A MODEL OF THE DIFFUSION OF BREAKTHROUGH PRODUCTS

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Abstract: Breakthrough products are innovative products that address a new set of needs and thus require the creation of a new market in order to succeed. Due to this requirement, the eventual success of breakthrough products is notoriously difficult to predict.

In this paper, a new model for the diffusion of breakthrough products is developed, based on the well-researched and widely accepted observation that new adopters need an acceptable reference before they actually adopt. This simple observation implies a reference network between potential adopters that can be modeled as a directed graph.

If the reference network is assumed to be a random graph, a first order differential equation can be derived. The solution of the differential equation predicts initial exponential growth, followed by a stage when new adoption gradually declines as saturation sets in, similar in form to qualitative adoption life cycles. The model requires only a small number of parameters.

The model assumes random networks but it is unlikely that many reference networks would be truly random graphs. Reference networks are examples of social networks and studies indicate that social networks typically are scale-free networks. Scale-free networks contain a few nodes with a very large number of incoming links, which means that a few reference sites are very widely acceptable. These special nodes are known as hubs in network terminology.

Closed form expressions for diffusion in scale-free networks are more difficult to obtain but computer simulation demonstrates a number of important observations. The major observations are that the shape of the adoption curve is very similar for random and scale-free networks but that adoption by a reference hub in a scale-free network is both necessary and sufficient for the rapid diffusion of a breakthrough product.

Keywords: Diffusion, Innovation, Life cycles, Reference networks.

1. INTRODUCTION

Breakthrough products are innovative products that address an unfulfilled set of needs and thus require the creation of a new market in order to succeed. Due to this requirement, the eventual success of breakthrough products is notoriously difficult to predict.

In this paper, a new model for the diffusion of breakthrough products is developed, based on the well-researched and widely accepted observation that new adopters need an acceptable reference before they actually adopt. This simple observation implies a reference network between potential adopters that can be modeled as a directed graph. Such a network is illustrated in Figure 1.

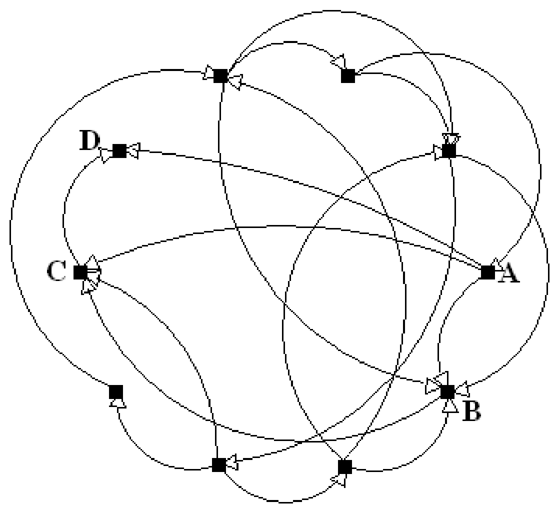

Fig. 1. Reference network of potential buyers

The nodes in Figure 1 represent potential adopters, while the directed links depict reference relationships. For example, node **A** in figure 1 considers node **B** to be an acceptable reference but the reverse is not true. Expressed differently, it means that if and when node **A** develops a buying impulse, it will definitely adopt the breakthrough product if node **B** has already adopted it. On the other hand, if neither node **B** nor any other acceptable reference node (nodes **C** and **D** in this case) has adopted, node **A** will not adopt, even if it currently has a strong buying impulse.

In the same manner, any node has a number of directed links that are connected to other nodes. Should any node adopt the breakthrough product, it in turn becomes a reference site for further potential adopters.

A breakthrough product is thus similar to a virus spreading through a population. Just as a virus needs contact between members of the population for an epidemic to occur, a breakthrough product needs an interconnected network of potential adopters to be able to penetrate the market.

If we plan to introduce a breakthrough product, the market for it invariably is a new market. In such a case it means that we will experience difficulty in selling the product without reference sites. This is a universal difficulty experienced with diffusion of innovation in social networks (Rogers, 1995).

