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Abstract: Electricity Real Time Pricing (RTP) tariffs inter alia have Demand Side Management (DSM) as a main aim. Customers are billed according to hourly fluctuating energy prices. Essentially RTP pricing signals can be seen as control inputs to change customer load usage. It is suggested that optimised pricing signals sent to RTP customers can lead to a much more efficient DSM initiative. This paper describes the methodology followed, and results obtained, of such an optimisation process.

Keywords: RTP, DSM, Optimisation, electricity, IP

1. BACKGROUND

Electricity Real Time Pricing (RTP) tariffs inter alia have Demand Side Management (DSM) as a main aim. RTP Customers are billed according to hourly fluctuating energy prices (David and Li, 1991).

Both one and two part RTP tariffs are currently in use in South Africa. Two Part RTP is based on a fixed price for energy usage at the agreed upon Customer Base Load (CBL). For energy use above this base the customers pay the varying RTP price. The customer receives a rebate, based on the RTP price, if less than the base load is used.

Under One Part RTP the customer is billed against the varying energy prices for his entire usage.

The reasoning behind these tariffs, from a DSM perspective is clear. High prices indicate an over demand/under supply situation. The theory is that if the clients are exposed to the high prices they will temporarily cut back on their demand, thus relieving strain on the network.

Industry feedback indicate that these tariffs do indeed go a long way towards achieving their goal. A short coming of the tariff is that the prices sent to the customers do not take their price sensitivity into account. This leads to a sub-optimal system.

This paper proposes a technique to optimise the pricing signal sent to customers. The signal is optimised in such a way that maximum DSM reaction is obtained at minimum cost to the utility.

The above optimisation problem can be solved within the Hybrid Systems modelling framework, utilising Integer Programming (IP). The process inter alia involves writing the customer load response models in Mixed Logical Dynamical format (Williams, 1993), and using Branch-and-Bound techniques to find the optimal solution.

This project forms part of a larger whole, the ultimate goal of which is to use the MLD models developed, and techniques described by Bemporad, Morari, and various other authors (Bemporad and Morari, 1999; Bemporad *et al.*, 2000a; Bemporad *et al.*, 2000b) to design an optimal controller. The results described here will be used as a benchmark.

2. CUSTOMER RESPONSE MODELS

2.1 Introduction

After extensive investigation, it was found that RTP customers respond to short term price changes, rather than to absolute prices. Due to the nature of the customers' processes, they tend to react to prices that differ from the average, rather than to the increase of the average price. The customers have to keep their processes running on the long term, so if the average cost of electricity increases they have to absorb it, or "go under". But, they do respond very well to daily price spikes away from the average.

This study was conducted on 5 Two Part RTP customers.

It was found that the hourly price compared to a moving price average over 168 hours (1 week) is a

realistic input to the system. The output is then the hourly load compared to a 168 hour moving load average. In other words, to obtain the input to the plant for a certain hour, the RTP prices for the previous 168 hours are averaged, this average is then subtracted from the RTP price in the particular hour, with the result referred to as Δ RTP:

$$\Delta RTP_{T_0} = RTP_{T_0} - \left\{ E\left(RTP\left(t\right)\right) \middle\| t \in \left[T_0 - 169, T_0 - 1\right] \right\}$$

Where T_0 refers to the current hour, and E() is the average operator.

In the same way the plant output, $\Delta Load_{T_0}$, can be defined as:

$$\Delta Load_{T_0} = Load_{T_0} - \{E(Load(t)) | t \in [T_0 - 169, T_0 - 1]\}$$

The subscript T_0 will be dropped from $\Delta Load_{T_0}$ and ΔRTP_{T_0} for the rest of this document.

2.2 Typical Customer Response

Figure 1 shows a typical customer's Δ Load vs. Δ RTP, as well as the standard deviation of Δ Load. This customer was selected because its Δ Load response is representative of that of the other customers on the tariff.

It is clear that the load response for a typical customer is non-linear. However, closer inspection reveals that the response can be divided into distinct control "regions". Within these regions linear approximations can be used to describe the reaction.

This makes it possible to write the models within the MLD framework, and use IP to find an optimal solution.





2.3 Modelled Response

Linear regression was used to obtain models describing the Δ Load vs. Δ RTP in the identified regions.

For the customer introduced in the previous section, the following model was obtained:

$$\Delta Load = 0 \text{ with } \Delta RTP \in [0, 15]$$
(1)

$$\Delta Load = -A \text{ with } \Delta RTP \in (15, 50]$$
(2)

$$\Delta Load = -A + m \cdot (\Delta RTP - 50)$$

with $\Delta RTP \in (50, 70]$ (3)

$$\Delta Load = -B \text{ with } \Delta RTP \in (70, \infty]$$
(4)

The average standard error, defined as the difference between the actual and modelled response is 6%.

Figure 2 shows the modelled vs. actual response for the customer in question.





Fig. 2. Typical Customer Modelled vs. Actual response

Although the Δ Load response model for only one customer is presented in this section, the responses of all 5 customers were used for the optimisation procedure presented in the next section.

3. OPTIMISATION

3.1 Introduction

The structure of the models presented lend themselves to optimisation in the Integer Programming (IP) framework (Williams, 1993).

Once the reaction models had been derived, the next step was to find a transform that would change the input Δ RTP price (RTPInput) to an optimal RTP price (RTPOutput).

It was important to not only optimise for maximum DSM reaction, but also to bring the cost of the

reaction into the utility function to be optimised. The reason for this is that if optimising for DSM reaction only, then the output price would always be the highest possible price, as this would ensure that all the customers reacted to their maximum capability. This would however mean extremely high cost to the utility, and is also not viable as it would cause all the prices over all hours to saturate at their maximum allowed level.

