{z}_{ij,i}$ ($k$) to neighbors, and receive $\mathbf{z}_{ij,j}$ ($k$), $j \in N_i$ from neighbors;

Step 4: Solve (22) to obtain $\hat{\mathbf{z}}_i(k+1)$;

$$
\hat{\mathbf{z}}_i(k+1) = \underset{\hat{\mathbf{z}}_i \in D_i}{\text{argmin}} \left( E_i + \sum_{j \in N_i} \alpha_i \left\| \mathbf{z}_{ij,j}(k) - \mathbf{z}_{ij,i}(k) \right\|^2 \right) \qquad (22)
$$

Step 5: If $\left\| \hat{\mathbf{z}}_i(k+1) - \hat{\mathbf{z}}_i(k) \right\| < \varepsilon$ , then go to Step 6;

else ( $\left\| \hat{\mathbf{z}}_i(k+1) - \hat{\mathbf{z}}_i(k) \right\| \ge \varepsilon$ ), go to Step 2;

Step 6: $\alpha_i = \alpha_i \cdot \beta$ ; if $\alpha_i > M$ , then stop; else go to Step 2.

***Remark 1***: The feasible region of component $i$'s sub-optimization problem (22) only depends on component $i$. The objective of (22) is also only involved with energy consumption of component $i$ ($E_i$) and the decision variables of its neighbors. ($\mathbf{z}_{ij,j}$, $\forall$ $j \in N_i$). Therefore, each component only need the information of itself and its neighbors.

***Remark 2***: In practice, the delay of communication and calculation makes decentralized methods difficult to implement synchronously. Algorithm I is a synchronous iteration method. In fact, numerical results show that this decentralized iteration method can also be implemented in an asynchronous manner, which is practical.

***Remark 3***: The value of $M$ determines that the penalty term goes very close to zero or not. For the optimization of HVAC system $M$=$10^5$ is large enough to guarantee that the penalty term is less than $10^{-6}$. The value of $M$ will be studied theoretically to make this algorithm applied widely.

In the decentralized method proposed in this paper, the only connection between sub-optimization of component $i$ and the others is $\mathbf{z}_{ij,j}, \forall j \in N_i$ , which is part of the optimal solution of component $j$. Thus, $\mathbf{z}_{ij,j}$ aims to minimize the objective of component $j$, i.e. $E_j + \sum_{i \in N_j} \alpha_i \left\| \mathbf{z}_{ij,i} - \mathbf{z}_{ij,j} \right\|^2$ . So that the energy consumption of component $j$ is also reflected in the objective of component $i$'s sub-optimization problem(22). When $\alpha_i$ is small, in extreme case, $\alpha_i = 0$ , the optimal solution minimize the energy consumption of component $i$ without considering the energy consumption of component $j$, i.e. without considering $\sum_{i \in N_j} \alpha_i \left\| \mathbf{z}_{ij,i} - \mathbf{z}_{ij,j} \right\|^2$ . When $\alpha_i$ increases, the optimal solution considers reducing $\left\| \mathbf{z}_{ij,i} - \mathbf{z}_{ij,j} \right\|^2$ , i.e. reducing the energy consumption of component $j$. If $\alpha_i$ approaches infinity, then $\mathbf{z}_{ij,i}$ = $\mathbf{z}_{ij,j}$ and the solution of (22) minimizes the energy consumption of the entire system.

## IV. Case Study

The HVAC system shown in Fig. 1 is studied. There are 3 chillers and 3 pumps. Assume that the chillers have the same parameters, as do the pumps. The building has 60 rooms, belonging to 10 risers. For each riser there are 6 floors and for each floor there is only one room. To simplify, the 10 risers are assumed to be identical.

The decentralized algorithm in this paper is applied to this case and the results of terminals are shown in Table I. For comparison, the optimal results obtained by centralized methods are listed in the same table. As the results for each riser are the same. Table I only shows one riser's results, where 'D' means the decentralized optimization results, 'C' means the optimal results obtained by the centralized method, and 'S' means the setpoint. Denote the decentralized optimization result and centralized optimization result of any variable $x$ as $x_D$ and $x_C$, then the relative difference between $x_D$ and $x_C$ is defined as $\Delta = \|x_C - x_D\| / \|x_C\|$ .Table I shows that the decentralized results almost converge to the optimal results. Table I shows that the valve opening of floor 2 is '1', which means that the valve of floor 2 is fully opened. This is the same as the experience of HVAC system control in practice, i.e. there is at least of valve fully opened. The wider the valve opened, the smaller the resistance of valve will be. Therefore, at least one valve is fully opened can reduce the resistance of valve and the energy consumption of pump.

TABLE I. RESULTS OF TERMINALS

| | **T(℃)** | | | **$K_v$** | | **$f_{ahu}$ (Hz)** | |
|---|---|---|---|---|---|---|---|
| | *S* | *C* | *D* | *C* | *D* | *C* | *D* |
| Floor 1 | 25.2 | 25.2 | 25.2 | 0.29 | 0.28 | 21.0 | 22.2 |
| Floor 2 | 24.9 | 24.9 | 24.9 | 1.00 | 1.00 | 37.9 | 37.2 |
| Floor 3 | 25.0 | 25.0 | 25.0 | 0.30 | 0.28 | 21.4 | 22.6 |
| Floor 4 | 25.0 | 25.0 | 25.0 | 1.00 | 1.00 | 35.2 | 34.5 |
| Floor 5 | 24.9 | 24.9 | 24.9 | 0.38 | 0.40 | 29.0 | 27.9 |
| Floor 6 | 25.2 | 25.2 | 25.2 | 0.99 | 0.98 | 36.9 | 37.5 |
| Δ | | 0 | | 0.027 | | 0.034 | |

The decentralized optimization results of chillers and pumps are shown in Table II and Table III respectively, where 'D' means the decentralized optimization results, 'C' means the optimal results obtained by centralized methods, $A$ is the on/off status of the component ($A$=1 means the component is working, otherwise $A$=0), $\mathcal{T}_{ch}^{w}$ is the outlet temperature of the chilled water through the chiller, $f_p$ is the frequency of the pump. Table II and III show that the decentralized results almost converge to the optimal results, and the decentralized method can solve the optimization problem including the discrete variable $A$.

TABLE II. RESULTS OF CHILLERS

| | **$A_{ch}$** | | **$\mathcal{T}_{ch}^{w}$ (℃)** | |
|---|---|---|---|---|
| | *C* | *D* | *C* | *D* |
| Chiller 1 | 1 | 1 | 5.35 | 5.38 |
| Chiller 2 | 1 | 1 | 5.35 | 5.38 |
| Chiller 3 | 0 | 0 | // | // |
| Δ | 0 | | 0.0056 | |

TABLE III. RESULTS OF PUMPS

| | **$A_p$** | | **$f_p$ (Hz)** | |
|---|---|---|---|---|
| | *C* | *D* | *C* | *D* |
| Pump 1 | 1 | 1 | 25.00 | 25.39 |
| Pump 2 | 1 | 1 | 25.00 | 25.39 |
| Pump 3 | 0 | 0 | // | // |
| Δ | 0 | | 0.016 | |

## V. Conclusion

This paper presents a decentralized optimization algorithm based on the penalty function method. Numerical results show that this algorithm can solve optimization problems that are nonlinear, nonconvex, and have both discrete and continuous decision variables. In practice, it is usually difficult to prove the convexity of a problem or to obtain the global system knowledge. The algorithm proposed in this paper has a broad application potential since it does not demand convexity of an optimization problem or the global system knowledge.

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