It is important to track evolving microstructure during the simulation of a variety of systems, including grain structure formation and evolution in thin films [1]. The interfaces in such systems, which can be thought of as n-phase systems (n≥2), can be tracked using multiple level-set fields [2] that are evolved independently and then reconciled as shown by Merriman et al. [3]. This method has the advantage that if n fields are used for tracking n materials, there is a natural correspondence between each level-set function and each material. However, this approach suffers from one setback: Recovering an explicit geometry by extracting the zero level sets is not trivial because the n level sets over-determine the interfaces. At points near triple lines and higher order junctions, the explicit triangulations extracted from each level set often disagree, even topologically. At such points it is difficult to construct a consistent surface or volume mesh without gaps or holes in order to perform a desired simulation on the explicit structure. Consistent meshes are required for an entire class of simulation tasks, including stress-strain and electromigration calculations on grain structures. Bloomfield et al.[1] proposed a partial solution for this problem which employed rectilinear voxels and robustly arrived at extracted structures that were always consistent. However, these structures (as shown in Figure 1) were problematic in that they had surface triangles with only a small number of normal directions (i.e., the structures were limited to “Manhattan geometries”) In this paper we demonstrate an algorithm that extends the voxel approach by removing the limitation of producing Manhattan geometries. By employing a volume mesh generator, we produce voxels that conform to the natural interfaces far from higher order junctions, and produce intuitive, topologically consistent, explicit triangulations at higher-order junctions. Our method works in both 2D and 3D, and requires an area mesh or a volume mesh generator respectively, that can accept internal constraints. In this work we describe the algorithm, analyze its computational complexity and expense, and show results for sizeable 2D systems for and simple 3D systems. 1. M.O. Bloomfield, D.F. Richards, and T.S. Cale, Phil. Mag., (83) 31-24, p. 3549 (2003). 2. S. Osher, and J. Sethian, J. Comput. Phys., (79) p. 12 (1988). 3. B. Merriman, J.K. Bence, and S.J. Osher, J. Comput. Phys., (112) p. 334 (1994).