ROBUST CONTROL OF CONTINUOUS FERMENTATION PROCESSES DESCRIBED BY MONOD-TYPE MODEL WITH DELAY

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Abstract**:** Discrete nonlinear problem for control synthesis of continuous fermentation processes is formulated and method for robust control synthesis is developed. The method consists of an optimal robust control design and an input/output linearization via nonlinear state feedback. The obtained total control is a robust one. It linearizes the nominal nonlinear system, determines global asymptotic stability of the closed loop system and guarantees the performance bound cost corresponding to a used quadratic criterion. The method is demonstrated by numerical simulations on the example of a Saccaromyces cerevisiae fermentation, described by Monod-type model with delay.

Key Words. Fermentation processes, uncertainties, time delay, nonlinear control design, robust control, guaranteed cost control, Riccati equation.

1.INTRODUCTION

Fermentation control has become an active area of research in the recent years. This is particularly attributable to the fact that fermenters can be extremely difficult to be controlled. Their dynamic behavior is nonlinear and the model parameters vary in an unpredictable manner (Abulesz and Lyberatos, 1989), (Andersen and Jorgensen, 1988), (Bastin and Dochain, 1990), (Tsoneva and Patarinska 1995). Accurate process models are rarely available due to the complexity of the processes. Moreover, lack of reliable sensors make the process state very difficult to be measured. Since the fermentation processes are inherently nonlinear and their parameters are uncertain, a general theory for the design of nonlinear feedback controllers (Henson and Seborg, 1991), (Isidory,1995) has to be applied. Using non-linear coordinate transformations and non-linear state feedback, the original nonlinear model can often be transformed into an equivalent linear model and linear control techniques can then be used to achieve optimality or robustness of the closed loop linearized system.

Continuous fermentation process control synthesis is considered in this paper.The purpose is to produce a total robust nonlinear controller design method in order to stabilize the continuous growth process at its optimal steady state by manipulating the influent flow rate. To cope with both parameter uncertainty and the structured non-linearity of the plant, the proposed method consists of a linear guaranteed cost regulation that ensures robust optimal control and also of a nonlinear compensation that achieves the exact input/output linearization, Fig.1.The approach is based on the following ideas:

- the model of the fermentation process is described on the basis of its nominal parameters and a model of uncertainties bounded by their minimal and maximal values;
- the problem for nonlinear control synthesis is solved for the nominal model in such a way that the closed loop system has a given performance and is linearized;
- the problem for guaranteed cost control is formulated for the linearized uncertain system in such a way that the outputs reach the desired, previously determined state set points.

2.CONTINUOUS FERMENTATION PROCESS DESCRIPTION

It is assumed that the fermenter has a constant volume, its contents are well mixed and that the feed is sterile. The dilution rate is available as a manipulated input. The biomass and substrate concentrations are the process state variables. The usual state space representation of a bacterial growth system by nonlinear mass balance equations is considered (Bastin and Dochain, 1990):

$$\dot{x}(t) = [\mu(s) - k_D]x(t) - D(t)[x(t) - x^0]\,,\ x(0) = x_{0,} \quad (1)$$

$$\dot{s}(t) = -\varepsilon(s)x(t) - D(t)[s(t) - s^0]\,,\ s(0) = s_0\,, \quad (2)$$

where $x \in R$ *[g/l]*, $s \in R$ *[g/l]* are the biomass and substrate concentrations, $D \in R$ $[h^{-1}]$ is the dilution rate, μ $[h^{-1}]$ is maximum growth rate,ε $[h^{-1}]$ is the specific limiting substrate consumption rate, k_D $[h^{-1}]$ is a disappearing constant, x^0, s^0 *[g/l]* are the inlet biomass and substrate concentrations, $v_0, v = x, s$ *[g/l]* are the initial concentrations. Here the Monod law expression is adopted for the bacterial growth rate. It is generally assumed that the specific growth rate of the micro-organisms is a function of the limiting substrate concentration during some time interval in the past as the bacteria growth is a function of the past values of the substrate. Then the specific growth rate is a time delay function of the substrate

$$\begin{aligned}\mu(s) &= \mu_m s(t-\bar{\tau})/[k_s + s(t-\bar{\tau})],\\ \varepsilon(s) &= \mu_m s(t)/Y[k_s + s(t)]\end{aligned}\ , \quad (3)$$

where τ *[h]* is time delay, Y *[g/g]* is an yield coefficient, k_s *[g/l]* is Michael's-Menten parameter.