Let us, for the moment, assume that our breakthrough product has been sold to the first adopter and is operating successfully. Our next concern is to persuade as many potential adopters as possible to adopt the product. This process is known as diffusion. We will develop a mathematical model for the diffusion of breakthrough products in the rest of this paper.

2. ADOPTION LIFE CYCLE

A first-time buyer of a breakthrough product is known as an adopter. A buyer can obviously adopt a breakthrough product for the first time only once. Should a buyer buy a similar product again, it is called a replacement. We are only interested in first time adoptions here.

The starting time, $t = 0$, is defined to occur when the first potential buyer adopts. Initially, relatively few other potential buyers are linked to the first adopter and the next sale only occurs when one of those linked nodes develops a strong buying impulse. The number of adopters grow very slowly in the beginning, but with increasing adoptions, the likelihood of further adoptions increases exponentially.

Over time, with a large percentage of potential buyers becoming adopters, the number of non-adopters declines accordingly. This slows down new adopter growth until it virtually stops. The resulting curve that is traced by the number of new adopters over time is known as the adoption life cycle. We develop a mathematical model for the adoption life cycle in this section.

As shown in the previous section, a potential buyer must develop a buying impulse **and** have access to a reference site before adoption occurs. For simplicity, we assume that the occurrence of buying impulses is independent of the reference site network structure.

This is a reasonable assumption if we accept that buying impulses occur due to particular problems encountered by potential adopters and not in the first place as a result of external communication. This assumption is most valid in cases where buying impulses originate from actual needs rather than fashion trends. Our model will thus be more useful for sales to companies than to consumers.

Another way to describe this scenario is to think of attending a cocktail party where you expect to know only a few people. If you are like me, you do not like standing around in a room full of strangers. When you arrive, you peep in and only enter when you see somebody that you know, otherwise you go for a walk and return after a while to see if an acquaintance has arrived in the meantime.

In this simple scenario, the arrival times at the door and your network of acquaintances could be said to be independent (assuming that you do not all agree to arrive at the same time).

We designate the total buyer population as N, the average number of links per node as k and the number of buyers who have already adopted as A. The next step is to calculate the probability that, given that a non-adopter node is linked to another node, the other node is an adopter or non-adopter:

$$P\{\text{a link is to an adopter}\} = \frac{A}{N}$$
$$P\{\text{a link is to a non-adopter}\} = 1 - \frac{A}{N}$$

$$\begin{aligned}\text{where } k &= \text{average number of links per node}\\ A &= \text{number of adopters}\\ N &= \text{total number of potential adopters}\end{aligned}$$

A potential adopter who develops a buying impulse will buy, according to our network reference model, if the buyer's associated node is linked to at least one adopter. Conversely, the potential adopter will not buy, even with a strong buying impulse, if the associated node is not linked to an adopter.

Assuming that there are A adopters at any specific stage, the probability that a potential buyer that develops a buying impulse will actually adopt is [1] :

$$\begin{aligned}&P\{\text{adoption} \mid \text{buying impulse}\}\\ &= P\{\text{link to an adopter}\}\\ &= 1 - P\{\text{all links to non-adopters}\}\\ &= 1 - P\{\text{a link is to a non-adopter}\}^k\\ &= 1 - \left(1 - \frac{A}{N}\right)^k \qquad (1)\end{aligned}$$

The notation $P\{\text{adoption} \mid \text{buying impulse}\}$ is a conditional probability and it reads as follows: the probability of an *adoption*, given that a *buying impulse* occurs. The reason why we may take the k^{th} power of the probability of a link to a non-adopter is because of our assumption that the buying impulse is independent of the links in the network. Strictly speaking, it is only valid for $k << N$.

Now that we know what the probability is for a potential adopter with a buying impulse to actually adopt, all that remains is to calculate the number of clients with a buying impulse at any given stage. Since we are only considering new adopters, the number of remaining potential adopters is $N-A$. If the probability that a specific potential adopter develops a buying impulse during a unit time interval is b, the expected number of buying impulses, $N_{impulse}$, occurring during a unit time interval is:

$$N_{impulse} = b\,(N - A) \qquad (2)$$

where b is the probability of a client developing a buying impulse during a unit time interval.

