

A Simplified Analysis of Temperature Swing Adsorption in Honeycomb Rotor Dehumidifiers

Tsutomu Hirose

5

Advanced Adsorption Technology, 19-7, 4 Chome, Miwadaai, Higashi-ku, Fukuoka, Japan

Abstract

10 Honeycomb rotor dehumidifiers have been used widely because of large surface area and rapid thermal response. Mass and heat transfer behavior for the performance evaluation is rather complicated and a system of simultaneous partial differential equations of 8 variables must be solved under a cyclic initial condition for the complete simulation.

15 With high efficiency dehumidifiers, however, a relative heat capacity $\lambda (= \rho_c c_{pc} L / \rho_g c_{pg} U t_c)$ is smaller than unity and a relative adsorption capacity $l (= \rho_c q^*_{s0} L / \rho_g x_{s0} U t_c)$ is higher than unity. When the cycle time is selected properly under the above condition, each step of adsorption/desorption can be divided into two different time zones, i.e. a transition stage appearing just after switching the step until $t/t_c < \lambda_s$ and a subsequent thermal equilibrium stage during $\lambda_s < t/t_c < 1$. Sensible heat transfer is predominant in the transition region and reaches the thermal equilibrium within this stage. Thus, the mathematical solution becomes free from the cyclic condition to save the iteration. In the
20 thermal equilibrium region, heat liberated by adsorption/desorption is balanced to warm/cool the streaming air, and the enthalpy of air is constant everywhere and equal to that of the feed/regeneration air. Since the amount adsorbed changes linearly, the time average values of temperature, humidity etc. can be calculated by a simple integration of the reciprocal of driving force, based on the short cycle time approximation or the continuous countercurrent flow model.

25 The above simplified model is proposed together with a graphical solution method and the result is compared with the more rigorous simulation.

1. Introduction

30 An adsorptive dehumidifier or a desiccant rotor adsorbs water vapor at room temperature to produce dry air and desorbs water vapor having been adsorbed at an elevated temperature. It is based on a principle of thermal (or temperature) swing adsorption (TSA) that is a kind of periodical mass transfer operations. This typical example of simultaneous mass and heat transfer process with periodic boundary conditions has been analyzed by a computer simulation by Tsujiguchi and
35 Kodama [1] and others. Heat/mass transfer between wet air and an adsorbent rotor consists of two stages which have different purposes, i. e. cooling/heating for switch of the process step and adsorption/desorption for treatment of water vapor. Each stage is distinguished clearly in a well-designed desiccant rotor. Heat/mass transfer can be analyzed more simply by obtaining an asymptotic solution to each stage. Contributions of each stage to the overall efficiency of
40 dehumidification are discussed based on this asymptotic analysis.

2. Analysis of Heat/Mass Transfer

2.1. Definition of Variables and Heat/Mass Balance

45

Figure 1 shows a schematic diagram of a 2-step TSA operation in which a desiccant rotor of length of L is rotated slowly and encountered periodically with adsorption (designated with a

subscript i=s) and desorption (designated with a subscript i=e). Relevant variables are expressed in terms of dimensionless variables as defined by Eqs. (1)-(7). They are Z for axial position z [m], τ for process time t [s], g for gas phase temperature T_g [K], f for solid phase temperature T_f [K], P for humidity x [kg/kg] and Q for amount adsorbed q [kg/kg]. Subscripts 0 and 1, respectively, refer to the inlet end of feed and the exit end of product in the adsorption stream.

$$Z = z / L, \quad (1)$$

$$\tau = t / t_{c,s} \quad (0 < \tau < 1) \quad (2)$$

$$\tau = 1 + (t - t_{c,s}) / t_{c,e} \quad (1 < \tau < 2) \quad (2')$$

$$g_i = \frac{T_{g,e1} - T_{g,i}}{T_{g,e1} - T_{g,s0}} \quad f_i = \frac{T_{g,e1} - T_{f,i}}{T_{g,e1} - T_{g,s0}} \quad (3), (4)$$

$$P_i = x_i / x_{s0} \quad (5)$$

$$Q_i = q_i / q_{s0}^* \quad Q_i^* = q_i^* / q_{s0}^* \quad (6), (7)$$

in which equilibrium amount adsorbed q_i^* is a function of temperature of adsorbent $T_{f,i}$ and humidity of air x_i and given by an adsorption isotherm.

