Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

New approach to adaptive vibration control

Maciej Niedźwiecki and Michał Meller
Faculty of Electronics, Telecommunications and Computer Science
Department of Automatic Control, Gdańsk University of Technology
ul. Narutowicza 11/12, Gdańsk, Poland
maciekn@eti.pg.gda.pl , michal.meller@eti.pg.gda.pl

Abstract— **The paper presents a new approach to rejection of sinusoidal disturbances acting at the output of a discrete-time linear stable plant with unknown dynamics. It is assumed that frequency of the sinusoidal disturbance is known, and that the output signal is contaminated with wideband measurement noise. It is not assumed that a reference signal, correlated with the disturbance, is available. The proposed solution combines the coefficient fixing technique, used to "robustify" self-tuning minimum-variance regulators, with automatic adaptation gain tuning. Simulation experiments confirm that, under Gaussian assumptions, the closed-loop system converges in the mean to the optimal one.**

Index Terms: Adaptive control, system identification, disturbance rejection.

I. INTRODUCTION

Consider the problem of cancellation of a narrow-band disturbance $d(t)$, with known frequency ω_o, corrupting the output of a discrete-time, stable linear plant of unknown dynamics governed by

$$y(t) = K_o(q^{-1})u(t-1) + d(t) + v(t) \tag{1}$$

where $t = \ldots, -1, 0, 1, \ldots$ denotes normalized time, q^{-1} is the backward shift operator, $y(t)$ is the system output, $K_o(q^{-1})$ denotes unknown transfer function of the controlled plant, $u(t)$ is the system input and, finally, $v(t)$ denotes a wideband measurement noise.

We will assume that the disturbance signal can be modeled as

$$d(t) = a_1(t)\sin\omega_o t + a_2(t)\cos\omega_o t = \boldsymbol{\alpha}^T(t)\mathbf{f}(t) \tag{2}$$

where $\boldsymbol{\alpha}(t) = [a_1(t), a_2(t)]^{\mathrm{T}}$, $\mathbf{f}(t) = [\sin\omega_o t, \cos\omega_o t]^{\mathrm{T}}$ and $a_1(t)$, $a_2(t)$ denote unknown, slowly-varying weighting coefficients.

Narrow-band disturbances are usually generated by rotating elements of electro-mechanical systems and their elimination may be a very important control task, determining quality of the underlying technological processes such as turning, milling, grinding etc.. In some cases a reference sensor can be placed close to the source of vibration, providing a signal that may be used for feedforward disturbance compensation. We will *not* assume that such reference signal is available.

The problem of narrow-band disturbance rejection was considered by many authors under different methodologies, such as internal model principle or the phase-locked loop based approach – see e.g. the recent work of Bodson and co-workers [1], [2], [3], [4], and Landau and co-workers [5], [6]. For an overview of different approaches see e.g. a tutorial paper [6].

An entirely new approach to cancellation of narrow-band disturbances, based on coefficient fixing and automatic gain tuning, was proposed and analyzed in [7], [8]. The new method was developed for complex-valued systems, i.e., for systems where $y(t)$, $u(t)$, $d(t)$ and $v(t)$ are complex-valued signals. In particular, it was assumed that disturbance has the form

$$d(t) = a(t)e^{j\omega_o t} \tag{3}$$

which can be considered a complex-valued counterpart of (2).

The main purpose of this paper is to extend the results presented in [7] to systems with real-valued input/output signals. This is a nontrivial task. We will show that for real-valued systems the analysis can be performed in a similar but not identical way as that carried for complex-valued systems. Such analysis requires different tools and leads to different quantitative results than those presented in [7].

II. OPEN-LOOP CASE

Since the control loop incorporates a transport delay of one sampling interval, when shaping the input signal at the instant t one needs an accurate one-step-ahead prediction of $d(t+1)$, further denoted by $\widehat{d}(t+1|t)$. Similarly as in the complex-valued case, we will base structure of the closed-loop predictor on the form of its open-loop analog.

