CONTROLLING A CLASS OF NONLINEAR PLANTS USING FUZZY GAIN SCHEDULING AND QFT

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Abstract: The work presented expands upon previous research into the uses of fuzzy gain scheduling (FGS). It combines FGS with the robust control design methodology, quantitative feedback theory (QFT). By combining FGS and QFT the advantages from both methodologies are achieved, namely the resulting controller synthesis methodology provides design transparency, because of QFT, and does not suffer from over design or from rapid open-loop variations, due to the FGS. This hybrid controller may be utilised to control a class of non-linear systems, where the plant may be expressed as a linear plant model with time varying parameters. The methodology for designing this hybrid controller is outlined by means of example. The example also experimentally compares a FGS with a simple QFT controller. The simulation results show that the hybrid FGS controller results in improved control over the plant, for little extra design effort.

Keywords: Fuzzy Control, Gain scheduling, Quantitative Feedback Theory

1. INTRODUCTION

Fuzzy gain scheduling is a control method which alters its parameters on-line to counter the effects of changes that occur in a plant.

It has been demonstrated that fuzzy gain scheduling (FGS) can be used to successfully control many systems [1, 2, 3, 4, 5, 6]. Experiments conducted, comparing FGS with other control methodologies, show that FGS often results in improved system control, reducing both control energy and command chatter [5]. In many cases the performance of a FGS controller is similar to that of adaptive learning algorithms. FGS, however, requires a far simpler design and architecture.

The majority of research performed on the advantages of FGS has been in conjunction with PID control [3, 6].

This work expands upon previous research by combining FGS with more general transfer functions to control a class of non-linear plants. FGS is used to reduce the variations and uncertainty in the non-linear plant. By doing so, the effects of those non-linearities are reduced.

QFT, originally proposed by Horowitz [7], has been shown to provide more robust control than most other linear controller design methodologies [8, 9, 10], as it directly addresses the reduction of plant uncertainty in an optimum manner. Thus, QFT is used in conjunction with FGS throughout this paper.

2. BACKGROUND TO FUZZY GAIN SCHEDULING

To use FGS it is essential to understand its two components, namely gain scheduling and fuzzy control.

2.1. Gain Scheduling

Gain scheduling (GS) [11, 12, 13] is a method of varying a linear controller so as to adapt its parameters as the plant's states change. This is useful in the control of plants where changing parameters can be measured. Most commonly, the parameters range is separated into several regions of operation. For each of these regions a linear controller is designed to provide satisfactory control within that region.

Whilst the plant states remain in a specific region, the linear controller, associated with that region, is utilised to control the plant. As the parameter changes, moving the plant into a different operating region, the control is switched to the linear controller associated with the new region.

Most gain scheduling controllers change controller's parameters where the controller's structure remains constant.

The sudden switching of parameters often leads to unsmooth transitions and, in some cases, limit cycles. If, instead of suddenly switching from one value to another, the controller's parameters' values were to vary in a smooth manner, these unsmooth transitions can be reduced or even eliminated.

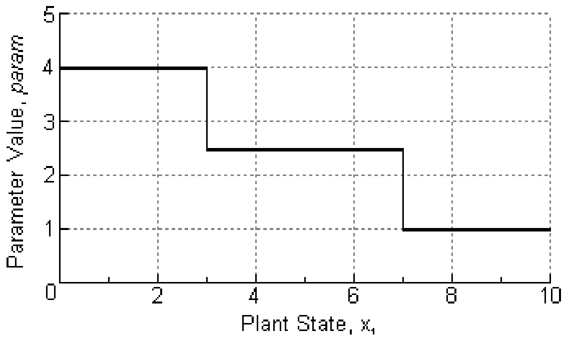

Figure 1: Graph demonstrating a crisp relationship between plant state and controller

Figure 2: Graph demonstrating a smoothed relationship between plant state and controller

Consider the graphs, in figure 1 and figure 2, of parameter value versus plant state. Figure 1 shows a crisp relationship between a plant's state, x_1, and a controller's parameter, *param*. Figure 2, shows a smoothed example of the relationship. Due to the fact that no sudden changes occur in the system model, this smoothed relationship is likely to reduce any unsmooth transitions that may exist as the plant moves from operating region to another.