Using equations 1 and 2, we find the total number of *new* adopters, ΔA, at any given time by:

$$\Delta A = b\ \Delta t\ \ (N - A)\left(1 - \left(1 - \frac{A}{N}\right)^k\right) \qquad (3)$$

$$\begin{aligned}&where\\ \Delta A &= \text{number of new adopters}\\ \Delta t &= \text{duration of infinitesimal interval}\\ A &= \text{number of adopters}\\ b &= \text{buying impulse probability}\\ N &= \text{total number of potential adopters}\\ k &= \text{average number of links per node}\end{aligned}$$

Equation 3 enables us to develop a feeling for the dynamic behaviour of a breakthrough product in a new market. Figure 2 shows a typical example.

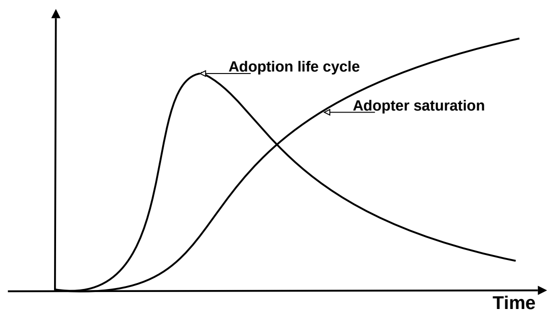

Fig. 2. Typical adoption life cycle, $\frac{\Delta A}{\Delta t}$, and adopter saturation, $\frac{A}{N}$, curves plotted against time

Figure 2 shows the adoption life cycle, $\frac{\Delta A}{\Delta t}$, versus time, as well as adopter saturation, $\frac{A}{N}$, versus time. The adoption life cycle is fundamental to our study in marketing. The adoption life cycle curve shows how the number of new adopters, ΔA, initially increases rapidly (exponentially) and then declines as the adopter population becomes saturated.

It is already obvious that there are two major stages during the adoption life cycle. We call the period before the peak in the adoption life cycle the early market and the period after the peak, the mature market.

Equation 3 can be written as a differential equation by noting that the variable A, the number

[1] This derivation is due to Professor Johan du Preez of the University of Stellenbosch, South Africa.

of adopters at a certain time, t, is a function over time, more accurately written as $A(t)$. In the remainder, we will write $A(t)$ when we want to explicitly indicate the time dependence, but we will also use A in cases where it helps with making the equations less cluttered. Anyway, we mean exactly the same thing by A and $A(t)$.

$$\frac{dA(t)}{dt} = b\left(N - A(t)\right)\left(1 - \left(1 - \frac{A(t)}{N}\right)^k\right) \quad (4)$$

We note that A/N is the adopter saturation, i.e. the ratio between adopters and the total adopter population, N. This is a useful parameter. However, to further reduce clutter, we define the remaining adopter capacity as v, where:

$$v = \left(1 - \frac{A}{N}\right) \quad (5)$$

Substituting equation 5 in equation 4 and using the differentiation chain rule, we obtain:

$$\frac{dv}{dt} = -bv(1 - v^k) \quad (6)$$

Equation 6 describes the diffusion dynamics of a breakthrough product from the time of successful launch. It is interesting to note that, for the special case of $k = 1$, this equation is identical to the *logistic growth equation* first proposed by Belgian mathematician Pierre Verhulst (1838) and widely used in biological population growth studies (Kingsland, 1982).

Equation 6 is also similar to the so-called Bass model (Mahajan and Bass, 1990). The crucial difference between our model and the Bass model is that the latter does not take any networking effects into account and hence implicitly assumes $k = 1$.

In any marketing effort, it is important to be able to predict where the peak in the adoption life cycle occurs. To calculate this, we differentiate the adoption life cycle curve, equation 6, with respect to v, the remaining adopter potential, and set the result equal to zero, in order to obtain:

$$\frac{A_{max}}{N} = 1 - \frac{1}{(k+1)^{\frac{1}{k}}} \quad (7)$$

This result is surprising at first glance. The adoption life cycle peak, as a function of adopter saturation ($\frac{A}{N}$), is only dependent on the average number of links per node in the network. However, if we remember that we can view the diffusion of a breakthrough product into a new market as a virus spreading through a population, it is less surprising that the interconnection rate should play an important role.

