OPTIMAL DESIGN OF BATCH-STORAGE NETWORK UNDER JOINT UNCERTAINTIES

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ABSTRACT

This study sought to find analytic solutions to the problem of determining the optimal capacity of a batch-storage network to meet demand for finished products in a system undergoing joint random variations of operating time and batch quantity. The raw material purchasing flow and final product demand flow are susceptible to joint random variations in the order cycle time and batch size. The production processes also have joint random variations in production cycle time and product quantity. Waste regeneration or disposal processes are included into the network to treat the spoiled materials from failed batches. The objective function of the optimization is minimizing the expected total cost, which is composed of setup and inventory holding costs as well as the capital costs of constructing processes and storage units. A production and inventory method, the PSW (Periodic Square Wave) model, provides a unique graphical method to find the upper/lower bounds and average of random flows, which are used to construct terms of the objective function and constraints of the optimization model. The advantage of this model is that it provides a set of simple analytic solutions while also maintaining a realistic description of the random material flows between processes and storage units; as a consequence of these analytic solutions, the computation burden is significantly reduced. The proposed method has the potential to rapidly provide very useful data on which to base investment decisions during the early plant design stage. It should be particularly useful when these decisions must be made in a highly uncertain business environment..

Introduction

The production and inventory analysis method known as the periodic square wave (PSW) method was recently developed and used to determine the optimal design of a parallel batch-storage system.[1] Subsequently, the method was extended to handle a sequential multistage batch-storage network (BSN),[2] and further modified to handle a non-sequential network structure that can deal with recycle material flows in the network.[3] The key advantage of the PSW model over other models lies in its simple analytical sizing and timing equations. This advantageous characteristic has been exploited in an analysis of an integrated financial and production system.[4, 5] The range of physical process structures for which the PSW model can be used has expanded from batch processes to multitasking batch or semi-continuous processes[6] and a multi-site batch production/transportation network (i.e., a large-scale supply chain system)[7]. A BSN basically resembles a state task network (STN) in which batch corresponds to task and storage corresponds to state; thus, BSNs provide an effective representation of supply chains such as purchasing, production, transportation and demand processes.

One of the challenging problems in supply chain optimization is dealing with uncertainty. Major business uncertainties arise from product demand forecasting, production equipment malfunctions, off-spec materials, changes in the prices of raw materials or finished products, and raw material supply shortages. A promising approach to accounting for such uncertainties is to include a computational mechanism to mitigate such uncertainty effects into the supply chain optimization model. However, the models developed to date are unsuitable for use in real applications due to severe computational complexity arising from the treatment of uncertainty as well as unknown probabilistic parameters. Given this situation, the present study aims to develop a compact analytical solution to optimize the design and/or operation of large-scale supply chains with batch processes. In this study, a novel optimization model resulting in simple analytical solutions with negligible computational burden is introduced.

A previous study using the PSW model on a BSN developed analytical solutions of supply chain optimization to deal with uncertainty[8]. The sources of uncertainty were batch size and cycle time variations in raw material purchasing, batch production, transportation and finished product demand processes. The processes were classified into three types according to their random characteristics: (1) processes that possess uncertainty only in the cycle time; (2) processes that possess uncertainty only in the batch size; and (3) processes that possess joint uncertainty in both the cycle time and batch size. In modern society, batch material losses associated with raw material purchasing and transportation processes occur infrequently; therefore, these processes were exclusively considered as type 1 processes in the previous study. Production processes may be either type 1 or type 2 processes. Mixing or blending processes do not usually involve batch material loss, and thus were considered to type 1 processes. Batch material losses do, however, occur in many reaction processes; hence these processes were considered type 2 processes. Most processes had joint uncertainties in cycle time and batch size (type 3 process); however, such joint uncertainties were excluded in the previous study due to the complexity of handling them. In the present study, an approach is developed that overcomes the complexity of handling joint uncertainties of cycle time and batch size. Here it is considered that all processes are subject to joint uncertainties of cycle time and batch size; therefore all processes are type 3 processes. Under this scheme, type 1 and 2 processes are subsystems of type 3 and hence do not need to be considered separately. In spite of the increased problem complexity associated with this approach, analytical solutions for the optimization problem are still available. These analytical solutions greatly reduce the computation time.