A discrete version of the model is obtained (Tsoneva and Patarinska 1995).

$$x(k+1) = \{1 + \Delta t[\mu_m s(k-\tau)/[k_s + s(k-\tau)] - k_D]\}x(k) -$$
$$- \Delta t D(k)[x(k) - x^0],\ x(0) = x_{0,} \quad (4)$$

$$s(k+1) = \{1 - \Delta t \mu_m x(k)/Y[k_s + s(k)]\}s(k) -$$
$$- \Delta t D(k)[s(k) - s^0(k)]\,, \quad (5)$$
$$s(0) = s_0, s(k) = \varphi_s(k), k \in [-\tau, -1],$$

$$y_x(k) = x(k), y_s(k) = s(k)\,, \quad (6)$$

$$\bar{\tau}/\Delta t = \tau, \Delta t << \frac{k_s + s(k-\tau)}{[\mu_m - k_D - D(k)]s(k-\tau) - k_s[k_D + D(k)]} \quad (7)$$

$$r_{\min} \le r \le r_{\max}, r = \mu_m, k_s, k_D, Y, x^0, s^0,\ r \in \Omega \quad (8)$$

where *Δt[h]* is the period of discretization, , φ_s *[g/l]* is an initial function for the time delay period, $r = m_m, k_s, k_D, Y, \tau, r \in \Omega \in R^4$ are closed regions of the parameter values, $y_v \in R, v = x, s$ *[g/l]* are outputs. The equations (7) and (8) are obtained using the second method of Lyapunov for stability of the process model under different values of the sampling period and different values of the model parameters. Throughout the paper it is assumed that the dilution rate is the control input, the biomass and substrate concentrations are measurable states and outputs, the influent substrate concentration is an external constant input, the desired steady state is determined (Tsoneva and Patarinska 1995), the model parameters vary within determined bounds and only their nominal and bound values are known. This model is a discrete nonlinear one with a state delay, affine control and uncertain parameters. It can be presented using the following description:

$$x(k+1) = (f_x + \Delta f_x)x(k) - \Delta t D(k)(g_x + \Delta g_x)\,,$$
$$x(0) = x_{0,} \quad (9)$$

$$s(k+1) = (f_s + \Delta f_s)s(k) - \Delta t D(k)(g_s + \Delta g_s)\,,$$
$$s(0) = s_0, s(k) = \varphi_s(k), k \in [-\tau, -1], \quad (10)$$

$$\Delta f_{v\min} \le \Delta f_v \le \Delta f_{v\max}, \Delta g_{v\min} \le \Delta g_v \le \Delta g_{v\max}, v = x, s \quad (11)$$

$$g_x = x(k) - x^0, g_s = s(k) - s^0,$$

$$f_x = 1 + \Delta t[\frac{\mu_m s(k-\tau)}{k_s + s(k-\tau)} - k_D],$$

$$f_s = 1 - \Delta t \frac{\mu_m x(k)}{Y[k_s + s(k)]}, \qquad (12)$$

where $f_v, g_v, v = x, s$ are scalar continuous and continuously differentiable functions. The minimal and maximal values of the functions in equation (11) are calculated using the minimal and maximal values of the parameters.