$$q_i^* = q_i^*(T_{f,i}, x_i) \quad (8)$$

Heat/mass balance and heat/mass transfer rate are expressed in terms of these variables for both adsorption step (subscript i=s) and desorption step (subscript i=e) as follows

$$\partial g_i / \partial Z = \pm \kappa_i (g_i - f_i) \quad (9)$$

$$\partial P_i / \partial Z = \pm N_i (Q_i^* - Q_i) \quad (10)$$

$$\lambda_i \partial f_i / \partial \tau = \kappa_i (g_i - f_i) - r N_i (Q_i^* - Q_i) \quad (11)$$

$$l_i \partial Q_i / \partial \tau = N_i (Q_i^* - Q_i) \quad (12)$$

A complex sign \pm takes $-$ for adsorption step (i=s) and $+$ for desorption step (i=e). Ten dimensionless parameters are grouped out in a course of normalization. They are number of heat transfer units κ_i , number of mass transfer units N_i , relative heat capacity λ_i , relative adsorption capacity l_i , relative heat of adsorption r , humidity ratio γ .

$$\kappa_i = \frac{h_i a L}{\rho_g c_{pg} U_i} \quad N_i = \frac{k_i a L q_{s0}^*}{\rho_g U_i x_{s0}} \quad (13), (14)$$

$$\lambda_i = \frac{\rho_c c_{pc} L}{\rho_g c_{pg} U_i t_{c,i}} \quad l_i = \frac{\rho_c q_{s0}^* L}{\rho_g U_i x_{s0} t_{c,i}} \quad (15), (16)$$

$$r = \frac{x_{s0} \Delta H}{c_{pg} (T_{g,e1} - T_{g,s0})} \quad \gamma = x_{e1} / x_{s0} \quad (17), (18)$$

Boundary and periodic conditions applied are as follows

$$g_s = g_{s0} = 1 \text{ at } Z = 0, \quad g_e = g_{e1} = 0 \text{ at } Z = 1 \quad (19)$$

$$P_s = P_{s0} = 1 \text{ at } Z = 0, \quad P_e = P_{e1} = \gamma \text{ at } Z = 1 \quad (20)$$

$$f_e \Big|_{\tau=1} = f_s \Big|_{\tau=1}, \quad f_s \Big|_{\tau=0} = f_e \Big|_{\tau=2} \quad (21)$$

$$Q_e \Big|_{\tau=1} = Q_s \Big|_{\tau=1}, \quad Q_s \Big|_{\tau=0} = Q_e \Big|_{\tau=2} \quad (22)$$

95

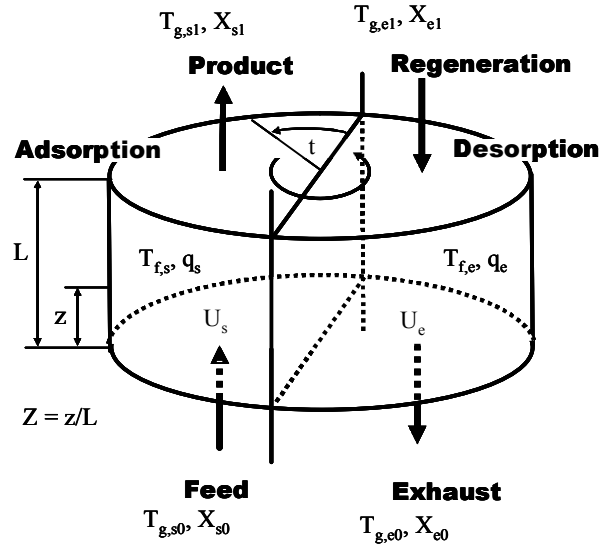


Fig. 1 Schematic diagram of a honeycomb rotor and relevant nomenclature

2.2. Transition Stage and Thermal Equilibrium Stage

In a well designed desiccant rotor, the relative heat capacity λ is necessary to be small for a rapid thermal response while the relative adsorption capacity l be large for a delayed breakthrough, i. e. the following equation

$$\lambda_i \ll 1 \ll l_i \quad (23)$$

has to be fulfilled.