When a batch production process is susceptible to random failures, the volume of on-spec product material fluctuates randomly for a given feed volume. In other words, a random amount of waste material is produced, as the amount of waste material equals feed volume minus on-spec product volume. Therefore, the system that deals with material quantity uncertainty should include waste material storage units and waste disposal processes. In this study, the BSN is modified to include a waste material stream in a batch process connected with a waste material storage unit and a waste disposal process connected with a waste material storage.

A multi-period formulation to treat long-term trends of variables and parameters that was introduced in the previous study[8] is omitted in this study and, therefore, the derivation corresponds to single-period formulation that accounts for short-term variations of variables and parameters. The single-period formulation presented here can be easily expanded to the multi-period formulation, as was done in the previous study.[8]

Upper and Lower Bounds of a Flow with Joint Uncertainties

Figure 1 shows three types of uncertainties: (a) uncertainty only in cycle time, (b) uncertainty only in batch size and (c) joint uncertainties in both cycle time and batch size.

The focus of the present study is joint uncertainties. The random properties of joint uncertainties are characterized by two random variables, $\mathbf{B}_{(l)}$ and $\boldsymbol{\omega}_{(l)}$, as shown in Figure 1(c), where subscript **(l)** represents the sequence of batch occurrence. It is not necessary to know the exact distribution functions of $\mathbf{B}_{(l)}$ and $\boldsymbol{\omega}_{(l)}$. It is assumed that $\mathbf{B}_{(l)}$ has a symmetrical distribution function with $\underline{\underline{B}} \le \mathbf{B}_{(l)} \le \overline{\overline{B}}$ and $\boldsymbol{\omega}_{(l)}$ has a non-symmetrical distribution function with $\underline{\underline{\omega}} \le \boldsymbol{\omega}_{(l)}$. The mean values of $\mathbf{B}_{(l)}$ and $\boldsymbol{\omega}_{(l)}$ are $\overline{B} \equiv \frac{\overline{\overline{B}} + \underline{\underline{B}}}{2}$ and $\overline{\omega}$ respectively. Two design parameters, time availability α and size availability β, are introduced such that $\alpha \equiv \frac{\underline{\underline{\omega}}}{\overline{\omega}}$ and $\beta \equiv \frac{\underline{\underline{B}}}{\overline{B}} \equiv 2 - \frac{\overline{\overline{B}}}{\overline{B}}$, where $0 < \alpha \le 1$ and $0 \le \beta \le 1$. As α and/or β approach 1, the process becomes more deterministic. The concept of availability, defined as minimum value without failure divided by average value with failure, comes from failure modes and effects analysis (FMEA). Note that the process with $\beta = 1$ and $0 < \alpha < 1$ is type 1, and the process with $\alpha = 1$ and $0 < \beta < 1$ is type 2 in the previous study.[8]

