ROBUST "LINEAR TIME INVARIANT EQUIVALENT" DESIGN FOR A NON-LINEAR MAGNETIC LEVITATOR

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Abstract: Horowitz' linear time invariant equivalent method for quantitative feedback design is successfully applied to achieve robust tracking performance on a non-linear magnetic levitation system. The system is used for teaching and demonstrating control system design. This paper reports on the application in the form of a tutorial.

Keywords: Quantitative feedback design, QFT, Magnetic levitation, Control engineering education, Non-linear control.

1. INTRODUCTION

This paper reports on an application of Horowitz' linear time invariant equivalent (LTIE) method for quantitative feedback theory (QFT) design of non-linear systems (Horowitz, 1981a, 1981b, 1981c, 1982, 1993; Yaniv, 1999; Horowitz and Baños). The paper is tutorial in nature and illustrates the successful application of the LTIE method to robustly control tracking of a non-linear magnetic levitation system used for teaching and demonstrating control system design.

The magnetic levitation control problem is of considerable practical engineering and academic interest and has been investigated by a number of authors using different approaches (for example, Yang and Tateishi, 2001; Lin and Tho, 1998, Hurley and Wölfe, 1997).

Section 2 presents the model and data used in the study. Section 3 presents an overview of the LTIE method, assuming some familiarity with the QFT method for single-input, single-output systems. Section 4 presents the design details and measured results for the tutorial example.

2. MAGNETIC LEVITATOR MODEL

Assuming a linear electrical circuit, the dynamic equations of the magnetic levitator illustrated in Fig. 1 are given by,

$$\frac{d}{dt}\begin{pmatrix} v \\ x \\ i \end{pmatrix} = \begin{pmatrix} -g + f(i,x)/m \\ v \\ (V(t) - iR)/L \end{pmatrix} \qquad (1)$$

where, v is the velocity, x is the position, i is the coil current, V is the applied coil voltage, R is the coil resistance, L is the coil inductance, m is the mass of the object to be levitated, and *f(i,x)* is the levitating force developed as a function of current and distance.

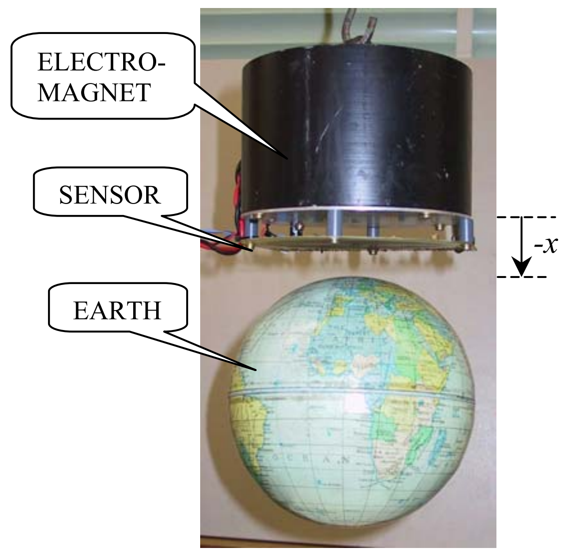

Fig. 1 – Magnetic Levitator

The electromagnet shown in Fig. 1 was designed and built as part of an undergraduate project. Our set-up levitates a hollow steel moon (mass, m_m=0.236 kg, diameter, ϕ_m=150mm) or earth (mass, m_e=0.333kg, diameter, ϕ_e=180mm). The coil inductance is $L \in [80, 100]$ mH and $R \in [3, 3.5]$ Ω, depending on operating conditions.

The sensor shown in Fig. 1 is an inductive proximity sensor that was built by our workshop. It consists of a spiral inductor etched on a printed circuit board that is excited by a tuned circuit at around 1 MHz. The proximity of the steel ball detunes the circuit, reducing the sensed voltage. The distance is evaluated from calibration data via a look-up table. (An undergraduate project has built and commissioned an optical sensor based on a linear array of charge-coupled devices.)

The magnetic force is often approximated by,

$$f(i,x) = k(i/x)^2 \quad (2)$$

The measured force to current and position is shown in Fig. 2, and k=0.0017 roughly fits the measured data shown. With the QFT design method, there is freedom to use a mix of theoretical and empirical relationships for the non-linear plant model.

