

ESTIMATING THE HEART INSTANTANEOUS FREQUENCY USING THE EARM ALGORITHM.

Marcio de Oliveira Santos^a, Allan Kardec D. Barros^a,

^a*Depto. de Engenharia de Eletricidade, Universidade Federal do Maranhao,
Sao Luis-MA/Brazil.*

Abstract

The cardiac signal (ECG) have some characteristics which we may understand the internal mechanisms of sympathetic and pathetic systems. The main caracatheristic is its quasi-periodic behavior. This behavior can be analysed through HRV – *Heart Rate Variability* formed by the time differences between two consecutive major peaks in a cardiac signal. Due to sample rate of HRV, alternatives methods was proposed to extract the HRV from ECG. Barros and Ohnishi [1] proposed the HIF Method – *HIF – Heart Instantaneous Frequency*, that use a driver function and wavelets to calculate the instantaneous frequency of ECG with small error in respect to HRV. In this paper, we verify the strength of our algorithm – EARM –, comparing the results obtained from it to the ones obtained from HIF, using samples of regular cardiac signals and sleep apnea ones. The comparatives results show a small error, demonstrating that EARM method can be used as alternative to HRV.

Keywords: ARMA, Filter design, Nonlinear models, Uncertainty.

1 Introduction

A well-known characteristic of the heart is its rhythmical activity. This behavior can be analysed through the *HRV – Heart Rate Variability*. The HRV is calculated by the set of inverses of the temporal differences between two consecutive R waves in a ECG signal. The fluctuation in the ECG is a result of the interactions between the sympathetic and parasympathetic systems and pacemaker cells thus the HRV becomes an important tool for the analysis of the state of the autonomous nervous system - ANS.

Nevertheless, the HRV presents an inherent disadvantage: its sampling rate de-

pends on the sampling rate of the ECG, which are acquired values in the order of 128 Hz, for patients at rest; and, 500 Hz, for patients under exercise. Due to this fact, some researchers decided to propose alternative methods for the calculation of the HRV. These methods, according to what (Lovell *et al.* [8]) shows, need to have a good time-frequency resolution. Hence, these new methods basically use the STFT and the Wavelets as analysis tools. Among the authors that proposed methods using *STFT – Short Time Fourier Transform*, are (Kwok and Jones [7]), (Cohen and Lee [2]), (Kootsookos *et al.* [6]) and (Tanaka *et al.* [10]). The disadvantage of these methods is its prohibitive computational weight, which makes more difficult

an on-line implementation.

Other stronger measure is given by the *HIF* method – *Heart Instantaneous Frequency*, formulated by Barros and Ohnishi [3]. It consists of two steps:

- Elaboration of a Driver function that will provide a rough estimation of the frequencies present in each time interval;
- Calculation of the Heart Instantaneous Frequency, using a band-pass filter around the central frequency obtained in the last step. To construct this filter, Wavelets were used.

Initially our job was proposed for offline processing, like (Barros [1]) method . With the modifications introduced in this paper, we managed to produce a version of the HIF for extraction in real time. These modifications are the new approach for the calculation of the Driver function, which is obtained by using a modified model of the Auto-regressive model proposed by (Kato and Ozaki [5]); and the optimization of the band-pass filter, reformulating the Wavelet used in [1].

2 The Method

The construction of our algorithm follows two main steps: estimation of the Driver function, as in the HIF method. This will provide us with an indication of the main frequency found in the signal; and, the modification of the Wavelet implemented in [1], in a way that the joint resolution in the time-frequency domain is as good as possible, according to what (Gabor [3]) established.

2.1 Estimation of the Driver Function

The first part of our task is to formulate a driver function that will provide an indication of the frequencies present in the

signal. To do so, we modified the PFNAR model, proposed by (Kato and Ozaki [5]). This occurs by introducing exponential coefficients to the usual AR model. This yields a better performance in obtaining the instantaneous frequency and also give it a better resolution. We called this new model EARM – *Exponential Auto Regressive Model*. Our model is auto regressive because it depends on the values of the previous samples used in the calculation. In the classic auto-regressive model, the higher the order of regression the more precise the result will be. However, a higher progression order would result in more processing time and more computational load.