Consider the problem of one-step-ahead prediction/compensation of a signal governed by

$$s(t) = d(t) + v(t) \tag{4}$$

where $d(t)$ is a harmonic disturbance, described by (2), and $v(t)$ denotes white measurement noise obeying

> (**A1**) $\{v(t)\}$ is a sequence of uncorrelated, normally distributed random variables with zero mean and variance σ_v^2: $v(t) \sim \mathcal{N}(0, \sigma_v^2)$.

To proceed further we will have to make some assumptions on the way the weighting coefficients $a_1(t)$ and $a_2(t)$, appearing in (2), vary with time. We will assume that both coefficients evolve, independently of each other, according

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

to the random-walk (RW) model, namely[1]

$$\boldsymbol{\alpha}(t) = \boldsymbol{\alpha}(t-1) + \mathbf{w}(t) \tag{5}$$

where

(**A2**) $\{\mathbf{w}(t)\}$, independent of $\{v(t)\}$, is a sequence of uncorrelated, normally distributed random variables with zero mean and covariance matrix $\mathbf{W} = \sigma_w^2 \mathbf{I}$: $\mathbf{w}(t) \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I})$.

and $\mathbf{I}$ denotes a 2×2 identity matrix.

Even though pretty naïve from the practical viewpoint, such model of variation will allow us to determine the lower bound on the mean-squared cancellation error, and hence to evaluate statistical efficiency of the proposed disturbance rejection scheme in absolute terms, rather than relative terms (e.g. by comparing it with one of the existing schemes).

Combining (2), (4) and (5) one arrives at the following state-space equations

$$\begin{aligned} \boldsymbol{\alpha}(t) &= \boldsymbol{\alpha}(t-1) + \mathbf{w}(t) \\ s(t) &= \boldsymbol{\alpha}^{\mathrm{T}}(t)\mathbf{f}(t) + v(t) \ . \end{aligned} \tag{6}$$

Denote by $\mathcal{S}(t) = \{s(1), \ldots, s(t)\}$ the set of measurements available at instant t. The optimal, in the mean-square sense, one-step-ahead predictor of $s(t)$ has the form [11]

$$\widehat{s}(t|t-1) = \mathrm{E}[s(t)|\mathcal{S}(t-1)] = \widehat{d}(t|t-1) = \widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t|t-1)\mathbf{f}(t)$$

where $\widehat{\boldsymbol{\alpha}}(t|t-1) = \mathrm{E}[\boldsymbol{\alpha}(t)|\mathcal{S}(t-1)]$ is a one-step-ahead predictor of $\boldsymbol{\alpha}(t)$. The mean-squared prediction error can be expressed in the form

$$\mathrm{E}\{[s(t) - \widehat{s}(t|t-1)]^2\} = \mathrm{E}[c^2(t)] + \sigma_v^2$$

where

$$c(t) = d(t) - \widehat{d}(t|t-1) = [\boldsymbol{\alpha}(t) - \widehat{\boldsymbol{\alpha}}(t|t-1)]^{\mathrm{T}}\mathbf{f}(t)$$

will be further called cancellation error.

Under assumptions (A1) and (A2) the optimal estimates of $\boldsymbol{\alpha}(t)$ can be computed recursively using the celebrated Kalman filtering (KF) algorithm

$$\begin{aligned} \widehat{\boldsymbol{\alpha}}(t|t) &= \widehat{\boldsymbol{\alpha}}(t|t-1) + \mathbf{g}(t)\varepsilon(t) \\ \widehat{\boldsymbol{\alpha}}(t|t-1) &= \widehat{\boldsymbol{\alpha}}(t-1|t-1) \\ \varepsilon(t) &= s(t) - \widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t|t-1)\mathbf{f}(t) \\ \mathbf{g}(t) &= \frac{\mathbf{P}(t|t-1)\mathbf{f}(t)}{\sigma_v^2 + \mathbf{f}^{\mathrm{T}}(t)\mathbf{P}(t|t-1)\mathbf{f}(t)} \\ \mathbf{P}(t|t-1) &= \mathbf{P}(t-1|t-1) + \sigma_w^2\mathbf{I} \\ \mathbf{P}(t|t) &= \mathbf{P}(t|t-1) - \frac{\mathbf{P}(t|t-1)\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{P}(t|t-1)}{\sigma_v^2 + \mathbf{f}^{\mathrm{T}}(t)\mathbf{P}(t|t-1)\mathbf{f}(t)} \end{aligned} \tag{7}$$

where $\widehat{\boldsymbol{\alpha}}(t|t) = \mathrm{E}[\boldsymbol{\alpha}(t)|\mathcal{S}(t)]$ denotes the filtered estimate of $\boldsymbol{\alpha}(t)$, while $\mathbf{P}(t|t-1)$ and $\mathbf{P}(t|t)$ are the a priori and a posteriori error covariance matrices, respectively.