If the magnetic force satisfies eq(2) (without uncertainty) or is available in empirical form, a simple non-linear approach to control the levitation height, x, would be to use say a linear controller operating on the error to generate a force set-point as its output (eq(1) is linear if the magnetic force can be controlled). The required current is calculated from the height measurement and force demand. Finally a current controller is used to achieve the required current – see Fig. 3. The scheme is a cascaded (inner and outer) controller with ratio control on the inner loop and it is appealing and works. Students should be alert to the possibility of using heuristic schemes. There are many successful applications gain scheduling (ratio control) in the process industry when the dynamics are process dependent in a benign way. Examples include mixing vessels where the lag depends on the flow-rate and systems with variable but measurable transport lag). This is not the main subject of this paper and will not be pursued further.

Local linearisation of eq(1) and eq(2), around positive i_0 and negative x_0, with $k\,i_0^2/x_0^2 = mg$ for steady state, gives some insight into the behaviour of the system and has often been used in design studies,

$$\frac{d}{dt}\begin{pmatrix}\Delta v\\ \Delta x\\ \Delta i\end{pmatrix} = \begin{pmatrix}0 & -2g/x_0 & 2g/i_0\\ 1 & 0 & 0\\ 0 & 0 & -R/L\end{pmatrix}\begin{pmatrix}\Delta v\\ \Delta x\\ \Delta i\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1/L\end{pmatrix}V(t) \quad (3)$$

This system has two real eigenvalues with the same magnitude located on either side of the imaginary axis and a third associated with the electrical circuit,

$$\lambda_{1,2,3} = \pm\sqrt{-2g/x_0}\,, -R/L \quad (4)$$

$\lambda_{1,2}$ become very fast as the distance to the magnet becomes small. The gain from current to magnetic force becomes small with increasing x_0.

An obvious problem with using a single local linearisation for control design is that it is not valid for tracking or if the levitated object is perturbed too far away from the operating point.

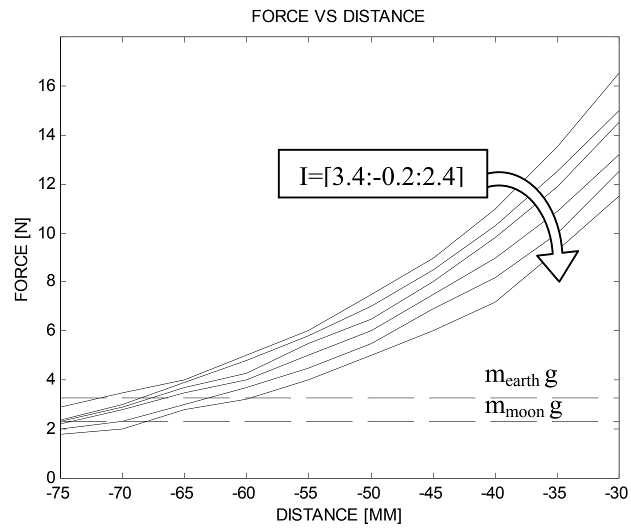

Fig. 2 – Relationship between force and distance at various values of coil current

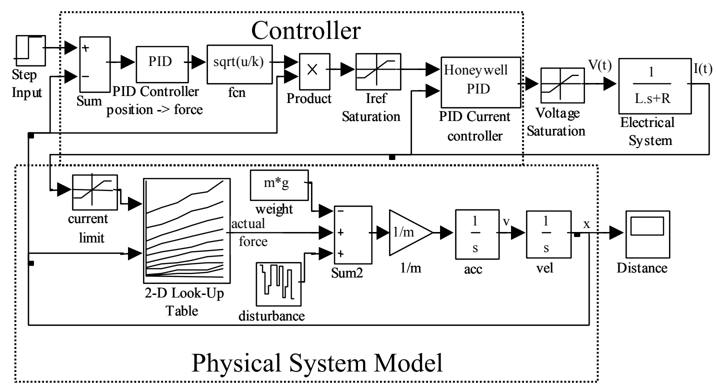

Fig. 3 – Simulink block diagram of control scheme with gain scheduling and square root extraction

3. LTIE DESIGN METHOD

Horowitz' LTIE design method provides an *exact* but *restricted* approach to non-linear design. This section summarises the method, but the reader is referred to the literature for the details.

Given a non-linear plant described by the state space equations, $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)$, $y = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},t)$, where $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{u}$ are the state-, output- and input-vectors respectively, and $\boldsymbol{f}$ and $\boldsymbol{g}$ are non-linear, possibly uncertain mappings of appropriate dimensions. Given also (by the client) is an ordered, enumerated set of acceptable outputs, $y(t) \in \{\mathcal{Y}\}$ for a particular given set of external signals (in this paper, the reference positions, $r(t) \in \{\mathcal{R}\}$).