It should be noticed that the crisp relationship in Figure 1 is easy to implement as it requires the following logic:

If ($x_1 < 3$) **then** *param* is 4
If ($x_1 \geq 3$) and ($x_1 < 7$) **then** *param* is 2.5
If ($x_1 \geq 7$) **then** *param* is 1

2.2. Fuzzy Control

Originally proposed by Lotfi Zadeh in 1965, fuzzy logic is a method of classifying a quantity by stating that it is neither “Big” nor “Small” but to assign a value to the “Bigness” or “Smallness” of the quantity.

Fuzzy logic was later expanded into a decision making process [15] which gave rise to fuzzy control.

In a fuzzy controller, inputs are fuzzified into values denoting the certainty that the input belongs to a certain fuzzy set. A rule base, consisting of “IF *premise* THEN *consequence*" rules, is then applied to these fuzzified inputs. The degree of satisfaction of the premise then denotes the “on-ness” of each rule, known as the Degree of Satisfaction (DoS). The consequence of each of the rules is then combined in a weighted manner according to the DoS of each rule. This produces an output for the fuzzy controller, and is known as defuzzification.

Fuzzy logic may be viewed as a system to interpolate between values on a curve where only a few points have been defined on that curve.

3. COMBINING FUZZY CONTROL AND GAIN SCHEDULING

From section 2.1 it can be seen that GS allows for an easy method to design non-linear controllers to control plants with changing parameters. Furthermore, the governing logic of a crisp GS surface may be described by “IF *premise* THEN *consequence*” rules. This crisp surface is easy to generate, but a smoothed surface provides smoother control in the presence of changing operating regions. Furthermore, from section 2.2, fuzzy logic may be viewed as method of interpolating a surface between defined points on a curve. Thus, it follows that fuzzy logic may be utilised to govern GS. The result is known as fuzzy gain scheduling (FGS).

GS is utilised to design crisp surfaces which will control the plant adequately. Fuzzy logic is then utilised to smooth the crisp surface. The “IF *premise* THEN *consequence*” rules governing the GS surface constitute the rule base for the fuzzy controller, as well as defining the inputs and outputs of the fuzzy logic.

Using fuzzy logic and GS allows for the easy design of smooth gain scheduling surfaces. The structure of a hybrid FGS controller is shown in figure 3. This hybrid controller consists of two layers, a FGS layer and a *G*(*s*) controller layer. The bottom layer, the G(s) controller layer, is responsible for providing the actuator signal for the plant. The FGS determines the values of the parameters of the controller in the bottom layer.

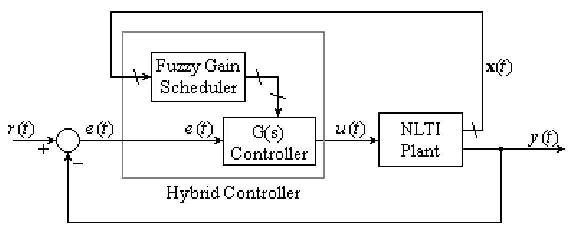

Figure 3: Structure of a control loop using a hybrid FGS controller with unity gain feedback.

The FGS layer may depend on either full or partial state feedback from the plant. Feeding back more states allows the controller to have greater knowledge of the plant. In turn, this allows the controller to be adapted to provide more effective control over the plant, by reducing any over design required.

4. THE USE OF FGS WITH QFT FOR CONTROLLING A PLANT WITH A VARYING PARAMETER: A CASE STUDY

To outline the advantages of FGS, a comparative study follows between QFT and FGS. Consider the plant described by the uncertain function:

$$a\frac{dy(t)}{dt} + y(t) = u(t) \tag{1}$$

where $y(t)$ is the plant output, $u(t)$ the plant input and a some measurable plant parameter restricted to between 1 and 5. Taking the Laplace transform of equation 1 gives:

$$P(s) = \frac{Y(s)}{U(s)} = \frac{a}{s+a} \tag{2}$$

For the controller structure shown in figure 3, assume the following one degree-of-freedom closed-loop specifications:

a) There must be no steady-state error for a step input.
b) The step response must have no more than 15% overshoot.
c) The step response rise time must be between 0.5 seconds and 5 seconds.

These specifications are non-astringent and so, may be approximated by a direct mapping to the frequency bounds. Many closed-loop systems can be approximated by second order prototypes up to the phase-crossover frequency [8, 9]. Therefore, the time domain specifications were translated to frequency domain bounds using second-order prototype models.

