MODELLING THE BREAKAGE OF SOLID AGGREGATES IN TURBULENT FLOWS

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Introduction

Breakage of small suspended aggregates (clusters, flocs) due to hydrodynamic stresses induced by fluid flow is crucial to both aggregation (coagulation, flocculation) and dispersion processes. The former finds broad application in solid-liquid separation where the transformation of particles in the colloidal size range into aggregates of a few micrometers to millimeters in size improves the performance of any separator. Aggregation is thereby usually performed in an agitated device where vigorous stirring leads to aggregate breakage which limits the formation of large aggregates. Regarding the reverse process, i.e., dispersing a solid into a liquid, aggregate breakage by vigorous stirring becomes the controlling mechanism.

Breakage Model

In this contribution we address the modeling of breakage kinetics of solid aggregates in a homogeneous turbulent flow [1]. It is generally assumed that breakage is a first order kinetic process. Accordingly, the population balance equation describing the evolution of the cluster mass distribution (CMD) reads as

$$\frac{\partial c(x,t)}{\partial t} = -K_B(x)c(x,t) + \int_x^\infty g(x,y)K_B(y)c(y,t)dy$$

where $c(x,t)$ is the CMD, x is the cluster mass normalized by the mass of the primary particle that forms the aggregate, $K_B(x)$ is the breakage rate function, and $g(x,y)$ is the fragment mass distribution (FMD). Hence, the number of fragments of mass (x, $x+dx$) formed by the breakage of a cluster mass y is $g(x,y)\,dx$.

A modeling strategy is proposed that allows us to derive expressions for $K_B(x)$ and $g(x,y)$ in a physically sound framework. It is believed that the first order breakage kinetics are governed by the turbulent fluctuations where only turbulent events that are violent enough lead to breakage. The magnitude of a turbulent event required to cause breakage is thereby determined by the properties of the aggregate and depends in particularly on the aggregate mass. To describe the turbulent fluctuations a multifractal model is adopted which provides a sufficient description of the fine scale turbulence [2,3]. The multifractal model accounts in particular for fine scale intermittency defined here as the strong and irregularly appearance of fluctuations in an else homogeneous flow. Further, a power law relation is used to relate the critical magnitude of a turbulent event that causes breakage to the mass of the aggregate.

The resulting breakage rate function differs substantially when compared to existing models (i.e., the power law breakage rate function [4] and exponential breakage rate function [5,6]) in as such that $K_B(x)$=0 and $K_B(x) \to \infty$ below and above a certain limiting aggregate size, respectively. Further, $K_B(x)$ exhibits a Reynolds number dependency due to intermittency that is missing in the present models [4-6]. It will be shown that when turbulence is modeled using a Gaussian velocity gradient, the presented modeling framework leads to an expression for $K_B(x)$ that in the limit of $x \to 0$ and $x \to \infty$ reduces to an exponential function and a power law that are formally identical to the breakage rate functions used in Ref [4] and [6], respectively.

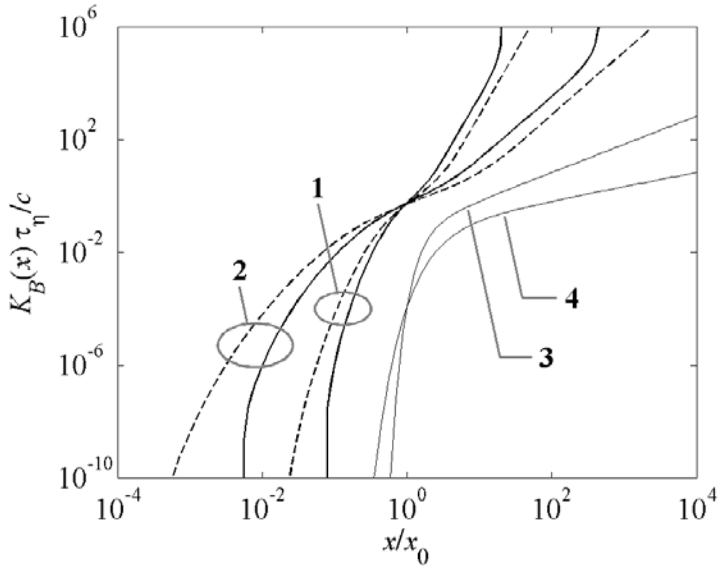

Figure 1. Breakage rate function $K_B(x)$ as a function of the aggregate mass x. Curve 1 and 2 refer to the novel model using a multifractal description for the fine scale turbulence whereas curve 3 and 4 are based on a Gaussian description. For the former, solid and dashed lines refer to R_λ = 137 and R_λ = 660, respectively (in normalized variables the Gaussian model exhibits no dependency on R_λ). Curve 1 (and 3) and curve 2 (and 4) refer to different aggregate strength exponents. In the axis, x_0 is a reference aggregate mass, c is a constant and $\tau_\eta = (\langle\varepsilon\rangle/\nu)^{1/2}$.

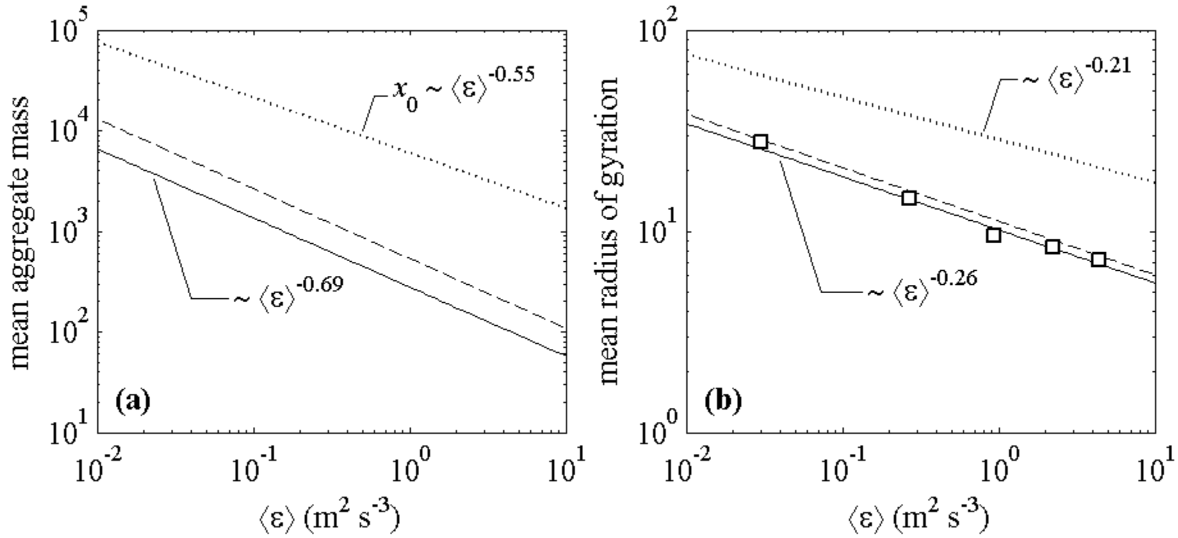

Figure 2. Asymptotic values of the mean aggregate mass $\bar{x}$ (*a*) and the mean radius of gyration ρ (*b*) as a function of the mean energy dissipation rate resulting from a pure breakage process. In both panels, the solid lines refer to the asymptotic values of the corresponding quantity. The dashed and the dotted line in (*a*) refer to asymptotic mass of the largest aggregates and the reference mass, respectively. In (*b*) the dashed and the dotted line refer to the corresponding aggregate sizes, respectively. The open symbols in (*b*) refer to the experimental values reported in Ref [7].

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