Figure 2 shows a typical example of a computer simulation for the time variation of humidity P_{s1} , temperature $(1-g_{s1})$, and the amount adsorbed Q_{s1} at the product end under the above condition of Eq. (23). A transition stage with a rapid change in humidity and temperature starts just after the switch to the adsorption step, and followed by a thermal equilibrium stage with a gentle change in them.

2.3 Thermal Equilibrium Stage

Heat transfer rates are balanced between the sensible heat the adsorption heat and reach a state of the dynamic steady state. Humidity increases with time while temperature decreases with time. Thus the enthalpy of air is kept constant independently of space and time. Amount adsorbed Q changes linearly with time and its slope decreases with decreasing value of N_i/l_i ($=k_i a t_{c,i}/\rho_c$). at a limit of $N_i/l_i \rightarrow 0$, amount adsorbed Q becomes time independent throughout the whole step of adsorption and desorption and a system of the basic equations, Eqs. (9)-(12) is reduced simply to a system of ordinary differential equations, resulting in a steady state solution.

$$(P_e - \gamma) = (l_e / l_s)(P_s - P_{s1}) \quad (24)$$

$$g_e - rP_e = -r\gamma, \quad (25)$$

$$g_s - rP_s = 1 - r \quad (26)$$

$$\kappa_s(g_s - f_s) = rN_O(Q_s^* - Q_e^*) \quad (27)$$

$$\kappa_s(g_s - f_s) = \kappa_e(l_e / l_s)(g_e - f_e) \quad (28)$$

$$Q_s = Q_e = \frac{Q_s^* + (N_e l_s / N_s l_e) Q_e^*}{(1 + N_e l_s / N_s l_e)} \quad (29)$$

$$dP_s/dZ = -N_O(Q_s^* - Q_e^*) \quad (30)$$

$$\frac{1}{N_O} = 1/N_s + (l_e / l_s)/N_e + R \quad (31)$$

$$R = \frac{(1+l_s/l_e)}{12} \left[\frac{(1+N_e l_s/N_s l_e)}{l_s(1+N_e/N_s)} \right]^2 \quad (32)$$

The term R defined by Eq. (32) is an additional resistance to mass transfer caused by a finite value of l_s and l_e . Since every variable of humidity, temperature or amount adsorbed changes linearly with time for a finite but small value of N_i/l_i , time-average values are identical with the above asymptotic solution. This is along with the concept of the short cycle time approximation [2] which has been applied to the analysis of pressure swing adsorption (PSA).

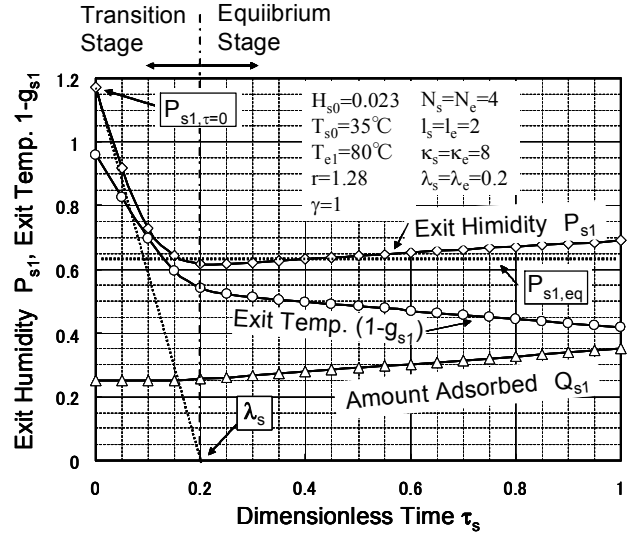


Fig. 2 Typical example of time variation of humidity, temperature and amount adsorbed at the product end

Humidity of the desorption step P_e is calculated by Eq. (24) for a given value of humidity of the adsorption step P_s between values at the inlet ($P_s=P_{s0}=1$) and the exit ($P_s=P_{s1}$). Air temperatures g_s and g_e corresponding to P_s and P_e are calculated from Eqs. (25) and (26), respectively. Equilibrium amounts adsorbed Q_s^* and Q_e^* given by an adsorption isotherm are a function of a set of the humidity P and adsorbent temperature f in the adsorption and desorption steps, respectively, and obtained by a trial and error solution of the simultaneous equations of Eqs. (27) and (28) together with f_s and f_e . This can be done straightforward by applying an excel tool of the goalseek. When the reciprocal of thus sought driving force of adsorption ($Q_s^* - Q_e^*$) is integrated,

$$N_O = \int_{P_{s1}}^1 \frac{1}{Q_s^* - Q_e^*} dP_s \quad (33)$$

the overall number of mass transfer units N_O , therefore the required length of the honeycomb rotor L can be calculated.