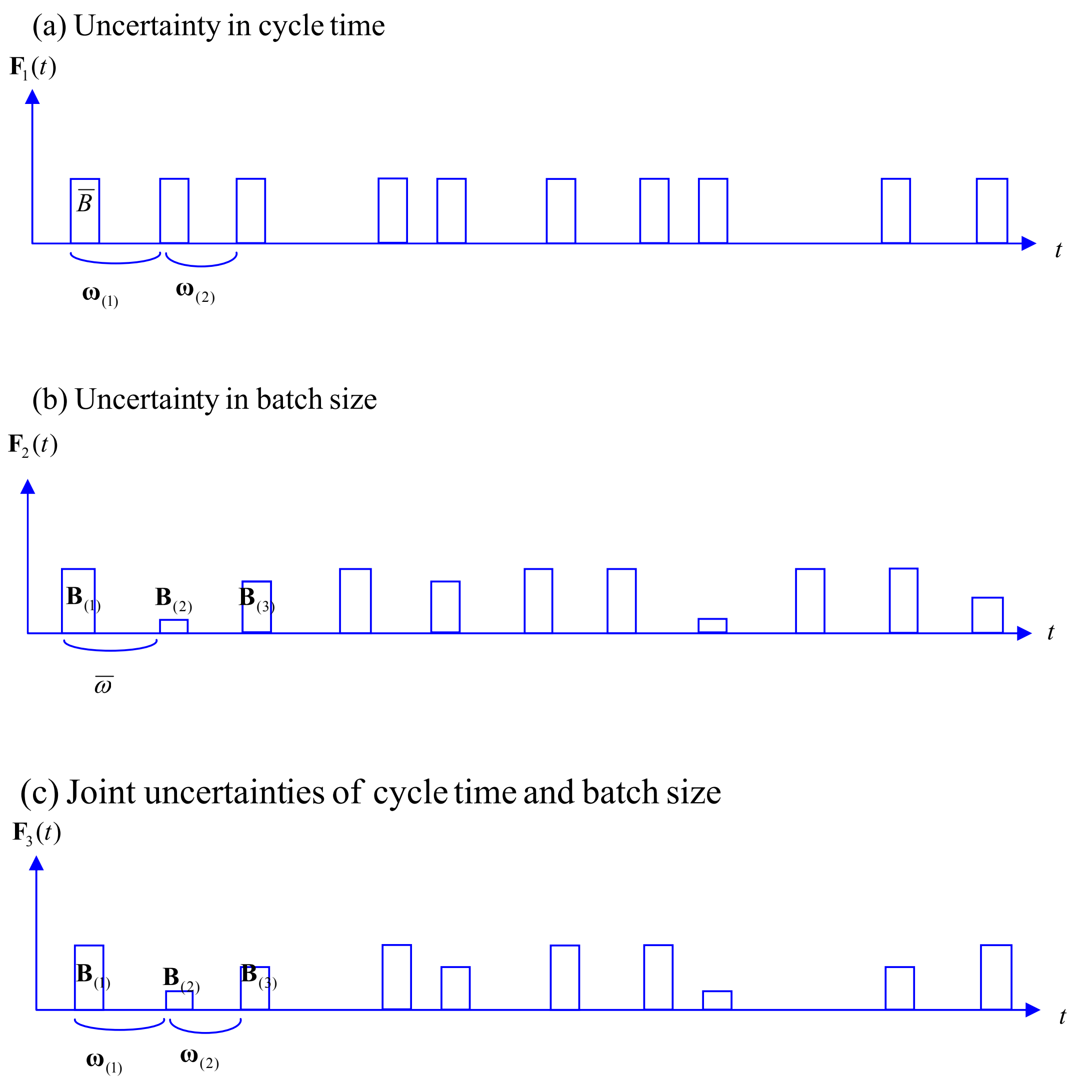
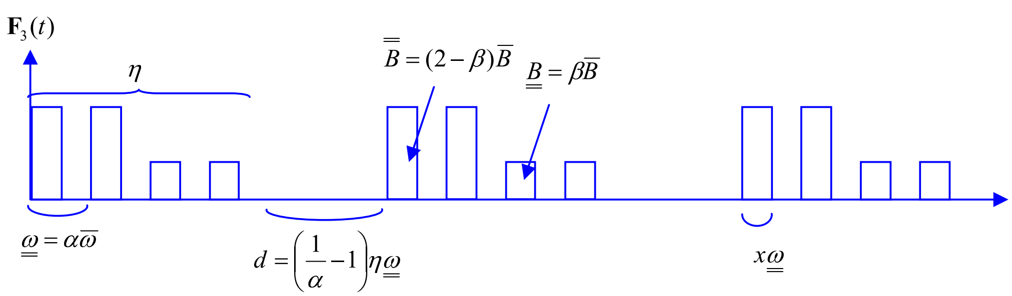


