



is common in the industry to process the data one scan at a time and to estimate the CD profile by exponential filtering of successive scans after first removing the scan average. The filtered profile is then taken as the CD estimate, the scan average taken as MD variation with the balance of the measured data assumed to be noise. Such an approach ignores the scanning geometry and makes no organized attempt to assign the spectral components of the scanned data to the MD, CD and noise categories. Early methods[1] for separating MD and CD included bootstrap algorithms using least squares methods to estimate CD and Kalman filtering to track MD changes. More recently wavelet analysis[2] has been applied to track the spectral content of the measurements in real time and to use this as a basis for MD/CD separation. There has also been significant work [3] on setting the scanning operation into a formal two dimensional sampling theory, leading to accurate reconstruction algorithms.

The approach presented in this paper draws on several of these themes. The idea of recursive bootstrap estimation is used to refine separation of estimated MD effects from the CD profile, wavelet filtering is used in order to identify noise components and, finally, periodic sampling theory is used to remove the effects of scanning geometry from the scanned data before it is processed.

The method has been tested on simulated data, in which case we have the advantage of knowing the true variations. Analysis has also been carried out on operating data, in which case the impact of the method can only be inferred from a comparison with the estimates provided by other filtering approaches. The acid test would come from analysis of the paper sheet in the laboratory after production, but the cost, logistics and experimental requirements of such a correlation mean that it is almost never feasible. We have no such data to present here.

## 2. Non-Uniform Sampling & Reconstruction

### 2.1. Periodic non-uniform sampling

The scanner measurement samples taken at each CD position on successive scans form a non-uniform but periodic sampling scheme, with the pattern of periodic sampling changing with CD position. In this case there are two samples per period. More generally there may be an arbitrary number of samples within each period before the pattern is repeated, as shown in Figure 2. It is well known that in such cases the relation between the bandwidth of a signal that is to be exactly reconstructed, and the minimum required Nyquist sampling rate of twice the bandwidth, holds true if the average sampling rate satisfies the Nyquist constraint. Thus, in principle, the variations in MD sampling pattern across the sheet are of

no concern and are equivalent for reconstruction purposes to the uniform sampling that occurs at the centre of the sheet. In practice, however, measurement noise effects on the reconstruction process are strongly influenced by the sampling pattern within each period. Samples obtained close to the sheet edges arrive in closely located pairs and do not allow reconstruction with the precision available from samples at the centre of the sheet.

To describe the general sampling pattern, samples are considered as  $N$  different uniform samples of the signal  $x(t)$  interlaced in time. Each set of samples is fitted into the pattern by a set of  $N$  delays  $\tau_k$ .  $T$  defines the average sampling time. Thus the sample times are:

$$t_{nk} = nNT + \tau_k, \text{ for } \begin{cases} k = 1, 2, \dots, N \\ n = 0, \pm 1, \pm 2, \dots \\ T = \frac{1}{2W} \end{cases}$$

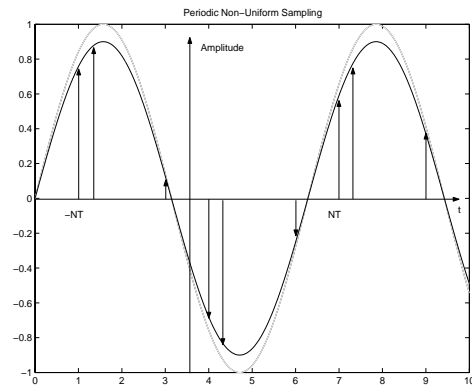


Figure 2 - Periodic Non-uniform Sampling

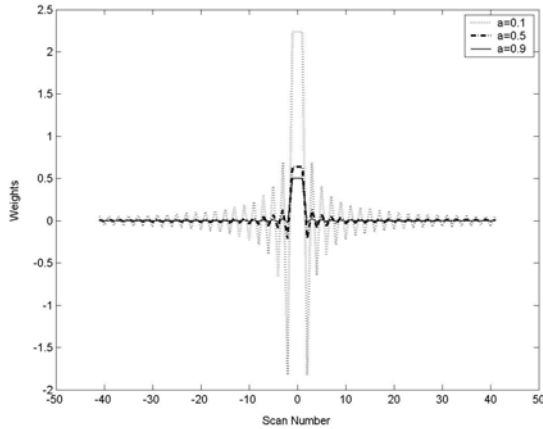
For periodic non-uniform samples of  $x(t)$ , in a special case when  $N=2$  and  $\tau_1 = -\tau_2 = \tau$ , the following interpolation formula holds. [4]

$$x(t) = \frac{\cos 2\pi W \tau - \cos 2\pi W t}{2\pi W \sin 2\pi W \tau} \times \sum_{n=-\infty}^{\infty} \left[ \frac{x(2nT + \tau)}{t - 2nT - \tau} - \frac{x(2nT - \tau)}{t - 2nT + \tau} \right]$$