2.4. Transition Stage

Sensible heat transfer is predominant in this stage since the temperature difference is large between gas and adsorbent. Time interval of the transition stage is close to the relative heat capacity λ_s ,

$$\tau_s = \lambda_s \quad (34)$$

The amount adsorbed $Q_{\tau=0}$ and adsorbent temperature $f_{\tau=0}$ at the start of the adsorption step are known in the above analysis of the thermal equilibrium stage and the exit humidity at the start $P_{s1,\tau=0}$ is given by integrating the equation

$$dP_s/dZ = -N_s(Q_s^* - Q_{\tau=0}) \quad (35)$$

A contribution $\Delta P_{s1,tr}$ of the transition stage to the exit humidity is given by the equation

$$\Delta P_{s1,tr} = (1/2)\lambda_s P_{s1,\tau=0} (1 - P_{s1,eq} / P_{s1,\tau=0})^2 \quad (36)$$

in approximating by a triangle as shown in Fig. 2. In the equation, the term $P_{s1,eq}$ is the exit humidity in the thermal equilibrium stage as obtained in the above section.

3. Performance Evaluation of a Desiccant Rotor

Figure 3 shows an example of the axial distribution of humidity P_s in the thermal equilibrium stage of the adsorption step and the result is compared between the numerical and the asymptotic solutions. Values at the midpoint $\tau_s = 0.5$ are plotted as representative value for the numerical solution since it is time-dependent. The equilibrium amount adsorbed q^* is assumed to be a function of a relative humidity $\phi = p/p_s$ and to be correlated by a Langmuir isotherm as

$$q^* = \frac{q_\infty K \phi}{1 + K \phi} \quad (37)$$

with Langmuir parameters of $q_\infty = 0.336$ kg/kg, $K = 2.18$. The asymptotic solution is a limiting solution as $N/l = Kat_c/\rho_c \rightarrow 0$ and is in complete agreement with numerical solution for a small value of N/l less than unity. A significant error

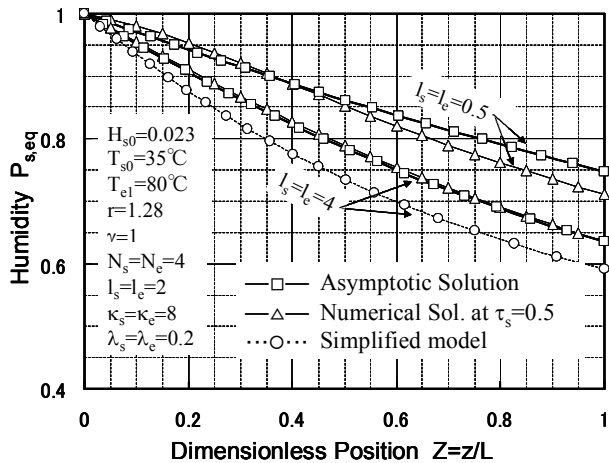


Fig. 3 Axial distribution of humidity analyzed by various methods of analysis

starts to appear for N/l beyond 10.

190 Figure 4 shows the result of the exit humidity when the relative adsorption capacity l_s is varied in keeping a ratio of relative heat to adsorption capacity l_s/λ_s constant at 10. Each contribution of the transition stage $\Delta P_{s1,tr}$ and the thermal equilibrium stage $P_{s1,eq}$ are plotted together with a sum of them, which is compared with the time average value of the numerical solution P_{s1} . Since the abscissa of this figure is inversely proportional to the adsorption step time t_c , the figure shows the effect of the rotation speed of a desiccant rotor. The contribution of the transition stage $\Delta P_{s1,tr}$ increases with increasing rotation speed while that of the equilibrium stage $P_{s1,eq}$ decreases and approaches to a limiting value with increasing speed. As a result, an optimum value of the rotation