Optimising for cost only will have the reverse effect, the prices would always be pegged at their minimum levels – having no positive effect on the customers' reactions.

The utility function used for optimisation purposes is made up of two components: a DSM Index that gives an indication of the reaction compared to the maximum possible reaction; a Cost Index, that gives an indication of the Cost of the reaction compared to the maximum possible cost. These indexes are defined as follows:

$$CostIndex = 1 - \frac{\Delta RTP^*}{\max(\Delta RTP)}$$

where

$$\Delta RTP^* = \Delta RTP + \alpha$$

Where ΔRTP is as defined previously, the difference between the input price, and the average input prices over the last 168 hours. ΔRTP^* is defined as the sum of ΔRTP and α .

With
$$\alpha \in [-\Delta RTP, \max(\Delta RTP) - \Delta RTP]$$

Max(Δ RTP) is defined as the maximum possible Δ RTP that will still elicit a further reaction from the participants (as can be seen from the customer model above, if Δ RTP goes above a certain level, the customers can no longer further react to it, and have to absorb the price).

$$DSMIndex = \frac{\Delta Load^*}{\max(\Delta Load)}$$

Where as above, $\Delta Load^*$ represents the difference between the clients' reactions, and their average (Nett) load over the past 168 hours, given the price signal ΔRTP^* . Max($\Delta Load$) is the maximum possible reaction (to a price signal larger or equal to max(ΔRTP)).

The utility function for this application is defined as follows:

UtilityFunction = CostIndex · DSMIndex

A commercially available IP solver (What's Best) was used to find the ΔRTP^* that gives the maximum UtilityFunction for each input ΔRTP .

In other words, for each input $\Delta RTP \in [0, \max(\Delta RTP)]$, a corresponding ΔRTP^* was found that maximises the UtilityFunction.

3.2 Results

The results of the above optimisation process is a mapping giving an optimal ΔRTP^* for each ΔRTP , as shown in Figure 3.

These results were stored in a look-up table. This means that one does not have to rerun the whole optimisation process each and every time one wants to find the optimal ΔRTP^* given a certain ΔRTP .

For example, lets say ΔRTP is 7c/kWh, and one wants to find the optimal ΔRTP^* , one can simply lookup the value (21 c/kWh in this case) from the table. Should the dynamics behind the models change, or one decides to change the Utility Function, it will be necessary to regenerate this look-up table.



Fig. 3. Optimal ΔRTP to ΔRTP^* mapping.

4. APPLICATION OF RESULTS TO HISTORIC RTP PRICE SERIES

4.1 Introduction

The optimisation results were now applied to historical RTP data, in order to find what the implication would be to historical DSM and revenue.

RTP prices for the period October 2002 to end September 2003 were used. These prices were used as inputs to the optimiser, and the outputs represented both the optimal ΔRTP^* for each hour, as well as the optimal client reaction. The assumption here is that the customers will respond as modelled.

An important parameter for this study is to decide at which level Δ RTP represents a strong DSM signal, as the price transform should only be used on prices representing such a signal.

A few remarks.

By setting the cut-off Δ RTP to 5c/kWh, 43% more DSM can be achieved, at an additional cost of 2.75%.

If the cut-off \triangle RTP is set to 6c/kWh or above, the system starts breaking even. Meaning the nett savings start becoming positive.

For example, if the cut-off Δ RTP is set to 10c/kWh, 12% more DSM reaction can be achieved, whilst realising a 9.3% cost saving compared to normal operation.

At a cut-off $\triangle RTP$ of 6 c/kWh, 33% more DSM reaction can be obtained, and a 1.5% cost saving realised. So the utility can essentially, at no cost, achieve 33% more DSM reaction.

4.2 Finding an Optimal Cut-Off ΔRTP.

As already mentioned, it is important to identify an optimal cut-off input ΔRTP . The optimisation transform should only be applied to prices above this cut-off price. It is relatively simple to find an optimal point for this cut-off price, as will be presented next.

In order find an optimal cut-off ΔRTP , the following indexes were used:

$$DSMIndex(COP) = \frac{AdditionalDSM(COP) + NormalDSM}{NormalDSM}$$

Where AdditionalDSM is the total DSM Reaction achieved under the normal price regime. AdditionalDSM(COP) is the additional DSM reaction achieved given the optimal price mapping, and the cut-off price (COP). $SavingIndex(COP) = \frac{NettSaving(COP)}{MaxSaving}$

Where NettSaving(COP) is the difference between the additional cost, and savings obtained, given a offoff price COP. MaxSaving is the maximum saving that can be achieved given any cut-off price.

A utility function, combining the two indexes above was now created:

TotalIndex(COP) = SavingIndex(COP) · DSMIndex(COP)

Figure 4 provides a graphical presentation of TotalIndex(COP).



Fig. 4. TotalIndex(COP).

TotalIndex has a maximum value (equal to 0.9929) at COP=16 c/kWh. At this off-off price, the nett saving is 12%, with the additional DSM reaction close to zero. This point thus also represents the scenario where one wants to keep the DSM reaction constant, and is only interested in obtaining savings.

5. CONCLUSION AND FUTURE WORK

The results obtained look promising. The techniques used lend themselves very well to modelling and optimising RTP customers' responses to different input prices.

This project forms part of a larger whole, the ultimate goal of which is to use the models developed, and techniques described by Bemporad, Morari, and various other authors (Bemporad and Morari, 1999; Bemporad *et al.*, 2000a; Bemporad *et al.*, 2000b) to design an optimal controller. The results described here will be used as a benchmark.

It is recommended that the implications of practically implementing such an optimised pricing scheme be investigated. Special attention should be given to the political implications (both within the utility, and as far as the National Energy Regulator is concerned).

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