Suppose that for each acceptable output, a unique input, $u(t) \in \{\mathcal{U}\}$ can be found. The LTIE plant set is then defined as $P=\mathcal{L}\{y\}/\mathcal{L}\{u\} \in \{\mathcal{P}\}$. As signals encountered in engineering applications have Laplace transforms, there is no problem in principle with this step. Note that we require ordering of r to u to y so that the LTIE plant set is defined for signals that may occur during the operation of the plant. Usually system identification is used to solve this step numerically (see for example Boje, 1991, Ljung, 1999) In the same way, the tracking specifications can be converted into an LTIE equivalent via $T_{spec}=\mathcal{L}\{y\}/\mathcal{L}\{r\} \in \{\mathcal{T}_{spec}\}$

The design method then makes use of the observation that under certain technical conditions, if a linear (or non-linear) controller can be designed so that the LTIE specifications are satisfied *exactly* by the LTIE plant set, for each given reference and one of the corresponding acceptable outputs, the controller will generate an input that results in exactly the same output from the non-linear process. One needs to show that the system output will indeed provide an acceptable output that gives in turn the corresponding required input, and the mechanism of proof is the Schrauder fixed-point theorem, discussed in the literature referenced above.

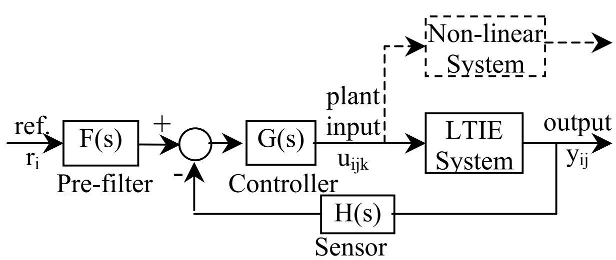

Fig. 4 – Linear time invariant equivalent method. Under conditions of the fixed point mapping, the non-linear system can replace the LTIE system with no change in the response

4. DETAILED DESIGN

4.1. Specifications

It is required that the system track step changes in the reference from –40 mm to –80 mm and back (2 elements in $\{\mathcal{R}\}$), roughly with second order dynamics with $\omega_n \in \{15,20,25\}$ rad/s and $\zeta \in \{0.6,0.7,0.8\}$ (9 elements in $\{\mathcal{P}\}$ for each reference – it is assumed that this will provide an adequate enumeration of the plant set). The current is limited to $i \in [0,5]$ amperes but to simplify the design, it is assumed that there is no rate limit as a separate, fast current controller will be used. In addition, to keep the discussion simple, plant uncertainty is assumed to be negligible, so each acceptable output is the result of only a single input.

4.2 System identification

Because of the possibility of current saturation, the responses to the specific reference step signals in the acceptable set were calculated based on a linear model but with saturation added as illustrated in Fig. 5. The system identification stage was fed with an acceptable output (taking saturation into account) and the corresponding current input, calculated using the full non-linear model. A second order, strictly proper linear model was used. Table 1 shows the identified poles and these values can be compared to the values of $\lambda_{1,2} = \pm 22.2$ at x_0 = -40 mm and $\lambda_{1,2} = \pm 15.7$ at x_0 = -80 mm for steady state operation.

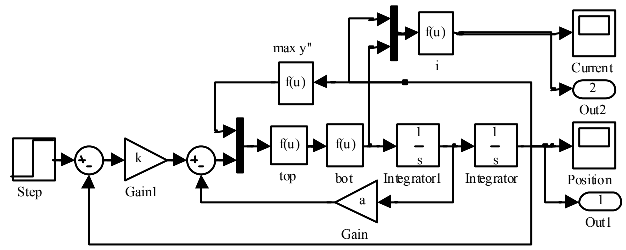

Fig. 5 – Identification data. Non-linear functions *up* and *down* generate current saturation limits

Table 1a – Identified poles, going up

λ_1	-22.7	-22.5	-21.5	-21.8	-21.7	-21.2	-21.1	-21.2	-20.9
λ_2	15.8	16.3	17.0	15.8	16.3	16.9	15.8	16.3	16.9

Table 1b – Identified poles, going down

λ_1	-15.7	-16.7	-17.5	-16.1	-16.8	-17.4	-16.4	-16.9	-17.3
λ_2	18.9	15.6	14.6	18.9	15.6	14.7	18.9	15.7	14.7