195

200

205

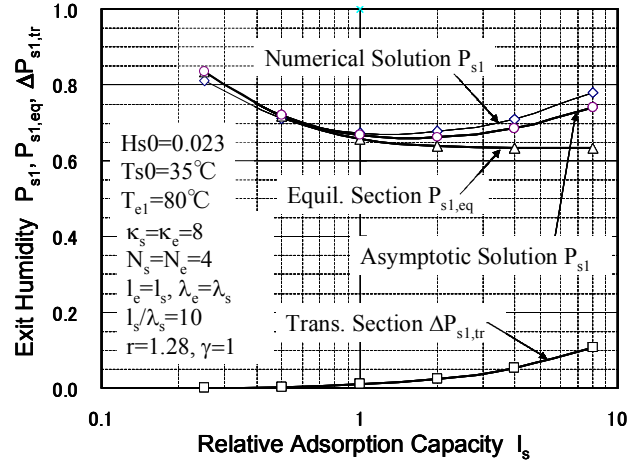


Fig. 4 Dependence of exit humidity on relative adsorption capacity

4. A Simplified Graphical Solution

210 Since heat transfer resistance ($1/\kappa_s$) in Eq. (27) is smaller than mass transfer resistance ($1/rN_o$), the temperature difference between air and adsorbent may be negligible. Equations (27) and (28) may be replaced with

$$f_s = g_s, \quad f_e = g_e \quad (38)$$

and the equilibrium amount adsorbed Q_s^* and Q_e^* can be evaluated simply for air temperature g_s and g_e but not adsorbent temperature. When Eqs. (24)-(26) and (33) are rewritten in a dimensional form as follows

215

$$x_e - x_{e1} = \frac{U_s t_{c,s}}{U_e t_{c,e}} (x_s - x_{s1}) \quad \text{Operating line} \quad (39)$$

$$h_s = h_{s0} = c_{pg} T_{g,s0} + x_{s0} \Delta H \quad \text{Iso-enthalpic line} \quad (40)$$

$$220 \quad h_e = h_{e1} = c_{pg} T_{g,e1} + x_{e1} \Delta H \quad \text{Iso-enthalpic line} \quad (41)$$

$$L = \frac{\rho_g U_s}{k_s a} = \int_{x_{s1}}^{x_{s0}} \frac{1}{q_s^* - q_e^*} dx_s \quad \text{Rotor length} \quad (42)$$

A set of Eqs. (39)-(41) can be expressed graphically as in Fig. 5. where the relative humidity ϕ is related to the humidity x with enthalpy h or air temperature T_g as parameter. A diagonal line is drawn for transforming between the ordinate and the abscissa. An adsorption isotherm Eq. (37) is plotted against the abscissa of relative humidity.

225

The state of air for given values of humidity x_{s0} , x_{s1} , x_{e1} is given by points s_0 , s_1 , e_1 on an h -line (iso-enthalpy line). The point A1 is a point at which a line turning on the diagonal line above the point e_1 intersects a line passing through a point s_1 . A line passing through the point A1 with a slope of $U_s t_{c,s}/U_e t_{c,e}$ is an operating line given by Eq. (39), which correlates the humidity between the adsorption and the desorption steps. When a state of air in the adsorption step is given by a point s on the h_s -line, the corresponding state of air in the desorption step is given by an intersection e between the h_e -line and a line turning twice on the operating line and the diagonal line. Equilibrium amount adsorbed q_s^* at the position of humidity x_s in the adsorption step is given by a point q_s^* where a line starting the point s and turning twice on the diagonal line and the adsorption isotherm

230

235

intersects a line above the point s . A corresponding amount adsorbed in the desorption step q_e^* is given by a point q_e^* by a similar graphical operation. A line segment $q_s^* q_e^*$ is the driving force of adsorption ($q_s^* - q_e^*$) for the given value of the humidity x_s . A graphical integration of Eq. (42) can be carried out to obtain the required length of the rotor L after seeking the driving force for several values of the humidity x_s between the feed x_{s0} and the product x_{s1} by repeating the same operation.

A broken line in Fig. 3 is an example of the axial distribution of the exit humidity obtained by this simplified graphical method. It is useful to find the overall trend in spite of less accuracy.

5. Summary

Simultaneous transfer of heat and water vapor in a desiccant rotor is analyzed by dividing the whole process into two time zones, i.e. the transition stage and the thermal equilibrium stage. By this simplification, the exit humidity can be calculated by a direct integration of the reciprocal of driving force for adsorption. The above asymptotic model is proposed together with a simplified graphical solution method and the result is compared with the more rigorous numerical simulation.

