Novel Objective Functions for Static Mixers

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Most static mixers are designed to maximize radial mixing to achieve compositional and thermal homogeneity. Flow inverters maximize heat transfer. Axial mixers that closely approximate an exponential distribution of residence times have been devised as well. More recent papers describe static mixers for surge dampening in industrial swing-cycle processes (Smith, Graham, and Palamara, 2006). An extension of this result allows complete attenuation of surges even when the component with periodic concentration variations is reacting by first order kinetics (Nauman and Smith, 2008) . Another class of motionless mixer is intended to maximize first appearance times in laminar flow, tubular reactors. Coiled tubes, especially with periodic changes in the direction of the principle axis have shown excellent results but are somewhat cumbersome in terms of spatial layout (Mridha, and Nigam, 2008). Baffles in a short, fat reactor used for sterilization have been shown effective. The general question is: what objectives functions are reasonable for static mixers and how can near optimal performance be achieved in practical hardware? The first question is addressed using the tools of residence time theory. The second question is usually addressed using commercial CFD packages, but these can be problematic. An alternative considered here is to simple, transparent code to test design concepts and to provide limiting cases for the verification of CFD code.

Laminar Flow Approximation to a Flat RTD

Following Smith, Graham, and Palamara (2006), a flat residence time distribution is

$$\begin{aligned} f(t) &= 0.5/t_p \text{ for } 0 < t < t_p \\ &= 0 \text{ for } t > t_p \end{aligned} \qquad (1)$$

This distribution will completely dampen a concentration having period t_p. This statement is true for any waveform. The flat RTD also does an does an excellent of attenuating sinusoidal inputs of other periods and typically outperforms a CSTR of equal volume. The washout function corresponding to Equation 1 is

$$\begin{aligned} W(t) &= 1 - t/t_p,\ t < t_p \\ &= 0,\ t > t_p \end{aligned} \qquad (2)$$

Note that $t_p = 2\bar{t}$ where $\bar{t}$ is the mean residence time. A previous paper (Nauman and Smith, 2008), showed how a flat RTD can be approximated by a shell-and-tube design when the tube flow is turbulent. The laminar flow case is actually a bit harder to do well. The straight line in Figure 1 is the desired $W(t)$ as given by Equation 2. The curved line in Figure 1 corresponds to five laminar tubes in parallel with equal flows per tube (20% of total). A numerical optimization, constrained by a constant $\bar{t}$, of tube volumes was based on sum-of-squares differences between actual and flat RTDs. A random optimizer was used, e.g. see Nauman 2008. Figure 2 shows the corresponding results for $f(t)$, the ideal curve being a flat line corresponding to Equation 1. Figure 3 shows the response of the five tube systems to sinusoidal disturbances. It outperforms a CSTR over a broad range of frequencies. Although not tested here, it is expected to perform better that a CSTR for disturbance of arbitrary waveform within this range of frequencies.

Maximization of First Appearance Times in Laminar Slit Flows

We consider the familiar parabolic velocity profiles for laminar flow in tubes and between flat plates. For a circular tube, the first appearance time is $t_{first} = \bar{t}/2$ and for flow between flat plates it is $t_{first} = 2\bar{t}/3$. For processes such a sterilizations, this makes a substantial difference in reactor volume and in the thermal time distribution (Nauman, 1977) and accounts for the common use of spiral and plate-type heat exchangers for processing heat sensitive materials. Figure 4 shows two s of a two-channel, slit flow geometry in which the relative channel widths can be varied. The velocity profile is parabolic in each , the pressure gradient down each channel is identical, and thus the mean velocities in the channels will be different except when the central baffle is located exactly at the center line, here denoted as $y = 0.5$. When there is only one , the optimal location for the baffle is at $y = 0.5$, and this gives $t_{first} = 0.667$, which is the same result that would be obtained without the central baffle. Suppose now that there are two s of equal length. An improvement to $t_{first} = 0.772$ can be achieved by placing the baffles at off-center locations in the two stages. Figure 5 shows results for systems up to 20 s. Table 1 gives the corresponding baffle locations. The optimization assumed each to be of the same length and ignored entrance effects at the transitions between stages, a justifiable assumption when the stage lengths are large. This assumption also means that the order of the various stages is unimportant.

Results reflecting entrance effects can be presumably be obtained from CFD codes, but caution is needed. Only recently has it been shown possible to generate a reasonably good washout function for diffusion free, laminar flow in a tube using Fluent (Tilton and Liu, 2008). Other postings on the Fluent website are inapplicable (e.g. for CSTR's where numerical diffusion would aid in approaching an exponential distribution) or show numerical diffusion large enough to grossly distort the first appearance time.