Here  $W$  is the bandwidth of the signal,  $T$  is the average sampling time, and it is assumed that we are sampling at exactly the Nyquist rate in relation to the bandwidth of the signal. Note that this interpolation is non-causal.

For the paper machine scanning arrangement, the value of  $\tau$  changes between different CD positions. This change will affect the reconstruction relationship, with the weighting amplitude assigned to the interpolation function associated with each sample increasing dramatically as  $\tau$  approaches the value of 1, which

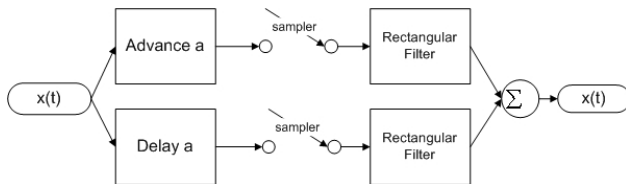
indicates the edge of the sheet. This is shown in Figure 3, where the individual interpolating functions are plotted. The variable,  $a$ , as indicated in Figure 1, varies from  $a=0$  at one side of the sheet to  $a=1$  at the other. At the centre, where  $a=0.5$ , the interpolating function becomes the familiar sinc function of uniform sampling theory.



**Figure 3 - Reconstruction Function Amplitudes for Different CD positions**

## 2.2. Reconstruction Using Filters

In Figure 3 [5] an arrangement of non-causal filters equivalent to the interpolation approach described in section 2.1 is shown. The periodic non-uniform sampling arrangement of the scanning sensor, where two samples are taken in each sampling interval of  $2T$ , is illustrated. One sample is taken at a point  $a$  seconds before the scan end and the other at a point  $a$  seconds after the scan end. The reconstruction of data from this sampling strategy, as shown in Figure 3, consists of one channel including a forward shift and the other with a backward shift.

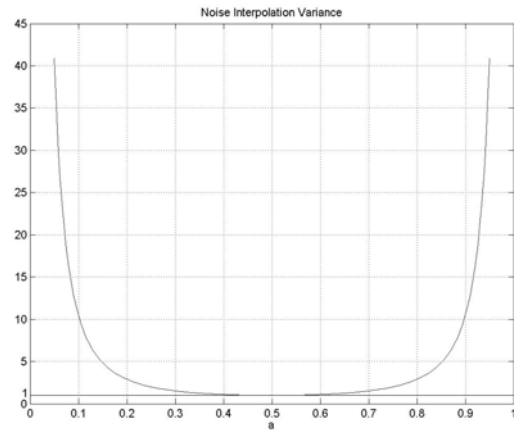


**Figure 4 - Data Reconstruction using Generalized Sampling Theorem**

Perfect reconstruction, is possible using this technique if we have a band limited signal. However, in reality the signal is highly corrupted with noise and is not band limited to the Nyquist rate of sampling. In the presence of white noise, the reconstructed signal from a set of periodic non-uniform samples can be shown to amplify noise power relative to the uniform sampling case by a

ratio dependant on the separation of the samples. For our case, with two samples, the noise amplification factor [6] will vary as shown in Figure 5, with the value of  $a$  lying between 0 and 1. For the case of periodic sampling  $a=0.5$ , at the sheet centre, the noise amplification factor is 1, as expected. The noise interpolation factor is the ratio of the variance of the noise of the reconstructed signal to that of the white noise added to the original signal.

$$A_e = \left( \frac{1 + \sin^2\left(\frac{\pi}{2} - \frac{W\tau}{2}\right) + \sin^2\left(-\frac{W\tau}{2}\right)}{2 \sin^2 \frac{W\tau}{2}} \right) = \frac{1}{\sin^2 \frac{W\tau}{2}}$$