Figure 1. Types of Uncertainty

Suppose that $\mathbf{B}_{(\mathbf{l})}$ and $\boldsymbol{\omega}_{(\mathbf{l})}$ have identical independent distribution functions with respect to **(l)**. For given convergence limits $0 < \varepsilon_1, \varepsilon_2 << 1$ and confidence levels $0 < \delta_1, \delta_2 << 1$, the weak law of large numbers says that there exists an integer η such that $P\left\{\left|\frac{1}{\eta}\sum_{l=1}^{\eta}\mathbf{B}_{(\mathbf{l})} - \overline{B}\right| < \varepsilon_1\right\} \geq 1 - \delta_1$ and $P\left\{\left|\frac{1}{\eta}\sum_{l=1}^{\eta}\boldsymbol{\omega}_{(\mathbf{l})} - \overline{\varpi}\right| < \varepsilon_2\right\} \geq 1 - \delta_2$. From Tchebycheff's inequality, $\eta \geq \frac{Var(\mathbf{B}_{(\mathbf{l})})}{\delta_1 {\varepsilon_1}^2}$ and $\eta \geq \frac{Var(\boldsymbol{\omega}_{(\mathbf{l})})}{\delta_2 {\varepsilon_2}^2}$, that is, $\eta = \max\left\{\mathrm{int}\left[\frac{Var(\mathbf{B}_{(\mathbf{l})})}{\delta_1 {\varepsilon_1}^2}\right], \mathrm{int}\left[\frac{Var(\boldsymbol{\omega}_{(\mathbf{l})})}{\delta_2 {\varepsilon_2}^2}\right]\right\} + 1$ if the least integer is chosen.[8] Where $Var(.)$ is variance operator and int[.] is a truncation function to make integer. The parameter η, called the occurrence number, should be an even number in order for 0.5η to be an integer. The time interval during which η batches occur is defined as the long cycle time $\tilde{\omega}$. Because the sample means of $\mathbf{B}_{(\mathbf{l})}$ and $\boldsymbol{\omega}_{(\mathbf{l})}$ converge to their mean values during the long cycle time according to the weak law of large numbers, $\tilde{\omega} = \eta\overline{\omega} = \frac{\eta}{\alpha}\underline{\underline{\omega}}$. Here, the long cycle time corresponds to the least period within which all random effects diminish with a given confidence level. Two more parameters are introduced for convenience, the average flow rate $D \equiv \frac{\overline{B}}{\overline{\omega}} = \frac{\alpha\overline{B}}{\underline{\underline{\omega}}}$ and the total dead time within a long cycle time $d \equiv \tilde{\omega} - \eta\underline{\underline{\omega}} = \left(\frac{1}{\alpha} - 1\right)\eta\underline{\underline{\omega}}$.

To generate the optimization formulation, the upper/lower bounds and average inventory level of storage units under joint uncertainties are needed. The upper bound of the inventory level will be used to compute the storage size; the lower bound of the inventory level will be used in the optimization constraint that ensures that the inventory level is always nonnegative; and the average of the inventory level will be used to compute the inventory holding cost of the optimization problem. If the upper/lower bounds and average of all flows coming into and going out of the storage units are known, the upper/lower bounds and average of the inventory level of the storage units can be easily identified. Note that the flow has a constant average flow rate D measured during a long cycle time. This means that in spite of the randomness, the total quantity processed during a long cycle time is constant. Two extreme cases of the flow with joint uncertainties exist-(a) the Upper bound case and (b) the Lower bound case-as shown in Figure 2 for the case of $\eta = 4$.

