Nonlinear traveling wave solutions to the Navier-Stokes equations in the plane Poiseuille geometry come into existence at a Reynolds number $Re$ very close to the experimentally observed value for transition to turbulence. These “exact coherent states” (ECS) quantitatively capture the dominant streamwise-aligned counter-rotating pairs of vortices of the near-wall buffer layer. The present work considers the effect of polymer additives on these states, using the FENE-P constitutive model of polymer solutions. These effects mirror many aspects of the experimental observations in fully turbulent flows near the onset of turbulent drag reduction: drag is reduced, streamwise velocity fluctuations increase, wall-normal velocity fluctuations decrease and length scales in all directions increase. The mechanism underlying these changes is suppression of streamwise vortices by the polymer forces exerted on the fluid. The existence threshold (critical Reynolds number) for these states increases with increasing Weissenberg number. In experiments with a given fluid, Reynolds and Weissenberg numbers ($Wi$) are linearly related. In this situation, we predict that for certain experimental conditions these states can come into existence at one value of $Re$, only to lose existence at a higher $Re$. Furthermore, there exist experimental conditions where these states cannot exist at all. Since polymer additives do not relaminarize turbulent flow, the present results imply that there must exist other nontrivial states in turbulent viscoelastic flow that exist at high $Wi$. 