Hence, to obtain more precise results, without needing to increase the regression order, we introduced exponential coefficients, resulting in the following formula:

$$\sum_{k=0}^M [a_k x(n-k) + b_k \cdot e^{j\Omega_0 n}] = \varepsilon(n) \quad (1)$$

where:

- a_k – auto-regressive coefficients;
- $x(n-k)$ – values of our input signal \mathbf{x} with a delay of \mathbf{k} time units;
- b_k – coefficients with the values of the amplitudes of the frequencies found in the ECG;
- Ω_0 – vector of the most representative frequencies found in a cardiac signal, which the values vary from 0.8 to 2.0 Hz;
- $e(n)$ – gaussian error.

The equation (1) can be rewritten in the following way so we can have the square error, from where we will derive the system of equations that will provide the instantaneous frequency of the signal that we are studying:

$$\varepsilon_k = d_k - A_k \cdot X_k + B_k \cdot E_k \quad (2)$$

With:

- ε – gaussian error;
- d – input signal;
- A – vector of auto-regressive coefficients;
- X – set of vectors with the values of the time-delayed input signals;
- B – coefficients of the frequency vector \mathbf{E} ;
- E – vector of the frequencies found in the cardiac signal;
- k – time index.

To obtain the equation for the estimation of the HIF in *Batch* mode and *on line*, we have to derive the square error which is given by the J_ω cost equation:

$$J_\omega = \varepsilon^2 = (d_k - A_k \cdot X_k + B_k \cdot E_k)^2 \quad (3)$$

Deriving (3) in relation to the vectors A and B and equalizing to zero, we obtain the system of equations used in *Batch* processing:

$$A = [dX + B^T EX] \cdot [XX^T]^{-1} \quad (4)$$

$$B = [dE + A^T XE] \cdot [EE^T]^{-1} \quad (5)$$

To extract the CIF in real time, we update the values of A_k and B_k using the following equations:

$$A_{k+1} = A_k + \mu \cdot (B^T E + X^T A - d)X \quad (6)$$

$$B_{k+1} = B_k + \eta \cdot (A^T X + E^T B - d)E \quad (7)$$

Where μ and η are the learning rate of the system.

2.2 Modification of the Wavelet

The next step of the EARM method is the reformulation of the Wavelet used in [1], so the relation in the time-frequency domain is optimized to the highest value assumed by the multiplication of the variance in the

time domain (σ_t) by the variance in the frequency domain of the signal (σ_ω). This is the formulation of the Heisenberg's uncertainty principle which was adapted to signal processing by (Gabor [3]).

Being the original Wavelet given by:

$$\psi(t) = e^{-\pi \left(\frac{t \cdot f_0}{\sigma}\right)^2} \cdot \sin(f_0 \cdot t) \quad (8)$$

The sine factor above can be rewritten as follows:

$$\sin(f_0 \cdot t) = \frac{e^{j \cdot f_0 \cdot t} - e^{-j \cdot f_0 \cdot t}}{2} \quad (9)$$

As our Wavelet is analytical, the negative complex exponential of the numerator equals zero, redefining (8) as being:

$$\psi(t) = e^{-\alpha \cdot t^2} \cdot e^{i \cdot f_0 \cdot t} \cdot \frac{1}{2} \quad \text{with } \alpha = \pi \cdot f_0^2 / \sigma^2 \quad (10)$$

Equation (10) belongs to a set of Gabor functions [3], except for the constant 1/2. The Gabor functions have the best time-frequency resolution, with joint variance in this domain equals 1/4 [3]. To verify the influence of the constant 1/2 in the final calculation of the variance of (10), we must calculate σ_t^2 and σ_ω^2 :

$$\sigma_t^2 = \frac{1}{4} \cdot Var [e^{-\alpha \cdot t^2} \cdot e^{i \cdot f_0 \cdot t}] = \frac{1}{4 \cdot \alpha} \quad (11)$$