[1]It is interesting to notice that when $\boldsymbol{\alpha}(t)$ obeys (5) the ratio $a_1(t)/a_2(t)$ slowly changes with time, which means that, strictly speaking, the instantaneous frequency of $d(t)$ is not constant but slowly varies around ω_o.

Let $\xi = \sigma_w^2/\sigma_v^2$. When the vector $\boldsymbol{\alpha}(t)$ changes sufficiently slowly, namely when

$$\sqrt{\xi} = \frac{\sigma_w}{\sigma_v} \ll 1 \tag{8}$$

and when the "period" of $d(t)$, equal to $T_o = 2\pi/\omega_o$, is sufficiently small, the matrix $\mathbf{P}(t|t-1)$ is in the steady state approximately constant

$$\begin{aligned} &\mathrm{E}_\infty\{[\boldsymbol{\alpha}(t) - \widehat{\boldsymbol{\alpha}}(t|t-1)][\boldsymbol{\alpha}(t) - \widehat{\boldsymbol{\alpha}}(t|t-1)]^{\mathrm{T}}\} \\ &\quad = \lim_{t\mapsto\infty} \mathbf{P}(t|t-1) \cong \mathbf{P}_\infty \end{aligned}$$

where E_∞ denotes the steady-state expectation (whenever it exists): $\mathrm{E}_\infty[x(t)] = \lim_{t\mapsto\infty} \mathrm{E}[x(t)]$. The limiting value of this matrix, denoted by $\mathbf{P}_\infty$, can be determined analytically using the deterministic averaging approach [9]. First, when condition (8) is fulfilled, it can be shown that for large values of t it holds that [10] $\mathbf{f}^{\mathrm{T}}(t)\mathbf{P}(t|t-1)\mathbf{f}(t) \ll \sigma_v^2$, leading to the following approximate relationship [cf. (7)]

$$\begin{aligned} &\mathbf{P}(t+1|t) \cong \mathbf{P}(t|t-1) \\ &\quad - \frac{1}{\sigma_v^2}\mathbf{P}(t|t-1)\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{P}(t|t-1) + \sigma_w^2\mathbf{I} \ . \end{aligned} \tag{9}$$

Second, since under the conditions specified above, variations in the covariance matrix $\mathbf{P}(t|t-1)$ are much slower than in the "regression" vector $\mathbf{f}(t)$, one can set

$$\mathbf{P}(t+1|t) \cong \mathbf{P}(t|t-1) \cong \ldots \mathbf{P}(t-T|t-T-1) \tag{10}$$

where T denotes the width of the local averaging window. Combining (9) with (10) one obtains

$$\mathbf{P}(t|t-1)\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_T \mathbf{P}(t|t-1) \cong \sigma_v^2\sigma_w^2\mathbf{I} \tag{11}$$

where

$$\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_T = \frac{1}{T}\sum_{i=t-T+1}^{t} \mathbf{f}(i)\mathbf{f}^{\mathrm{T}}(i) \ .$$

Note that

$$\lim_{T\mapsto\infty}\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_T = \left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_\infty = \frac{1}{2}\,\mathbf{I} \tag{12}$$

and that $\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_T$ can be closely approximated by $\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_\infty$ when $T \gg T_o$. This allows one to rewrite (11) in the form

$$\mathbf{P}(t|t-1)\mathbf{P}(t|t-1) \cong \mathbf{P}_\infty\mathbf{P}_\infty \cong 2\,\sigma_v^2\sigma_w^2\mathbf{I}$$

leading to

$$\mathbf{P}_\infty \cong \sqrt{2}\,\sigma_v\sigma_w\mathbf{I}$$

and to the following steady-state recursive estimation formula

$$\widehat{\boldsymbol{\alpha}}(t+1|t) = \widehat{\boldsymbol{\alpha}}(t|t-1) + h_\infty\mathbf{f}(t)\varepsilon(t) \tag{13}$$

where $h_\infty = \sqrt{2}\,\sigma_w/\sigma_v = \sqrt{2\xi}$.