(a) Upper Bound Case



(b) Lower Bound Case

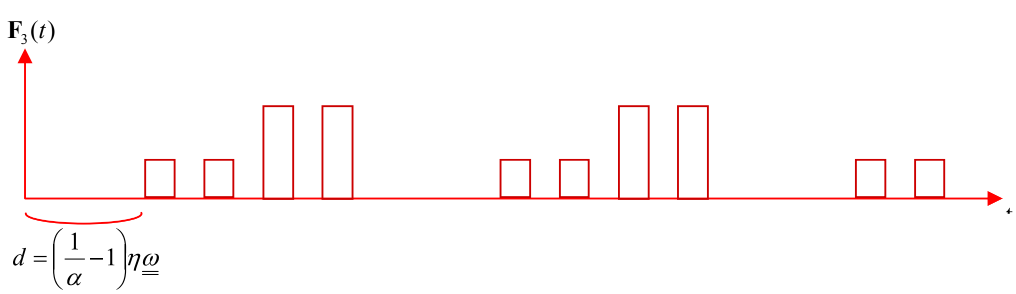


Figure 2. Two Extreme Cases of Flow

The upper bound case has 0.5η times of maximum batch size $\overline{\overline{B}}$ with minimum cycle time $\underline{\underline{\omega}}$, 0.5η times of minimum batch size $\underline{\underline{B}}$ with minimum cycle time $\underline{\underline{\omega}}$, and a total dead time d within repeated long cycle times. The lower bound case has total dead time d, 0.5η times of minimum batch size $\underline{\underline{B}}$ with minimum cycle time $\underline{\underline{\omega}}$, and 0.5η times of maximum batch size $\overline{\overline{B}}$ with minimum cycle time $\underline{\underline{\omega}}$ within repeated long cycle times.

Note that in spite of the greater difference between these two cases, the total quantity processed during a long cycle time of both cases is a constant, $D\tilde{\omega}$. Figure 3 shows the cumulative flow functions of the two cases.

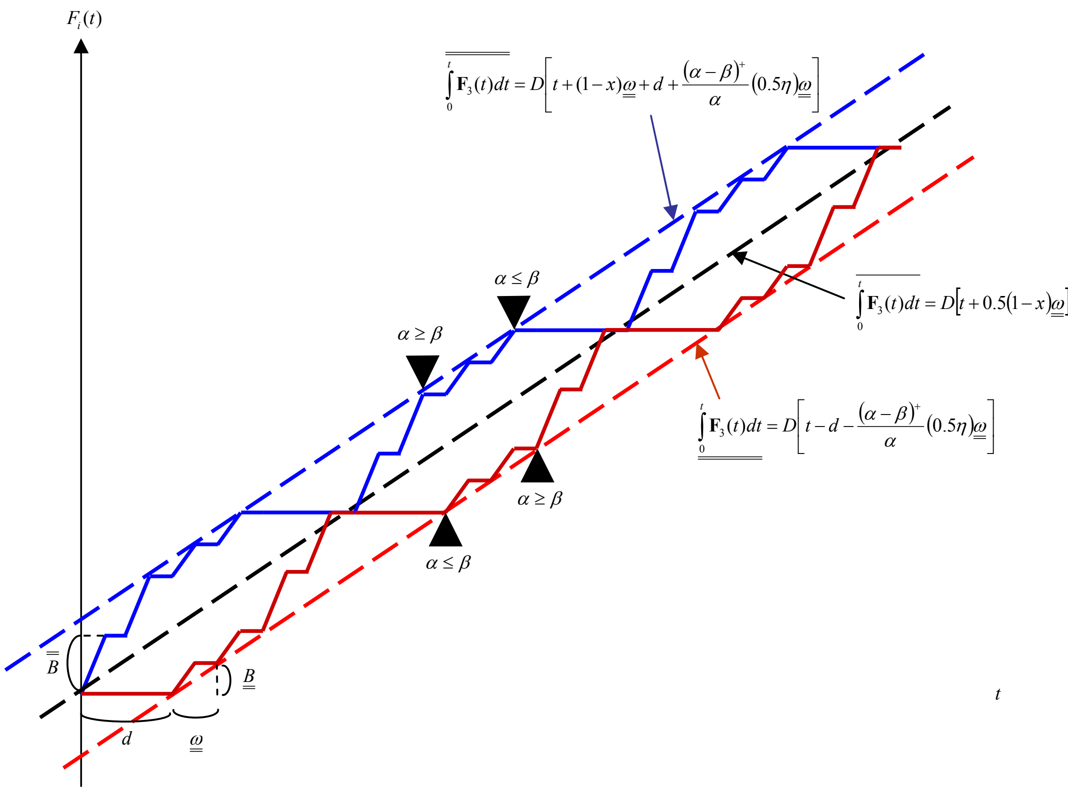


Figure 3. Cumulative Flow Functions for Two Extreme Cases.