3.FORMULATION OF THE TOTAL NONLINEAR CONTROL SYNTHESIS PROBLEM

The problem considered is one of regulating the biomass and substrate concentrations at prescribed steady state values x^*, s^*, despite the disturbance input and parameter variations, by acting on the dilution rate. This problem is solved on the basis of the input/output linearizing control concept (Henson and Seborg, 1991), (Isidory,1995), in which the optimal robust linear control synthesis is used to cope with parameter uncertainties (Chang and Peng, 1972), (Luo and Van DenBosch, 1993), (Tsoneva and Popchev 1997). The initial nonlinear control synthesis problem can be written as follows: Find a static state feedback control law of the form

$$D[.] = \alpha_x[.]x(k) + \alpha_s[.]s(k) + \beta_x[.]u_x(k) +$$
$$+ \beta_s[.]u_s(k), [.] = [x, s, r, \Delta t, x^*, s^*],$$
$$r = \mu_m, k_s, k_D, Y, \tau, \qquad (13)$$

where u_x, u_s are the external reference signals, α_x, α_s and β_x, β_s are smooth vector functions in a neighborhood of the set point and $\beta_x[.] \neq 0, \beta_x[.] \neq 0$, such that:

1)the linearizing control is calculated for the nominal system

$$x(k+1) = f_x x(k) - \Delta t D(k) g_x,$$
$$s(k+1) = f_s s(k) - \Delta t D(k) g_s,$$
$$y_x(k) = x(k), y_s(k) = s(k) \qquad (14)$$

on the basis of pre-specified stable linear differential equations called the reference model

$$x(k+1) = l_x x(k) + v_x(k),$$
$$s(k+1) = l_s s(k) + v_s(k),$$
$$y_x(k) = x(k), y_s(k) = s(k), \qquad (15)$$

where $l_x = const$, $l_s = const$ provide the desired dynamics,

2)the external reference signals are obtained as linear feedbacks for the linearized uncertain system

$$u_x(k) = W_x(k)x(k) + W_{xs}(k)s(k) + V_x(k)h_x(k) +$$
$$+ V_{xs}(k)h_s(k) + T_x(k)x^* + T_{xs}(k)s^*,$$
$$u_s(k) = W_s(k)s(k) + W_{sx}(k)x(k) + V_s(k)h_s(k) +$$
$$+ V_{sx}(k)h_x(k) + T_s(k)s^* + T_{sx}(k)s^*, \qquad (16)$$

where the corresponding W an V are the linear controller matrices and h are the compensating vectors for the set-points.

3)the closed system (9)-(12),(13),(16) is locally stable around the steady state points $y_x^* = x^*, y_s^* = s^*$, and is robust towards the model parameters uncertainties.

4. NONLINEAR CONTROL SYNTHESIS

The nonlinear control synthesis problem here is to find, if possible, a nonlinear state feedback of the kind (13) for the system (14), such that the map between the external inputs $u_x(k), u_s(k)$ and the outputs $y_x(k), y_s(k)$ is linear Fig. 1. To solve this problem the right parts of equations (14) and (15) are equalized:

$$l_x x(k) + u_x(k) = x(k+1) = f_x x(k) - \Delta t D(k) g_x,$$
$$l_s s(k) + u_s(k) = s(k+1) = f_s s(k) - \Delta t D(k) g_s \quad (17)$$

and written in the form of vector matrix representation

$$\begin{bmatrix} l_x - f_x & \\ & l_s - f_s \end{bmatrix} \begin{bmatrix} x(k) \\ s(k) \end{bmatrix} + \begin{bmatrix} u_x(k) \\ u_s(k) \end{bmatrix} = - \begin{bmatrix} g_x(k) \\ g_s(k) \end{bmatrix} \Delta t D(k), \qquad (18)$$

After some algebraic transformations of (18) the linearizing nonlinear control is obtained

$$D(k) = -d \left\{ \begin{bmatrix} g_x(l_x - f_x) \\ g_s(l_s - f_s) \end{bmatrix}^T \begin{bmatrix} x(k) \\ s(k) \end{bmatrix} + \begin{bmatrix} g_x \\ g_s \end{bmatrix}^T \begin{bmatrix} u_x(k) \\ u_s(k) \end{bmatrix} \right\}$$

$$d = -\frac{1}{\Delta t[g_x^2 + g_s^2]}, \qquad (19)$$

This control is a function of the time delay in the expression for the specific grow rate. The closed loop system (9)-(12),(19) is linearized, uncertain and has the form:

$$z(k) = [x(k)s(k)]^T, z^* = [x^* s^*]^T, \qquad (20)$$

$$z(k+1) = A(k)z(k) + B(k)u(k) +$$
$$+ \Delta A(k)z(k) + \Delta B(k)u(k), \qquad (21)$$

where

$$\Delta A_{\min} \leq \Delta A(k) \leq \Delta A_{\max}, \Delta B_{\min} \leq \Delta B(k) \leq \Delta B_{\max}, (22)$$

$$A(k)=\frac{1}{g_x^2+g_s^2}\begin{bmatrix} g_s^2 f_x+g_x^2 l_x & g_x g_s(l_s-f_s) \\ g_x g_s(l_x-f_x) & g_x^2 f_s+g_s^2 l_s \end{bmatrix},$$

$$B(k)=\frac{1}{g_x^2+g_s^2}\begin{bmatrix} g_x^2 & g_x g_s \\ g_x g_s & g_s^2 \end{bmatrix},$$

$$\Delta A(k)=\frac{1}{g_x^2+g_s^2}\begin{bmatrix} \Delta f_x(g_x^2+g_s^2)+\Delta g_x g_x(l_x-f_x) \\ \Delta g_s g_x(l_x-f_x) \end{bmatrix}\ldots$$

$$\ldots\begin{bmatrix} \Delta g_x g_s(l_s-f_s) \\ \Delta f_s(g_x^2+g_s^2)+\Delta g_s g_s(l_s-f_s) \end{bmatrix},$$

$$\Delta B(k)=\frac{1}{g_x^2+g_s^2}\begin{bmatrix} \Delta g_x g_x & \Delta g_x g_s \\ \Delta g_s g_x & \Delta g_s g_s \end{bmatrix} \qquad (23)$$

$A\in R^{2\times2}, B\in R^{2\times2}$ are the state and control matrices of the linearized system, $\Delta A\in R^{2\times2}, \Delta B\in R^{2\times2}$ are the state and control matrices of uncertainties, $z=[x,s]^T\in R^2$ is the state vector of the linearized system, $u\in R^2$ $[h^{-1}]$ is the linear robust control input. The values of matrices *A(k)* and *B(k)* are calculated for $r=r_{nom}$. The values of the bounds of the matrices *ΔA(k)* and *ΔB(k)* are calculated respectively for

$\Delta r_{\max}=r_{\max}-r_{nom}, \Delta r_{\min}=r_{nom}-r_{\min}$.

The guaranteed cost control can now be used to find the external outputs $u_x(k), u_s(k)$, such that certain quality features of the total closed loop system are reached.

5. SYNTHESIS OF AN OPTIMAL LINEAR GUARANTEED COST CONTROL

In order to obtain the optimal external reference signal $u(k)=[u_x(k)u_s(k)]^T$ the following problem for optimal control design of a linear discrete time system is considered: Find the control $u(k)=u(k,z,z^*), k=\overline{0,K-1},$ which minimises the functional

$$J=1/2\{\|z(K)-z^*\|_S^2+\sum_{k=0}^{K-1}[\|z(k)-z^*\|_Q^2+\|u(k)\|_R^2]\},$$

$$K\to\infty, K\neq\infty \qquad (24)$$

and satisfies the constraints

$$z(k+1)=[A(k)+\Delta A(k)]z(k)+[B(k)+\Delta B(k)]u(k),$$

$$z(0)=z_0, \qquad y(k)=z(k) \qquad (25)$$

$$\Delta A_{\min}\le\Delta A(k)\le\Delta A_{\max}, \Delta B_{\min}\le\Delta B(k)\le\Delta B_{\max} \qquad (26)$$

where J is the performance index, $Q\ge0, Q\in R^{2\times2}, R>0, R\in R^{2\times2}$ are the weighting matrices of the performance index, K is the number of steps in the optimization horizon. The solution of the optimal control problem is based on the functional of Hamilton