For the calculation of σ_ω^2 , we used the result of the Fourier Transform of (10),

$$\hat{\psi}(\omega) = \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot e^{-\frac{(\omega \cdot f_0)^2}{2\alpha}} \quad (12)$$

So, σ_ω^2 equals:

$$\sigma_\omega^2 = \frac{1}{4} \cdot \frac{2\pi}{\alpha} \cdot 2\alpha = \pi \quad (13)$$

To satisfy the condition $\sigma_t^2 \cdot \sigma_\omega^2 = \frac{1}{4}$, we will have $\alpha = \pi$. Transposing this result to (8),

we obtain $\sigma = f_0$, resulting in,

$$\psi(t) = e^{-\pi \cdot t^2} \cdot \sin(f_0 \cdot t) \quad (14)$$

3 Results

The strength of the EARM algorithm was tested only for batch signals. The first signal that we used as a test was the senoidal signal composes by waves of 3.5 and 7 Hz, of different amplitudes to white noise was added. The steps to obtain the HIF are:

- Obtaining the sample of the input signal \mathbf{x} ;
- Composition of the vector of frequencies \mathbf{E} , composed by the component frequencies of the signal (3.5 and 7 Hz);
- Establishment of an arbitrary error of 10^{-7} and use of the formula 4, obtaining the wanted results.

After the calculation of the HIF via EARM, we calculate the instantaneous frequency through HIF. The results were compared among themselves through the mean square error formula:

$$\varepsilon = \sqrt{\frac{1}{N} \sum_{j=1}^N \left[\frac{fy(j) - hif(j)}{hif(j)} \right]^2} \quad (15)$$

Where:

- fy – vector of the samples produced by the EARM method;
- hif – vector of the samples produced by the HIF method.

The difference in percentage found for this first signal was 0,27%.

The next test was the analysis of real cardiac data. 10 cardiac signals were selected from normal people and 8 cardiac signals from sleep apnea carriers. From this point, we steps followed same as the previous analysis, compounding the vector E with

frequencies varying from 0,8 ~ 2,0 Hz. The values of the differences in percentage between the EARM and the HIF are given by tables 2.2 and 2:

Table 1

Comparison of the results obtained by the EARM and HIF methods for normal cardiac signals

Signal	Error in relation to HIF
s1	0.0815
s2	0.0056
s3	0.0234
s4	0.0064
s5	0.0193
s6	2.876e-4
s7	0.0318
s8	0.0101
s9	0.0122
s10	0.0177

Table 2

Comparison of the results obtained by the EARM and HIF methods for sleep apnea.

Signal	Error in relation to HIF
s11	0.0054
s12	0.0034
s13	0.0054
s14	8.1414e-5
s15	0.028
s16	0.0096
s17	7.2453e-4
s18	0.0751

A factor that influences decisively to the correct determination of the instantaneous frequency in the EARM is the number of samples in the signal used for the calculation. The figures 1,2 and 3 show the values of the EARM. – for inputs of 25.000, 50.000

and 100.000 samples, respectively – compared to HIF values, with errors in percentage between the methods shown in Table 2.2.

Table 3

Comparison of the results obtained for samples of different sample sizes.

Size of the input (points)	Error in relation to HIF
25.000	0.0251
50.000	0.0075
100.000	0.0056

4 Conclusion

The main characteristic of the natural signals is its temporal non-determinism, which is why they are considered stochastic processes. Spite of this, some signals have a aperiodicity that allow us to obtain its instantaneous frequency. A good example of these signals is the cardiac signals, which the instantaneous frequency allows us to evaluate the state of the Autonomous Nervous System. Some methods were proposed for the extraction of the instantaneous frequency, like the ones shown in (Kwok and Jones [7]), (Cohen and Lee [2]), (Kootsookos *et al.*[6]) and (Tanaka *et al.*[10]) – which are based on STFT – *Short Time Fourier Transform*. Because it presents a high computational cost, an on-line implementation is almost impossible.