1	2	3	4	5	6	7	8	9	10	15	10
0.500	0.458	0.439	0.427	0.418	0.412	0.407	0.403	0.399	0.396	0.391	0.382
	0.542	0.500	0.478	0.463	0.452	0.444	0.437	0.432	0.427	0.407	0.404
		0.561	0.522	0.500	0.485	0.473	0.464	0.457	0.450	0.428	0.420
			0.573	0.537	0.515	0.500	0.488	0.479	0.471	0.445	0.438
				0.582	0.548	0.527	0.512	0.500	0.491	0.460	0.439
					0.588	0.556	0.536	0.521	0.509	0.474	0.454
						0.593	0.563	0.543	0.529	0.487	0.465
							0.597	0.568	0.550	0.501	0.476
								0.601	0.573	0.514	0.486
									0.604	0.527	0.496
										0.541	0.507
										0.556	0.517
										0.574	0.526
										0.591	0.537
										0.608	0.549
											0.560
											0.575
											0.587
											0.587
											0.615
0.667	0.712	0.735	0.751	0.763	0.771	0.778	0.783	0.787	0.793	0.800	0.807

Table 1. Optimal baffle positions for multi-stage, two-channel slit flow

The results to this point could conceivably be realized in real hardware and show a significant improvement in first appearance time. We now what appears to be an upper limit on the first appearance time in multi-stage laminar flow. We postulate a multi-channel device that can produce any desired reorientation of the fluid at the transition between stages. For a two-stage device, the optimal transformation appears to be complete flow inversion (Nauman, 1977) where the fluid is literally turned inside out so that what was nearest the wall in the first stage is at the centerline in the second stage and conversely. A conceptual design even exists for this (Nauman, 1977). A more complex transformation is needed between second and third stages, but the effects of the assumed-to-be-optimal transformation are easily simulated. The various streamlines emerging from the second stage are sorted according to residence time. This with the shortest residence times after the second stage will be placed near the wall in the third stage and conversely. The upper curve in Figure 5 shows the results. It is speculated that these represent an upper limit for first appearance times in multi-stage. The coiled tubes of Saxena and Nigam (1984) approach these numerical values but require the equivalent of many more stages.

Conclusions

The paper has discussed two problems which represent non-conventional goals for static mixers. The approach has been to select an objective function, either the entire washout function or the first appearance time of that function. Then a static mixer configuration was specified, and the design parameters of the configuration were optimized to using the simple random technique. This techniques is delightfully easy to program but computationally inefficient. They the fluid mechanics in the present examples were simple and could be quickly solved to any desired accuracy. Because of modern computing speeds, the approach has been used on much more complicated systems. For example, it might be applied to the functional optimization of wall temperatures in a heat exchanger in order to minimize the breadth of the thermal time distribution. The counterpart of this study applied to two-channel flows in a tube should also be feasible. The random techniques is unsuitable for full-blown CFD studies but can provide guidance for selecting parameters in detailed case studies.

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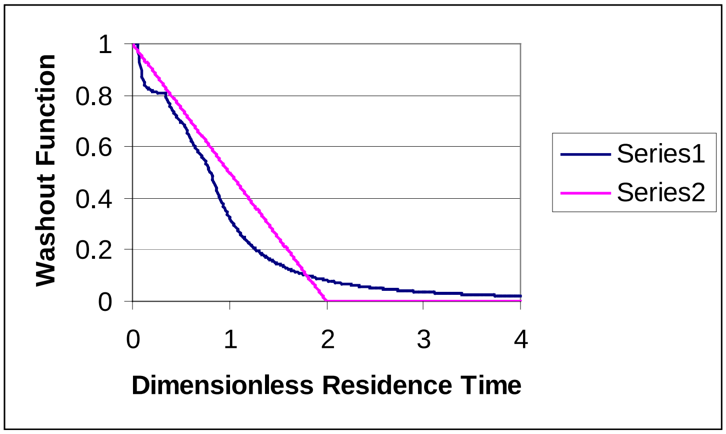