$$H=\sum_{k=0}^{K-1}1/2\{\|z(k)-z^*\|_Q^2+\|u(k)\|_R^2]+$$

$$p(k+1)^T\{[A(k)+\Delta A(k)]z(k)+[B(k)+\Delta B(k)]u(k)\}\}, \qquad (27)$$

using a guaranteed cost control approach. The following Riccati equations

$$G^{-1}(k+1)=A(k)[G(k)-Q]^{-1}A^T(k)-$$

$$-B(k)R^{-1}B^T(k)+U1(k),\ G(K)=S, \qquad (28)$$

$$G^{-1}(k+1)h(k+1)=-A(k)G^{-1}(k)[Q^{-1}G(k)-I]^{-1}+I]h(k)-$$

$$+I]h(k)-A(k)[Q^{-1}G(k)-I]^{-1}z^*+U2(k),$$

$$g(K)=Sz^* \qquad (29)$$

are solved, where *U1* and U2 are defined as

$$U1(k)=A(k)[G(k)-Q]^{-1}\Delta A^T(k)+$$

$$+\Delta A(k)[G(k)-Q]^{-1}A^T(k)$$

$$-\Delta B(k)R^{-1}B^T(k)-B(k)R^{-1}\Delta B^T(k)+$$

$$\Delta A(k)[G(k)-Q]^{-1}\Delta A^T(k)-\Delta B(k)R^{-1}\Delta B^T(k), \qquad (30)$$

$$U2(k)=-\Delta A(k)G^{-1}(k)[Q^{-1}G(k)-I]^{-1}+I]h(k)-$$

$$-\Delta A(k)[Q^{-1}G(k)-I]^{-1}z^*, \qquad (31)$$

$G\in R^{2\times2}$ is the matrix solution of the Riccati equation and $h=[h_x h_s]^T$ is the compensating vector. The upper and lower bounds of the uncertain parameters are used to calculate the boundary values of U1 and U2 and then to solve the Riccati equations. A theorem is proven (Tsoneva and Popchev, 1999)., which determines the kind of the guaranteed cost control and the value of the upper bound of the criteria. The optimal control is given by the equations:

$$u(k)=W(k)z(k)+V(k)h(k)+T(k)z^*,$$

$$W(k)=-R^{-1}B^T(k)[A^T(k)]^{-1}[G(k)-Q] \qquad (32)$$

$$V(k)=-R^{-1}B^T(k)[A^T(k)]^{-1}$$

$$T(k)=-R^{-1}B^T(k)[A^T(k)]^{-1}Q \qquad (33)$$

where $W\in R^{2\times2}, V\in R^{2\times2}, T\in R^{2\times2}$ are matrices of the optimal guaranteed cost control.

6.TOTAL FEEDBACK CONTROL AND SIMULATIONS

The total feedback control consist of two parts – a nonlinear one and a linear robust compensation. It is given by equations (19) and (16).

$$u_t(k) = D(k) + u(k), \quad u(k) = [u_x(k)\ u_s(k)]^T \qquad (34)$$

where u(k) is the vector of the linear control.

The method is demonstrated by a simulation on the example of the growth of a Saccaromyces cerevisiae fermentation process in a glucose limited medium. The nominal parameters (following (Abulesz and Lyberatos, 1989)) are $\mu_m = 0.5574, k_s = 2.0535$, $x^* = 0.204, s^* = 0.832$, $k_D = 0., Y = 0.175, \tau = 3.3, x^0 = 0.001, s^0 = 2.0$

The influence of the deviations from the set points of about 20% is shown in Fig.2.

7.CONCLUSION

A discrete nonlinear problem for the control of continuous fermentation processes is formulated and a method for robust control synthesis is developed. The method consists of an optimal robust control design and an input/output linearization via nonlinear state feedback. The obtained total control is a robust one.

The proposed method has some positive qualities that can be summarized as follows:

1)a specific growth rate modeled by an analytical time delay function is used, 2) the nonlinear control is simultaneously concerning the biomass and the substrate concentrations, 3) the total control consists of two loops – an inner one with input/output linearization via a sensitive nonlinear state feedback, and an outer one with an optimal linear guaranteed cost feedback control.