Specification (a) gives the required DC gain of the closed-loop system. To achieve zero steady state error, the gain must be 1 at DC. The damping ratio is given by (b) as 0.6 and the undamped natural frequency is given by (c) as lying between 0.8 and 4.

These translated specifications result in the following upper and lower specification bounds:

$$\left|\frac{16}{(j\omega)^2+4.8j\omega+16}\right| \le \left|T(j\omega)\right| \le \left|\frac{0.8}{(j\omega)^2+1.6j\omega+.64}\right| \tag{3}$$

where $T(s) = \left|\frac{L(s)}{1+L(s)}\right|$ and $L(s) = G(s)P(s)$ for controller $G(s)$ and plant $P(s)$.

4.1. Design of a Simple QFT Controller

A simple QFT controller was designed for the plant such that the specifications would be met. The nominal plant chosen for the design of the QFT controller was the case where $a = 1$:

$$P_N(s) = \frac{1}{s+1} \tag{4}$$

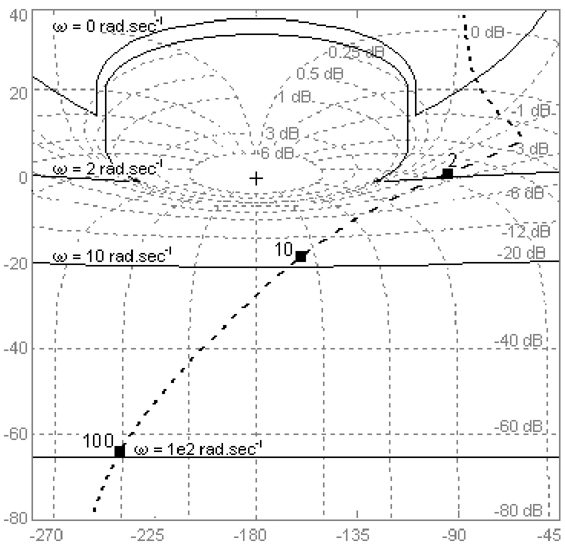

Figure 4: Nichols chart showing the nominal loop transmission function, $L_0(s)$, with the lower nominal loop transmission bounds superimposed.

The one degree-of-freedom control strategy produces closed regions on the Nichols Chart wherein the open-loop system must lie. To ensure that the Nichols Chart remains uncluttered, only the lower nominal loop transmission bounds have be superimposed onto the plot. This is justified by the fact that the controller was designed to have minimal gain.

The QFT controller was then designed to meet the specifications, i.e.

$$G_{QFT}(s) = 0.8 \cdot \frac{s \cdot \frac{1}{0.3} + 1}{s \cdot \left(s \cdot \frac{1}{5} + 1\right) \cdot \left(s \cdot \frac{1}{50} + 1\right)} \qquad (5)$$

This controller is sub-optimal in that the phase margin can be decreased by 40^0. This over design contributes to less peaking in the closed-loop output signal and increased bandwidth, which, for this demonstration, is not considered important.

4.2. *Design of the Hybrid FGS Controller*

Using the form of the QFT controller, a hybrid FGS controller was designed. The form of the FGS controller was thus:

$$G_{FGS}(s) = K_c \cdot \frac{s \cdot T_d + 1}{s \cdot \left(s \cdot T_i + 1\right) \cdot \left(s \cdot \frac{1}{50} + 1\right)} \qquad (6)$$

where K_c, T_d, and T_i are the controller parameters that are varied by the FGS layer.

To design the gain scheduling component of the FGS layer, the plant was considered at several values of the parameter *a*. The values chosen where 1 and 5, as these are the extremes of the parameters range, as well as 2.3, approximately the geometric mean of the range of *a*.

A controller of the same parametric structure as that of equation 6 was then designed for each of the plant models at these values of *a*, using an iterative, “by-hand”, refinement approach. These controllers where designed such that the closed-loop transfer function sat approximately mid-way between the upper and lower frequency specifications as defined in equation 3. This resulted in the controller parameters given in table 1.

Table 1: Table relating *a* to K_c, T_d and T_i.

	Value of *a*		
	1	2.3	5
K_c	0.6	0.6	0.5
T_d	1	2	3
T_i	.1	.157	.33

4.2.1. Designing the Fuzzy Rule Base

From table 1, the following fuzzy rule base was obtained:

1. **If** $a = 1$ **then** $K_c = 0.6$, $T_d = 1$ and $T_i = 0.1$.
2. **If** $a = 2.3$ **then** $K_c = 0.6$, $T_d = 2$ and $T_i = 0.157$.
3. **If** $a = 5$ **then** $K_c = 0.5$, $T_d = 3$ and $T_i = 0.333$.