Figure 1. Five tube approximation to a flat RTD

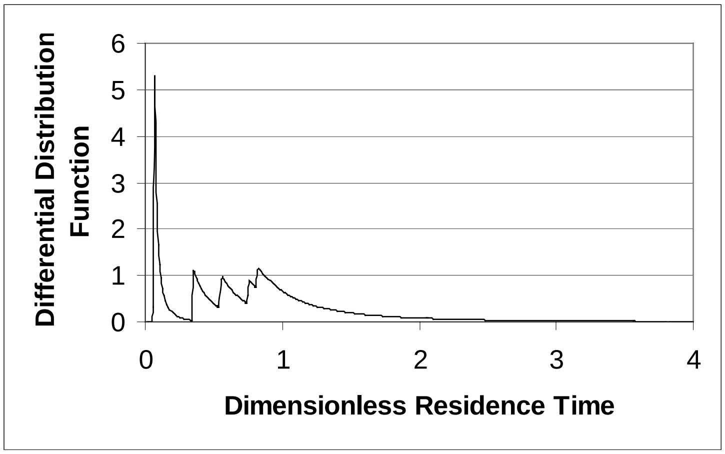

Figure 2. Density function, *f*(*t*), for the five tube approximation

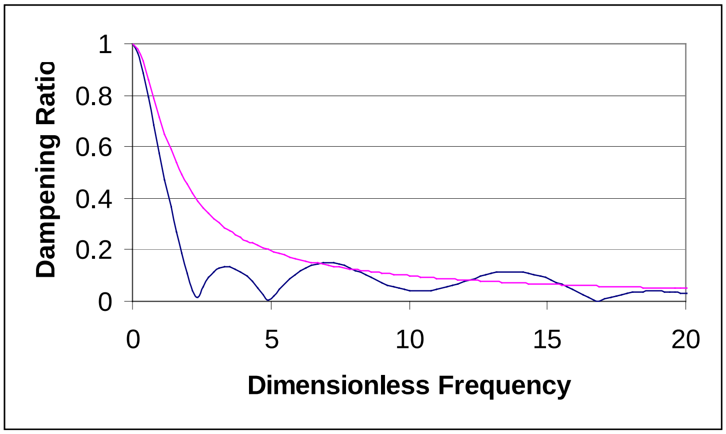

Figure 3. Response of the five tube approximation compared to that of a CSTR

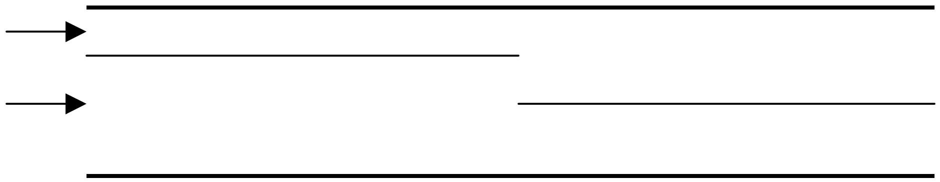

Figure 4. Two stages of the two-channel slit flow geometry

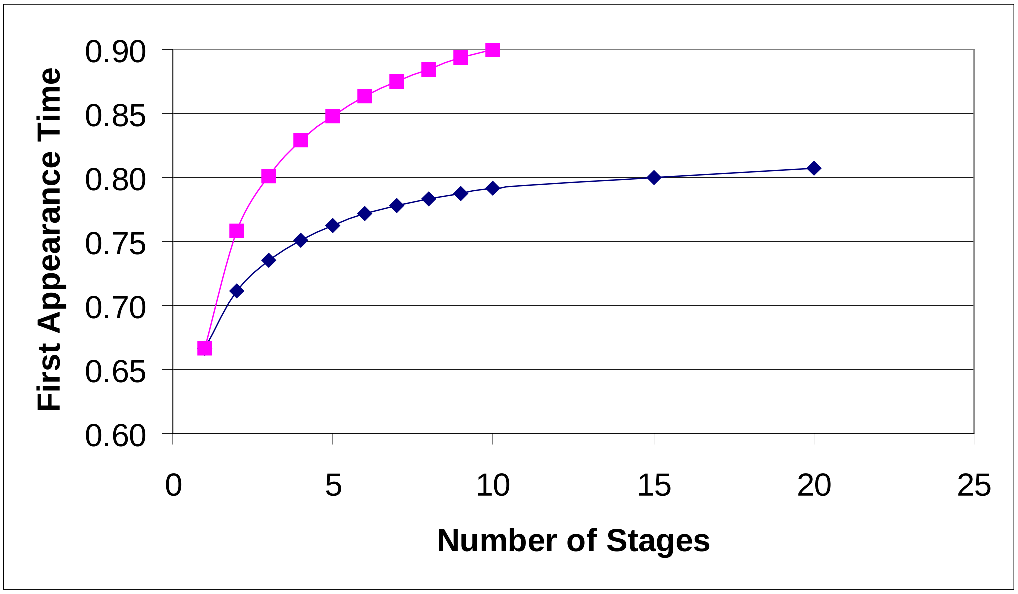

Figure 5. Optimized first appearance times for multi-stage, two-channel slit flow. The higher curve is speculated to be the upper limit possible in multi-stage slit flow.