The dotted lines are the upper and lower bounds of the two extreme cases. Note that there are two contacting points depending on the values of α and β. The integral of all flows with joint uncertainties exists between the dotted lines, that is, $\underline{\underline{UPSW}} \le \int_0^t \mathbf{F}_3(t)dt \le \overline{\overline{UPSW}}$, where

$$\underline{\underline{UPSW}}(t;D,\underline{\underline{\omega}},t',x,\theta) = D[t-t'-\theta\underline{\underline{\omega}}] \tag{1}$$

$$\overline{\overline{UPSW}}(t;D,\underline{\underline{\omega}},t',x,\theta) = D[t-t'+(1-x)\underline{\underline{\omega}}+\theta\underline{\underline{\omega}}] \tag{2}$$

Here

$$\theta = \left(\frac{1}{\alpha} - 1\right)\eta + \frac{(\alpha - \beta)^{+}}{\alpha}(0.5\eta) \tag{3}$$

where $(X)^{+} \equiv \max\{0, X\}$, x is called storage operation time fraction and t' is initial start-up time. The average inventory level is highly dependent on the random properties of failures. The exact value of the average inventory level cannot be obtained without defining the probability distribution functions of all random variables, which is a nontrivial task. In this study, an intuitive approach is taken. Specifically, the average of flow $\overline{UPSW} \equiv \overline{\int_0^t \mathbf{F(t)}dt}$ is selected as the line equidistant from the upper and lower bounds [8]:

$$\overline{UPSW}(t; D, \underline{\underline{\omega}}, t', x) = D\left[t - t' + 0.5(1 - x)\underline{\underline{\omega}}\right] \tag{4}$$

This selection could be the most probable [8].

The objective function for the design of the batch-storage network is to minimize the annualized expectation of total cost, which consists of the setup cost of processes, the inventory holding cost of storage units, and the capital cost of the processes and storage units for a given time availability, size availability and occurrence number in a long cycle time of each process. The optimization constraints are no depletion of all storage units

Solving Kuhn-Tucker conditions gives optimal cycle times:

$$\underline{\underline{\omega}}_k^j = \sqrt{\frac{\alpha_k^j A_k^j}{D_k^j \Psi_k^j}} \tag{5}$$

$$\underline{\underline{\omega}}_n^j = \sqrt{\frac{\alpha_n^j A_n^j}{D_n^j \Psi_n^j}} \tag{6}$$

$$\underline{\underline{\omega}}_i = \sqrt{\frac{\alpha_i A_i}{D_i \Psi_i}} \tag{7}$$

where

$$\Psi_k^j = \left[0.5H^j + b^j\right](1 - x_k^j) + \frac{(2 - \beta_k^j)a_k^j}{\alpha_k^j} + \left(H^j + 2b^j\right)\theta_k^j \tag{8}$$

$$\Psi_n^j = \left[0.5H^j + b^j\right](1 - x_n^j) + \frac{(2 - \beta_n^j)a_n^j}{\alpha_n^j} + \left(H^j + 2b^j\right)\theta_n^j \tag{9}$$

$$\Psi_i = \frac{(2 - \beta_i)a_i}{\alpha_i} + (1 - x_i)\sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)f_i^j + (1 - x_i')\sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)\frac{1}{2 - \beta_i}g_i^j + (1 - x_i')\sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)\frac{1 - \beta_i}{2 - \beta_i}\hat{g}_i^j$$

$$+\sum_{j=1}^{|J|}\left(H^j+2b^j\right)\left(f_i^j\theta_i+\frac{1}{2-\beta_i}g_i^j\theta_i'+\frac{1-\beta_i}{2-\beta_i}\hat{g}_i^j\hat{\theta}_i\right) \tag{10}$$

Example Plant Design

Suppose the plant that produces 3 finished products from 4 raw materials as shown at Figure 4. Figure 4 also includes most input data for computation. A design problem without process I7, storage J10 and J11 was studied in the previous study[3]. The waste materials of failed batches are collected in Storage J10. The waste material J10 can be disposed through the waste disposal process J10 or can be regenerated through the Process I7 to raw materials J2 and J4. The waste material of process I7 goes to storage J11. The waste material J11 is disposed through the waste disposal process J11.