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REFERENCES

Abulesz E. and G. Lyberatos (1989). Periodic operation of a continuous culture of baker's yeast, *Biotech. and Bioeng.*, **34**, pp.741-749

Andersen M. and S. Jorgensen (1988). Multivariable adaptive control of a simulated continuous yeast fermentation, *Proc IFAC Symposium on Adaptive Control of Chemical Processes*,Copenhagen, pp.109-114.

Bastin G. and D. Dochain (1990). *On-line estimation and adaptive control of bioreactors*, Elsevier, Amsterdam.

Chang S. and T.Peng (1972). Adaptive guaranteed cost control of systems with uncertain parameters. *IEEE Trans on Autom Control*, **AC-17**, No 4,. pp. 474-483

Henson M. and D. Seborg (1991). A critique of exact linearization strategies for process control. *J. Process Control*, **1**, pp. 122-139.

Isidory.A.(1995). *Nonlinear Control Systems*, Springer, London.

Luo J. and P. Van DenBosch (1993). Minimax guaranteed cost control for linear systems with large parameter uncertainties-Riccati equation approach. *Proc. of the XII IFAC World Congress*, **8**, pp. 51-54.

Tsoneva R. and T. Patarinska (1995). Optimal control of continuous fermentation processes. Decomposition methods for start-up, steady-state and minimum time problems. *Bioprocess Eng.*, **13**, pp.189-196.

Tsoneva R. and I. Popchev (1997). Guaranteed cost control for discrete time systems with parameter uncertainties, *Proc of the Fourth Intern. Symp. On Methods and Models in Automation and Robotics*, Miedzyzdroje, Poland, 141-144.

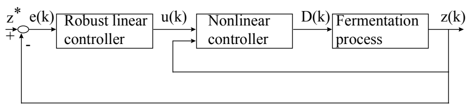

Fig. 1. The closed loop control system

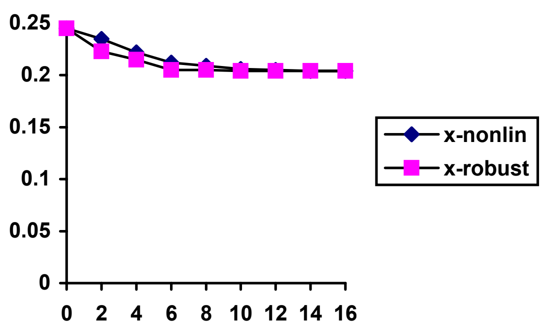

Fig 2a Biomass concentration of the closed loop system

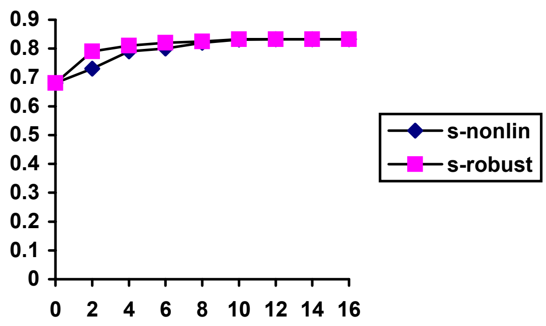

Fig 2b Substrate concentration of the closed loop system

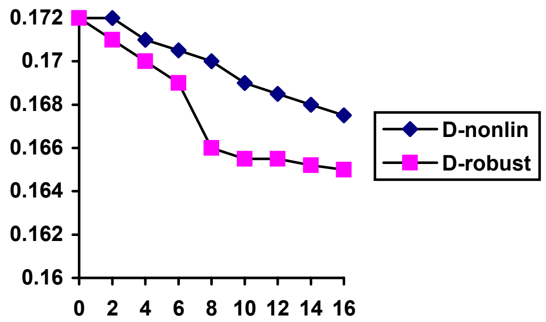

Fig 2c Dilution rate of the closed loop system