Links per node, k	**Adopter saturation, A/N**
1	50%
5	30%
10	21%
15	17%
20	14%

Table 1. Adopter saturation at life cycle peak for various average links per node in the reference network

From Table 1 it is clear that a higher interconnection rate in a market results in earlier market maturity, i.e. at a lower level of adopter saturation. This knowledge is of crucial importance to a fast growing company. An astute organisation can use this information to predict, by estimating the interconnection rate and adopter saturation in a market, when the market will change from an early to a mature market.

The observation that the adopter life cycle peaks at lower adopter saturation levels for higher market interconnectivity highlights a major difference with the Bass model, which implicitly assumes that k is always equal to 1. This, in turn, predicts that the adopter life cycle will always peak at 50% adopter saturation, which will normally be too late.

Note that we have determined what the adopter saturation is when the peak in the adopter life cycle occurs and that it depends only on the average interconnection density, k, in the market. We have not yet determined at what time the peak occurs. In order to do that, we need to solve the differential equation in equation 6. Luckily, this is straightforward and the result is:

$$\frac{A(t)}{N} = 1 - \frac{1}{\left(1 + e^{kbt + \ln\left(\left(\frac{N}{N-1}\right)^k - 1\right)}\right)^{\frac{1}{k}}} \quad (8)$$

While deriving equation 8, it is necessary to define $t = 0$. In line with our assumption so far, we defined $t = 0$ to occur when $A = 1$, i.e. when the first adoption occurs. From that time forward, the adoption life cycle starts its relentless curve, mostly independently of supplier actions.

The observation that the adoption life cycle is independent of supplier actions needs some qualification:

(1) The first adoption must have taken place, otherwise the market is still in $t < 0$ territory. This implies that at least one supplier must have a product offering that offers value for money to an adopter.

(2) Further potential buyers must also see value for money in the available product offering(s), otherwise they remain part of unsatisfied demand. Since one adopter saw value for money and continuous product improvements are usually taking place, it is not unrealistic to assume that other adopters with similar needs to the first adopter will see similar value for money in the available product offering.
(3) Potential adopters who do not see the same value for money proposition in available offerings have different needs to the first adopter, which implies that they are in a different market segment. It could thus be argued that the market segment in which they are has not yet reached time $t = 0$. This implies that closely related but not identical market segments might be in different stages of maturity.

Equation 8 is the desired result. If we know three things about a new market, we are able to predict the adopter saturation at any time after the first adoption. All we need to know is the number of potential adopters, N, the average interconnection rate, k, in the market and the probability that a client will develop a buying impulse in one unit of time, b.

The buying impulse probability, b, might seem to be a difficult parameter to measure. If the adopters are companies, the estimation of b is simplified significantly.

Companies typically work on an annual budget. The buying impulse frequency is thus not more frequent than once per year. Not all potential adopters develop a buying impulse every year. If we assume that one in x adopters develop a buying impulse in a year and our time is measured in weeks, b is calculated as:

$$b = \frac{1}{52x} \quad \text{per week}$$

We are further able to predict at what time, t_m, the adoption life cycle peaks by using equation 7 in 8. The result obtained is:

$$\begin{aligned} t_m &= \frac{1}{kb} \ln \left(\frac{k}{\left(\frac{N}{N-1}\right)^k - 1} \right) \\ &\doteq \frac{1}{kb} \ln(N) \end{aligned} \qquad (9)$$

The approximate solution in equation 9 is obtained with the help of Taylor series. From 9 we draw a few conclusions:

(1) N is always positive and bigger than one, thus $\ln(N) > 0$. The peak time, t_m, is thus always later than the starting time, $t = 0$, of the adoption life cycle. Naturally, this must be the case but stating it is useful as a sanity check on our result.
(2) For larger k, the adoption life cycle peaks earlier. This is consistent with our expectation, i.e. that a breakthrough product diffuses faster in a highly interconnected market.
(3) For larger b, the adoption life cycle peaks sooner. This is also intuitively satisfying.
(4) Equation 9 is an approximation to the exact solution but it is very accurate (better than 0.1% for typical parameters).