4.2.2. Designing the Input Fuzzification Membership Functions

The input to the fuzzy layer was identified as the measurable plant parameter a. Since the controllers were designed using three values of a, these same three values form the basis for the input fuzzification. These values where 1, 2.3 and 5. Thus the three membership functions (MF), each chosen to be triangular, are named "1", "2.3" and "5" each peaking to a certainty of 1 at the values 1, 2.3 and 5 respectively. Triangular MFs were chosen since they are both easy to design and not computationally intensive.

4.2.3. Designing the Output Defuzzification Membership Functions

Since K_c takes on the values 0.6 and 0.5, two output MFs where defined named "0.6" and "0.5". Similarly, for T_d three output MFs were defined, namely "3", "2" and "1". For T_i again three MFs were used, named "0.333", "0.157" and "0.1".

The FGS surfaces obtained are shown in figures 5 to 7.

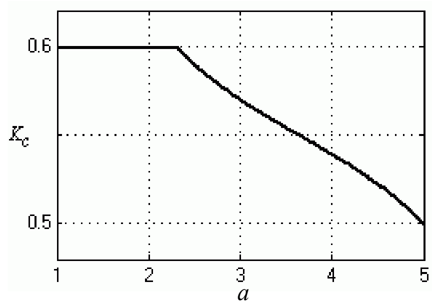

Figure 5: Fuzzy surface relating a to K_c.

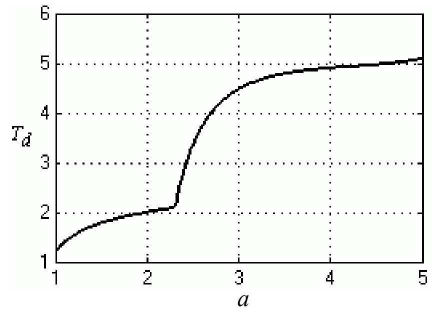

Figure 6: Fuzzy surface relating a to T_d.

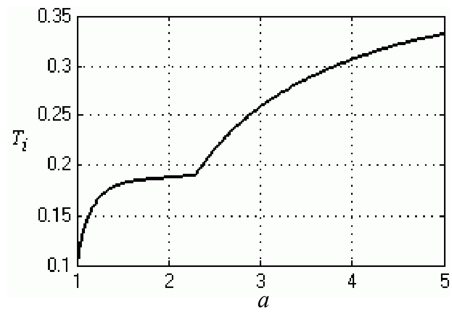

Figure 7: Fuzzy surface relating a to T_i.

4.3. Results of the Experiment

The resulting closed-loop non-linear system was simulated using Simulink. To ensure that the plant parameter a varies rapidly, a was varied as fast and with the same amplitude as the plant output, $y(t)$, i.e. $a(t) = y(t)$. This ensures that the variation of a is fast in comparison to the bandwidth of the system.

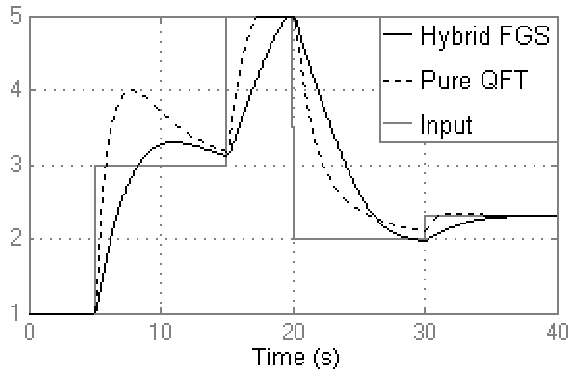

Figure 8: Simulated output of the experiment.
RMS power of actuation signal [QFT]: 20.30
RMS power of actuation signal [FGS]: 8.38

The results of the simulation are shown in figure 8. In addition to showing the output of the plant, the RMS actuator power required to control the system is also shown. This value for the power was obtained by calculating the RMS value of the actuator signal. This value is merely a comparative measure, not absolute, due to the nature of the simulation.

