A Comprehensive Model of Intracranial Dynamics of the Human Brain

Srinivasa Kondapalli¹, Michalis Xenos¹, MahadevaBharath R. Somayaji¹, Richard Penn² and Andreas A. Linninger¹,
(1) Departments of Chemical and Bio-Engineering, University of Illinois at Chicago, 810 S Clinton St., Chicago, IL 60607, (2) University of Chicago Hospitals, 5841 S. Maryland Avenue, Chicago, IL 60637

EXTENDED ABSTRACT

Summary

The objective of this work is to develop a mathematical model that describes the intracranial dynamics of the circulatory system in the human brain, based on the first principles of fluid dynamics and elastic deformation. Current approaches to imaging such as MRI, fMRI and CT scans can only provide flow field information without any pressure state information of the Cerebrospinal fluid (CSF) and blood flow inside the brain. Through this model we quantify the flow as well as the pressure fields of blood and CSF motion, along with the deformation of the parenchyma as a function of time in the cardiac cycle. Most of the published work on cerebral circulatory models so far, has been devoted primarily in studying only the dynamics of CSF flow under pathological situations such as Hydrocephalus. However, these models do not fully quantify the dynamics of the cerebral circulation with the interactions of blood and CSF with the solid brain (gray and white matter) as a whole. Further, earlier works on intracranial dynamics have been based mainly on very high level abstractions such as simplified compartmental or electric circuit models. Though many experimental studies based on MRI techniques have been performed to study the circulatory aspects of the brain, controversy exists on various aspects of cerebral circulation such as the timing and motion of CSF, the origin of CSF pulsations. Our model studies the circulatory dynamics of the blood and CSF through first principles approach based on non-linear equations of steady and unsteady flow conditions through distensible elastic vessels. In order to validate our model we will also present experimental evidence from the flow data obtained by cine phase contrast Magnetic Resonance Imaging (MRI).

Methodology

The flow of blood and CSF in the brain are causally related with various pressure gradients throughout a network of blood vessels and the CSF ventricles. A simplified geometry of brain with the system interactions is shown in figure 1. The blood flow or the vascular network (layer 1) in the model comprises of the arterial system with 20 major arteries and about 40,000 intracerebral arteries. The micro circulatory (capillaries) and the venous networks account for about 200,000 and 75,000 blood vessels respectively. The CSF flow occurs in the ventricular system (layer 3) within the four cavities of lateral ventricles (first and second) and the third and fourth ventricles. The pulsations caused from the arterial blood flow drive the motion of CSF, which is produced from the Choroid plexuses of the lateral and third ventricles. The CSF from
the ventricular cavities flow along the subarachnoid space and is reabsorbed back into the venous blood through the sagittal sinuses.

In order to simulate the flow of both blood and CSF through a complex network of distensible elastic vessels, the model assumes a one-dimensional fluid motion with the equations of the flow written using the Navier-Stokes equation of continuity and momentum, combined with the equation of distensibility for the vessel wall known as the ‘tube law’ equation. Blood and CSF are considered as viscous and incompressible fluids flowing through elastic vessels of circular cross-section.

The governing equations, written using the conservation of mass and momentum along with the equation of tube law are,

**Continuity:**

\[ \frac{\partial A}{\partial t} + \frac{\partial (AU)}{\partial x} = 0 \]  
\[ \text{(1)} \]

**Momentum:**

\[ \frac{\partial U}{\partial t} + \frac{\partial U^2}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + F = 0 \]

\[ F = \frac{8\eta U}{\rho A} \]  
\[ \text{(2)} \]

**Tube Law:**

\[ P = P_e + \frac{E_h}{2R_0} \left( \frac{A}{A_0} - 1 \right) \]  
\[ \text{(3)} \]

Where \( h \) is the thickness of the Tube and \( R_0 \) its initial radius. \( A_0 \) is the Area of the tube at rest (when no flow occurs) and \( A \) is the extended area at given flow conditions of velocity, \( U \) and pressure, \( P \). The model simulates for the velocity and pressure states of the fluid through the entire network with the inlet and out boundary conditions for pressure of the blood. The pressure at the inlet (ascending aorta) corresponds to an average arterial pressure of 100-120 mm Hg. The output pressure is taken to be constant with a pressure of 7 mm Hg which corresponds to the blood pressure in the Jugular veins.