3. REFERENCE NETWORKS AS SOCIAL NETWORKS

The existence of reference networks has been studied extensively (Rogers, 1995). Unfortunately such networks are invariably small (less than a hundred nodes) and do not provide reliable evidence of the network structure for networks with thousands of nodes.

It is important to note that reference networks are examples of social networks, since reference relations fundamentally are based on humans, even in strictly business to business markets. Some hints are provided by studying other kinds of social networks. Numerous examples, ranging from actors in Hollywood movies to sexual relations in Sweden to indicate that social networks tend to be scale-free (Barabási and Bonabeau, 2003).

A scale-free network is a network where the histogram of the numbers of links per node is a straight line on a logarithmic scale. In practice this means that there are many nodes with a small number of links while a few nodes have a huge number of links (known as hubs) and every value in between. There is thus no *typical* number of links per node.

The emergence of scale-free social networks can be motivated by noting that such networks grow with new nodes being added over time. Two factors lead to the ubiquity of scale-free networks:

(1) New nodes can obviously only link to older, existing nodes. Even if new nodes link randomly to existing nodes, older nodes will have time to collect more links.
(2) Newer nodes tend to prefer linking to existing nodes with more links. In terms of our reference paradigm it means that existing nodes that are already more widely referred to will, in general, be more acceptable references for newer nodes as well.

An example of a scale-free network constructed in this manner is shown in Figure 3. Note the high number of links at the oldest node at 3 o'clock.

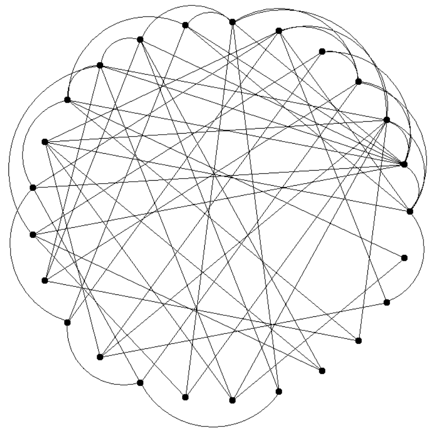

Fig. 3. Scale-free reference network with $N = 25$ and $k = 3$. The oldest node is at 3 o'clock. Newer nodes were added in an anti-clockwise order. Note that all links are directed from newer to older nodes.

It can be shown mathematically that networks grown under these conditions result in scale-free networks (Barabási and Albert, 1999).

In the derivation of the adoption life cycle equation in 6, the implicit assumption was that nodes are linked randomly, thus forming a random reference network. A similar mathematical derivation for scale-free networks is not easy to obtain in closed form, but the effect can be simulated. These simulations demonstrate the following aspects:

(1) The general shape of the adoption life cycle remains the same.
(2) The shape of $\frac{dA}{dt}$ plotted as a function of t may be dramatically different for random and scale-free reference networks but it is very similar when plotted against $\frac{A}{N}$.
(3) If an initial adoption node happens to be a recent addition with few links, such a node is most probably unacceptable as a reference for older nodes with more links. This means that full-scale exponential growth cannot start in practice until one of the hubs adopts.
(4) Once one of the hubs adopts, a large number of potential adopters are linked to the hub and they only require a buying impulse to adopt.

The existence of hubs in scale-free networks thus provides a necessary and sufficient condition for successful diffusion of breakthrough products into scale-free networks. This is consistent with kingpin adopters as described by Geoffrey Moore (Moore, 1991) and underlines the crucial necessity of cultivating such highly influential reference sites early on.

4. CONCLUSION

A mathematical model for the diffusion of breakthrough products into new markets was developed. The main novelty, compared to existing models, is that the concept of a reference network is highlighted. It was shown that, if the reference network is scale-free, like most social networks tend to be, a necessary and sufficient condition for rapid diffusion is adoption by one of the hubs in the network.

Apart from some interesting quantitive results, it is hoped that this paper will sensitize entrepreneurs to the existence of reference networks and the crucial importance of identifying reference hubs. The latter aspect is so important that it is fair to state that if a reference hub cannot be identified in a potential new market, it would be unwise to make any substantial investments in entering the market.

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