The most significant result from this experiment is that the hybrid FGS controller meets the specifications whilst the QFT controller does not due to the large over shoot encountered at 5 seconds. This overshoot may initially seem surprising, as one may say the QFT controller has been designed to control the plant for any plant parameter values, within the given range. However, an important distinction must be drawn. Correctly stated, this assertion would be that the QFT controller has been designed to control the plant for slowly changing (with respect to the plant bandwidth) plant parameter values, within the given range [8].

The effects of changing plant parameters are largely countered in the FGS controller by changing the controller's parameters at the same rate that a is changing. Doing so compensates for changes in the plant parameters such that the effective open-loop transmission function's parameters remain approximately unchanged. By doing so it is possible to reduce any effects of changing plant parameters.

It can also be noticed that hybrid FGS requires less actuator power than the QFT controller. This is generally true of a hybrid FGS controller as the gain-bandwidth product of controller is reduced. The risk of saturating the actuation signal is also decreased due to the reduced gain-bandwidth product of the controller.

5. COMMENTS ON THE USE OF FGS TO CONTROL NON-LINEAR PLANTS

From the above case study it has been shown that QFT controller does not account for rapidly varying parameters.

FGS allows for non-linearities to be easily designed for by treating the effects of the non-linearities as uncertainty in the plant model. To do so requires the plant and non-linearities present to conform to a form, such that the non-linearities can be interpreted as parametric uncertainty. It is here proposed that this requirement dictates that the plant must conform to the form of a linear system with time varying parameters.

5.1.1. Advantages of Utilising FGS as a Control Methodology

FGS may be utilised in conjunction with other non-linear control techniques. For example, the equivalent linear time invariant (ELTI) method [8], may be utilised to design a linear controller for a non-linear plant. Should the plant templates become too large to design effective controllers, FGS may be used. FGS would allow the plant templates to be broken into several smaller templates, and a controller to be designed for each of the reduced templates.

It is generally true that a hybrid FGS controller requires less actuation power, and suffers from less actuation signal saturation than a linear controller, due to the decreased gain-bandwidth product of the FGS compensated controller.

5.1.2. Disadvantages of FGS

It must however be noted that, presently, there is no design or analysis method which will guarantee stability and specification adherence throughout the entire range of operation of the FGS compensated system. This is an area which requires future research. However, designing a FGS controller does provide a more transparent synthesis methodology than pure fuzzy control.

Because the FGS layer only varies the parameters of the $G(s)$ controllers, the form of these controllers must remain constant. If this is not so then poles (or zeros) must move towards infinity, or so as to cancel out the effects of other zeros (or poles). For example, consider the two controller equations:

$$G_1(s) = \frac{1}{(s/12+1)(s/2+1)} \tag{7}$$

and

$$G_2(s) = \frac{1}{(s/5+1)} \tag{8}$$

It is evident that the controllers structures are not equal, however $G_2(s)$ can be interpreted as:

$$G_2(s) = \frac{1}{(s/5+1)(s/\lambda+1)} \tag{9}$$

where $\lambda \rightarrow \infty$. Therefore, the poles of $G_1(s)$ can move to the positions in $G_2(s)$ such that the pole at -2 moves to -5 and the pole at -12 moves toward $-\infty$. It should be noted that the poles (or zeros) need not be constrained to move exclusively on the real axis.

6. CONCLUSION

It has been demonstrated that, assuming accurate measurement of plant parameters and states, fuzzy gain scheduling can provide adequate control for a plant with rapidly varying parameters. Thus, it is proposed that, FGS can be used for any system which is in the form of a linear system with parametric changes.

It must be noted that FGS is not a universal panacea for all control problems but is merely another tool in the control engineer's toolbox. It is most useful when employed in conjunction with a robust controller synthesis technique. By combining FGS with QFT, the resulting non-linear controller has distinct advantages over its linear counter part. The resulting controller synthesis methodology provides design transparency, due to the use of QFT, and does not suffer from over design or from rapid open-loop variations, due to the FGS.

The advantages of FGS may be summarised by the statements that FGS a) reduces the gain-bandwidth product of a controller, thus reducing the power required to control the plant and the risk of saturation, and b) reduces changes in the open-loop system by countering the effects of changing plant parameters and so may improve specification conformance in transient regions. It is this second summarising statement which encapsulates the greatest benefit of FGS.

This is an idealised scenario demonstrating the concept of FGS. Additional research must be conducted into the stability, robustness of the resulting system, as well as the effects of noise, disturbances, measurement lag, and computation time.

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