Nomenclature

- c_{pc} : heat capacity of adsorbent, $J.kg^{-1}.K^{-1}$
- c_{pg} : heat capacity of air, $J.kg^{-1}.K^{-1}$
- c_w : heat capacity of water vapor, $J.kg^{-1}.K^{-1}$
- f : dimensionless temperature of adsorbent, Eq. (4)
- g : dimensionless temperature of air, Eq.(3)
- h : enthalpy of air, J/kg
- ha : volumetric heat transfer coefficient, $W.m^{-3}.K^{-1}$
- ΔH : heat of adsorption or vaporization, $J.kg^{-1}$
- K : Langmuir constant
- ka : volumetric mass transfer coefficient, $kg.m^{-3}.s^{-1}$
- L : length of rotor, m
- l : relative adsorption capacity, Eq.(16)
- N : number of mass transfer units, Eq. (14)
- N_O : overall number of mass transfer units, Eq. (31)

- p : vapor pressure, Pa
- p_a : atmospheric pressure, Pa
- P : dimensionless humidity, Eq.(5)
- ΔP_{s1} : contribution from transition section
- q : amount adsorbed, kg/kg
- Q : dimensionless amount adsorbed, Eq.(6)
- q^* : equilibrium amount adsorbed, kg/kg

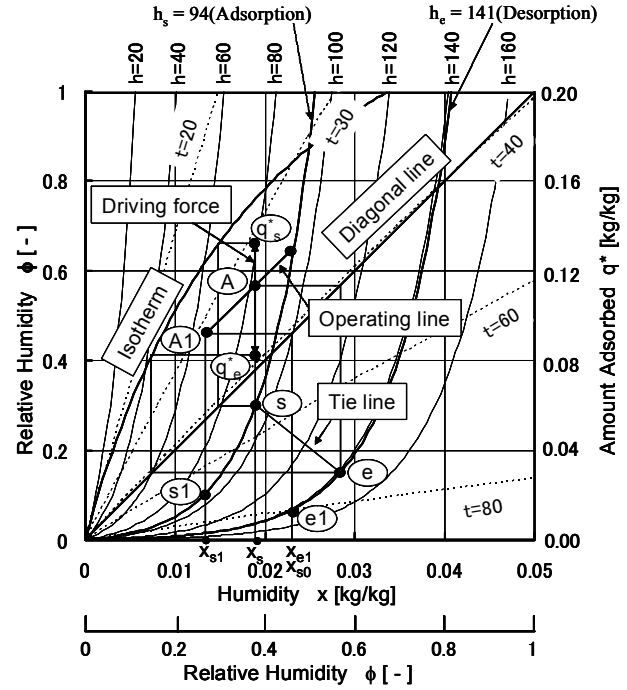


Fig. 5 Simplified graphical method of analysis

Q^*	: dimensionless value of q^* , Eq.(7)
q_∞	: Langmuir constant, kg/kg
r	: dimensionless heat of adsorption, Eq. (17)
R	: additional mass transfer resistance, Eq. (32)
t	: time from start of adsorption, s
t_c	: step time, s
T_f	: temperature of adsorbent, °C
T_g	: temperature of air, °C
U	: superficial velocity of air, m/s
x	: humidity, kg/kg
z	: distance from feed inlet, m
Z	: dimensionless distance, Eq.(1), $= z/L$
γ	: ratio of humidity, Eq.(18), $= x_{e1}/x_{s0}$
κ	: number of heat transfer units, Eq.(13)
λ	: relative heat capacity, Eq.(15)
τ	: dimensionless time, Eqs. (2),(2')
ρ_c	: bulk density of adsorbent, kg.m^{-3}
ρ_g	: density of air, kg.m^{-3}
ϕ	: relative humidity
subscripts	
0	: inlet end of feed
l	: exit end of product
e	: desorption step
i	: $i=s$ (adsorption) or $i=e$ (desorption)
s	: adsorption step

References

- [1] T. Tsujiguchi and A. Kodama : *Trans. of the JSRAE*, 22(4), 417(2005), 23(4), 467(2006) and 24(3), 205(2007).
 [2] T.Nakamura, et al. : *Adsorption*, 11, 609(2005)