**Figure 5 - Noise Amplification factor at different CD positions**

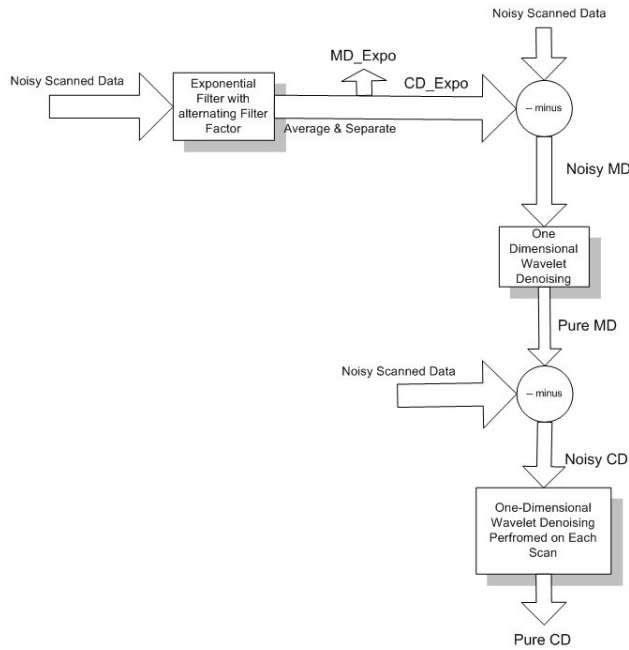
## 3. Recursive Filtering Algorithm

The algorithm presented here initially processes the scanned data so as to eliminate the periodic sampling effects discussed in Section 2. The data is then recursively processed using wavelet filtering in order to separate MD and CD variations from noise terms. In the examples that follow, scanned data has been processed in real time – i.e. causal filtering has been used. Significant improvements would be possible if batch processing of the accumulated data were possible, but the additional delay involved is not normally acceptable.

Wavelet methods have been shown to be effective for removing noise from the data. We initially assume that a CD profile estimate is available. Then this CD profile is subtracted from the measured noisy data, to leave a first estimate of the MD variations, which are treated as a one-dimensional data set. Wavelet de-noising is effective in removing noise components from this MD data. With this MD estimate, the recursion is initiated by returning to the raw scanned data and refining the estimated CD

profile, which will include noise terms. Wavelet filtering is again applied, now to the CD profile.

The block diagram shown in Figure 6 describes this procedure.



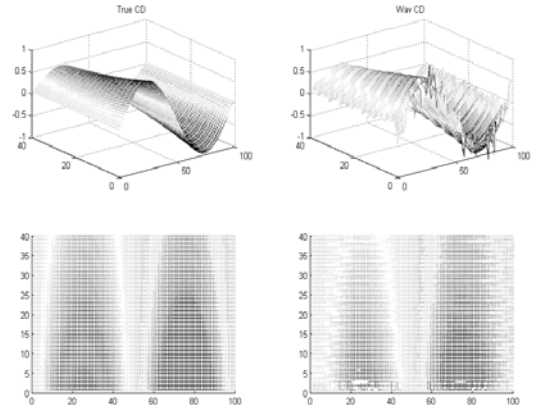
**Figure 6 - Recursive Filtering Algorithm**

Initially, a filter based on the periodic sampling theory described earlier, is applied to the scanned data. The non-causal filters can be approximated at this stage by causal ones for real time processing. The full filter is used for off-line batch applications. The reduced filter is similar to conventional exponential filtering, with the difference that the time constant is changed in recognition of the varying time intervals between the samples varies. In this way the sampling geometry of the scanner is recognized.

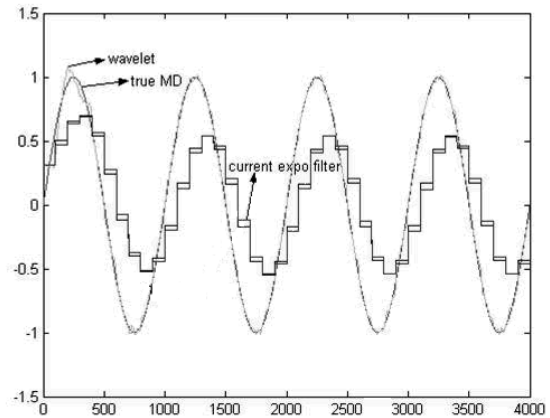
#### 4. Simulation Results

The algorithm has been tested on a variety of simulated data sets, and typical results are shown in Figures 7 and 8. The data was corrupted by white noise of standard deviation  $\sigma=0.2$  imposed on a signal of unit amplitude. It will be seen that excellent estimates are obtained in comparison with those generated by conventional filtering.