4.3 Feedback design

The controller design is shown in Fig. 6. It based on models from both the identified LTIE models and the steady state linearisation. The controller is a PI controller at low frequency with a high gain lead term at high frequency that is required to stabilise the system under all operating conditions specified.

$$G(s) = \frac{350(s/5+1)}{s} \frac{(s/20+1)}{\left((s/200)^2 + 2s/200+1\right)} \quad (5)$$

4.4 Pre-filter design

The pre-filter is a simple lag, $F(s) = 1/(s/4.8+1)$, as a result of the very high loop bandwidth required to meet the stabilisation and robust stability requirements. This ensures that step reference signals are heavily filtered before the plant input, leaving current margins for disturbance attenuation.

4.5 Results

Fig 7 shows a full non-linear simulation result using the measured model parameters, tracking a reference with force disturbances added (σ= 0.1N band limited noise, 10ms sampling).

5. CONCLUSIONS

This paper has presented a tutorial on the LTIE QFT design method applied to a magnetic levitation system. There is enough detail in our design for students to follow the design method. Simulation is "doomed to succeed" and unfortunately, measurements of the physical device with this controller were not available for this paper.

6. ACKNOWLEDGEMENTS

Mr G Väth designed and built the proximity sensor and Mr D Moodley designed and built the electromagnet. Mr Dovhani Makhado implemented an analogue lead controller that was used to take the photograph shown in Fig. 1.

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REFERENCES

Boje E, "Linear Model Identification for Multivariable Continuous Time Systems", *Identification of Continuous Time Systems*, editors: Sinha NK and Rao GP, Kluwer, 1991, ISBN 0-7923-1336-4

Boje E; "A tutorial on linear time invariant equivalent quantitative feedback design - PID control of a simple pH neutralisation process" *Quantitative and parametric feedback theory symposium*, Purdue University, Indiana, pp 94-105, August 1995

Horowitz, I, 1981a, "Improvement in non-linear quantitative feedback design by cancellation", *IJ Control*, vol. 34, no 3, 547-560

Horowitz, I, 1981b, "Non-linear uncertain feedback systems with initial state values", *IJ Control*, vol. 34, no. 4, 749-764

Horowitz, I, 1981c, "Quantitative synthesis of uncertain non-linear feedback systems with non-minimum phase inputs", *IJ Systems Science*, vol. 12, no. 1, 55-76

Horowitz I, 1982, "Feedback Systems with non-linear uncertain plants", *IJ Control*, vol. 36, no 1, 155-171

Horowitz I, 1993, *Quantitative Feedback Design*, QFT Publications, (4470 Grinnell Avenue, Boulder Colorado, 80303), ISBN 0-9635760-1-1

Horowitz I and Banos A, "Fundamentals of noninear QFT", in A. Banos, F.Lamnabhi-Lagarrigue, and F. Montoya (ed.), Advances in the control of nonlinear systems, LNCIS #264, pp- 63-134, 2001

Hurley and Wölfe, 1997, "Electromagnetic Design of a Magnetic Suspension System", *IEEE Trans. on Education*, vol. 40, no. 2, 124-128

Lin J-L and Tho B-C, 1998, "Analysis and μ-based controller design for an Electromagnetic Suspension System", *IEEE Trans. on Education*, vol. 41, no. 2, 116-129

Ljung L, 1999, *System Identification, Theory for the User*, Prentice Hall

Yang Z-J and Tateishi M, 2001, "Adaptive robust control of a magnetic levitation system", *Automatica*, vol. 37, 1125-1131

Yaniv O, *Quantitative feedback design of linear and nonlinear control systems*, Kluwer, 1999.