Our task was based in (Barros and Ohnishi [1]) work – which allows the extraction of cardiac instantaneous frequency (HIF) offline – and introduced exponential coefficients to the PFNAR Auto-Regressive model, by (Kato and Ozaki [5]). This new model, which was called EARM, will be used as Driver function. The filtering is based on the Wavelet of [1], which was modified to improve the relation $\sigma_t^2 \cdot \sigma_\omega^2$ to 1/4.

Due to the changing in the Driver function,

we developed two equation systems: one for the extraction of the FIC in *batch* and the other for the *on line* extraction.

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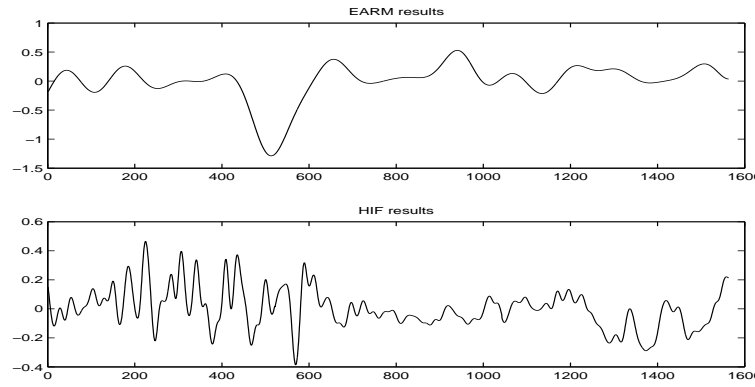


Fig. 1. Sinal de entrada com 25.000 pontos

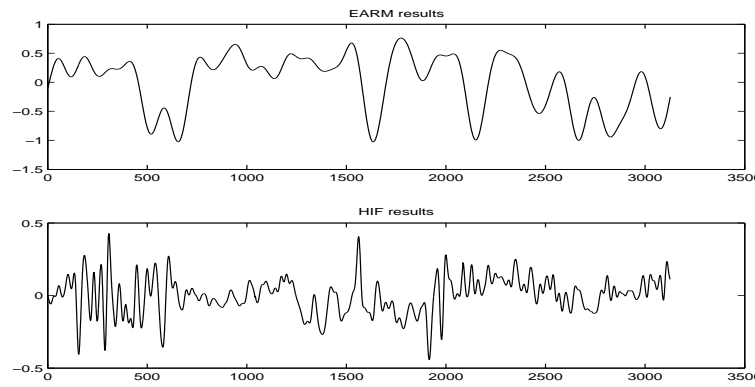


Fig. 2. Sinal de entrada com 50.000 pontos

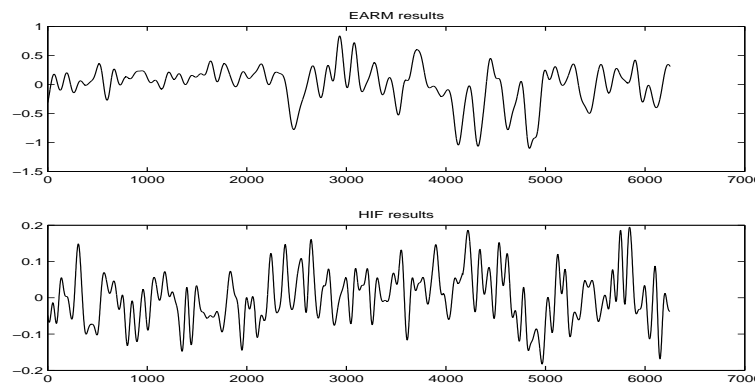


Fig. 3. Sinal de entrada com 100.000 pontos