MD signals are shown in Figure 8, which shows that using the wavelet algorithm described above gives excellent estimation of the MD signal compared to using the exponential filter, or scan average.



**Figure 7 - CD Profiles: Actual (left) & Filtered (right)**



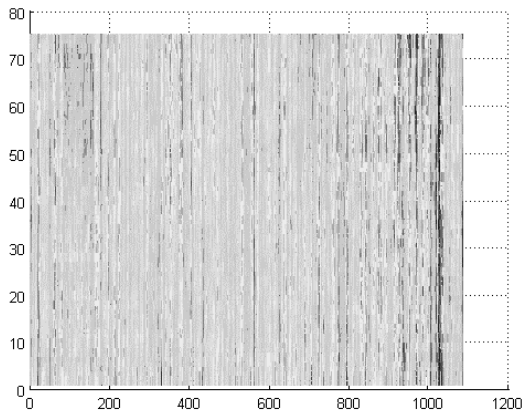
**Figure 8 – True & Estimated MD Signals**

Error variance analysis for the MD signals establishes that the wavelet filtered MD error is very minor compared to that of the exponential filter.

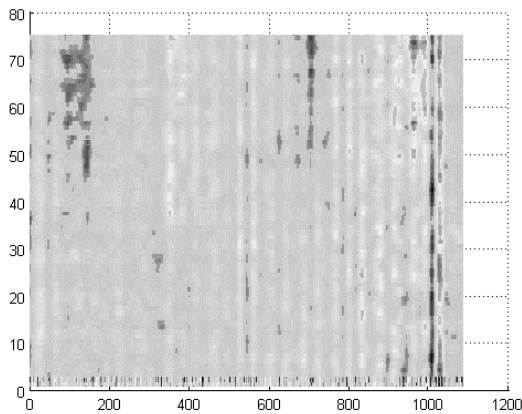
In the simulation the scan time needed for the scanner to move from one side of the sheet to the other, is assumed to be 20 seconds. The data has 100 measurement points, and 40 scans are taken into account. The true MD signal is a relatively slow pure sine wave; the CD is also a sine wave that decays in time.

#### 5. Industrial Results

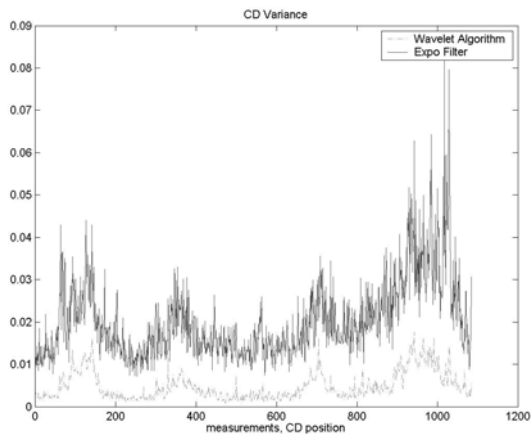
The proposed algorithm has been tested on an industrial set of moisture data, for different periods of time. Figure 9 shows the CD profile estimates obtained using conventional filtering, while Figure 10 show the CD profiles obtained using wavelet filtering.



**Figure 9 – Filtered CD Using Exponential Filter**



**Figure 10 - Filtered CD, Using Wavelet Algorithm**



**Figure 11 - CD Variance of the Filtered Data**

Figure 11 compares the CD error variance of the data, to the error variance of the CD profile if it was filtered using conventional methods.

## 6. Conclusion

It has been shown that recursive wavelet filtering can provide excellent reconstruction of both machine direction and cross machine process variations. Sampling geometry has been taken into account in filtering data before it is submitted to the wavelet processing. The proposed method combines approaches, each of which has been discussed individually elsewhere. This method is applied to the measured data following each scan, and so may be performed on line.

Important issues remain to be addressed. The aliasing effects of high frequency machine direction disturbances should be addressed in a formal manner, especially when these variations are close the scan frequency. Variable rate scanning has been proposed as a method to address these issues. It is also important to note that the full application of periodic sampling theory has not been implemented both because of noise issues at the sheet edges as indicated in Figure 5, and because of a need to restrict data processing to a scan-by-scan basis. Batch processing would allow more accurate results, as would a full analysis of causal non-uniform sampling filters.

## 7. References

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