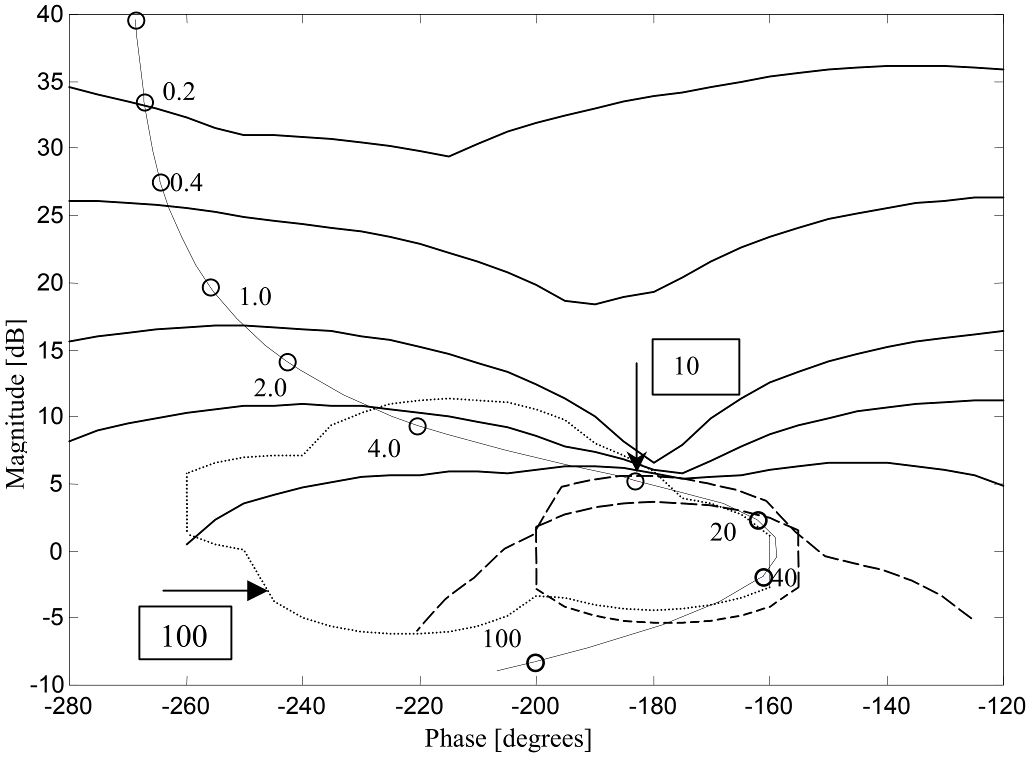

Figure 6- Nichols chart illustrating feedback design.
Solid lines – tracking tolerance specifications. Broken lines – Robust stability margins. Dashed lines – Robust stability and tracking tolerance at 10 rad/s

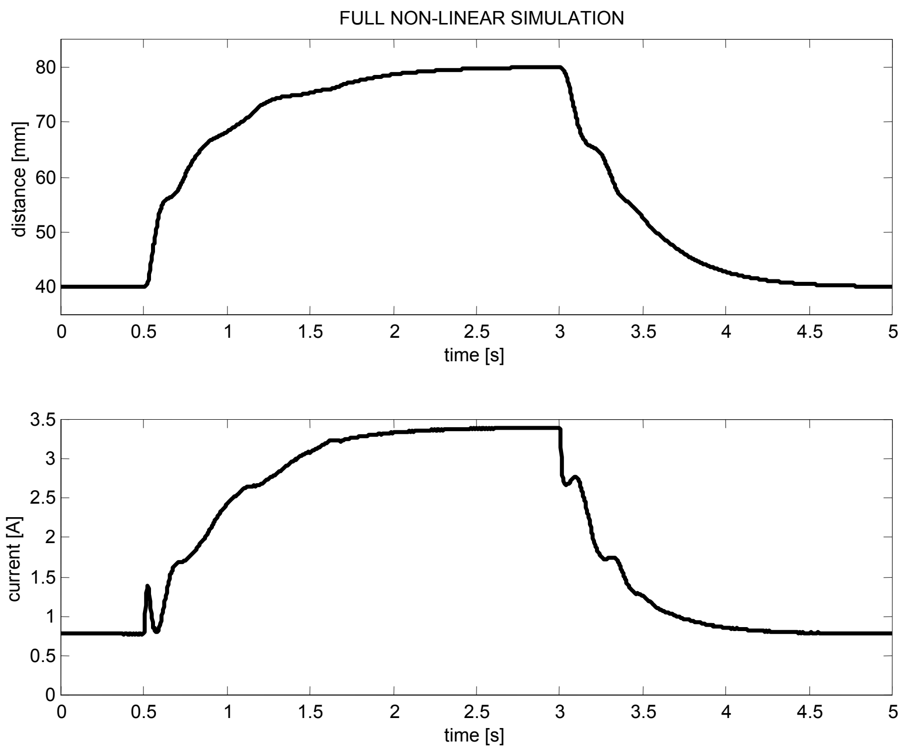

Figure 7 - Non-linear simulation for tracking specified step references and σ = 0.1N band limited noise (10ms sampling) added as a force disturbance