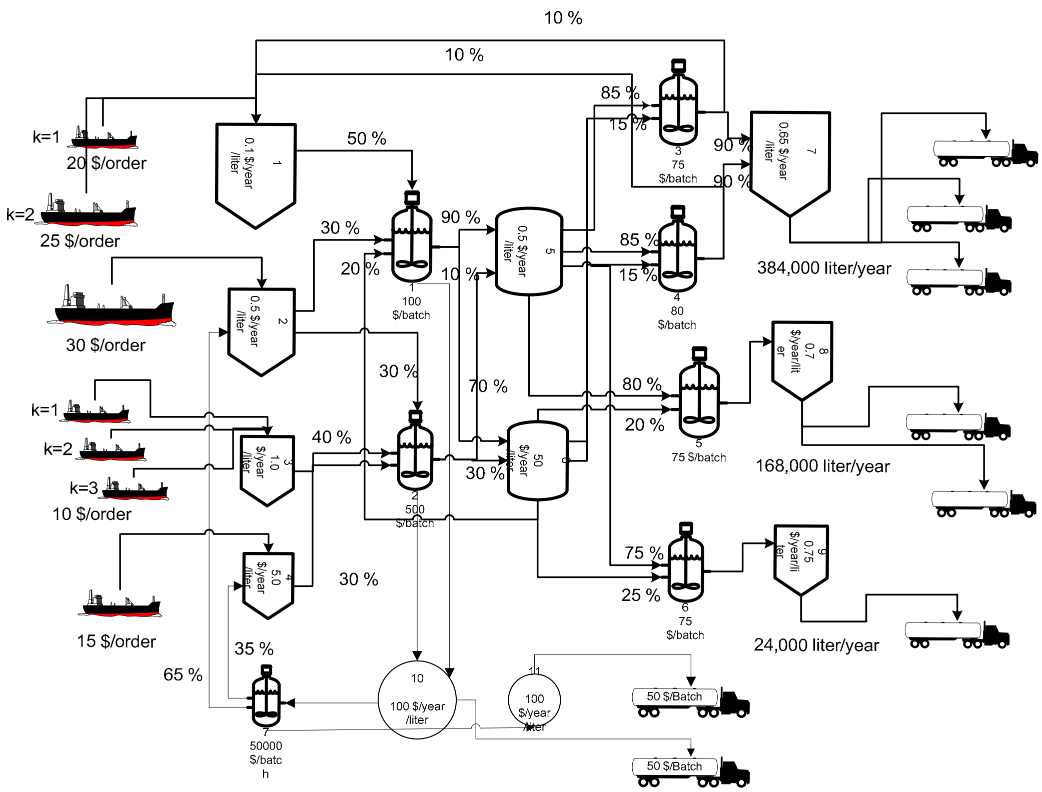


Figure 4. An Example Plant Design Problem

Conclusion

This study deals with determining the optimal sizes of batch processes and storage units interconnected in a general network structure when the processes are subject to joint uncertainties of operating time and batch quantity. Waste regeneration and disposal processes were included into the network to treat materials from failed batches. An unique graphical method was used to find the upper and lower bounds and average of material flows susceptible to short-term joint random variations in the cycle time and batch size. In the definition of the random properties, availabilities and occurrence number were introduced as input parameters instead of more widely used parameters such as the mean and variance. The availability is commonly used in process reliability analysis methods such as failure modes and effects analysis, and the occurrence number is proportional to the variance. These parameters were chosen as they are more practical and easier to estimate based on human perception. The optimization problem consisted of minimizing the expected sum of the setup cost, capital cost of processes/storage units, and inventory holding cost under the constraints of meeting random product demand and no depletion of storage materials. The stochastic version of the PSW model with unique graphical analysis provided analytical solutions of the optimization problem. These analytical solutions greatly reduce the computational burden, which is the major achievement of this study. The analytical optimal solutions made it possible to conduct sensitivity analysis with respect to the input parameters, time availability and size availability with two fixed values of occurrence number.

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