In an analogous way, assuming that the quantities $\boldsymbol{\alpha}(t)$ and $\widehat{\boldsymbol{\alpha}}(t|t-1)$ change slowly compared to $\mathbf{f}(t)$, one can compute

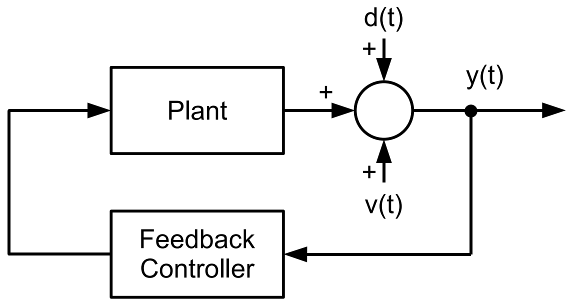

Fig. 1. Block diagram of the disturbance rejection system.

the steady-state mean-squared cancellation error yielded by the KF algorithm

$$\begin{aligned} \mathrm{E}_\infty[\langle c^2(t)\rangle_\infty] &\cong \mathrm{tr}\{\mathbf{P}_\infty \langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\rangle_\infty\} \\ &= \frac{1}{2}\,\mathrm{tr}\{\mathbf{P}_\infty\} = \sqrt{2}\,\sigma_v\sigma_w \ . \end{aligned} \tag{14}$$

Since Kalman filter is the optimal estimation algorithm, the right-hand side of (14) determines the lowest achievable value of the mean-squared cancellation error for the problem at hand, sometimes referred to as the Bayesian Cramér-Rao bound [12] (the classical Cramér-Rao bound does not apply to systems/signals with random parameters).

III. Adaptive feedback controller

We will look for the control signal that minimizes the mean-squared cancellation error for the system described by (1) – see Fig. 1. We will assume that the controlled plant is stable and has nonzero gain at the frequency ω_o:

(A3) $K_o(q^{-1}) = \sum_{i=0}^{\infty} f_i q^{-i}, \ \sum_{i=0}^{\infty} |f_i| < \infty,$
$K_o(e^{-j\omega_o}) \neq 0$

but we will *not* assume that its transfer function $K_o(q^{-1})$ is known.

Vaguely speaking, to cancel sinusoidal disturbance $d(t)$, one should generate such sinusoidal input signal $u(t)$ which, after passing through the plant, will have the same shape as $d(t)$ but opposite polarity. Note that the steady-state response of a linear system to the sinusoidal input signal $u(t) = \boldsymbol{\alpha}^{\mathrm{T}}\mathbf{f}(t)$ can be written in the form

$$K_o(q^{-1})\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{f}(t) = \boldsymbol{\alpha}^{\mathrm{T}}\mathbf{K}_o\mathbf{f}(t) \tag{15}$$

where

$$\begin{aligned} \mathbf{K}_o &= \begin{bmatrix} \mathrm{Re}\{K_o(e^{-j\omega_o})\} & \mathrm{Im}\{K_o(e^{-j\omega_o})\} \\ -\mathrm{Im}\{K_o(e^{-j\omega_o})\} & \mathrm{Re}\{K_o(e^{-j\omega_o})\} \end{bmatrix} \\ &= k_o \begin{bmatrix} \cos\phi_o & \sin\phi_o \\ -\sin\phi_o & \cos\phi_o \end{bmatrix} . \end{aligned}$$

The quantities $k_o = |K_o(e^{-j\omega_o})|$ and $\phi_o = \mathrm{Arg}[K_o(e^{-j\omega_o})]$ can be recognized as a true plant gain at the frequency ω_o and its true phase shift, respectively.

Therefore, had the matrix $\mathbf{K}_o$ been known, the following disturbance rejection rule could have been used

$$u(t) = -\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t+1|t)\mathbf{K}_o^{-1}\mathbf{f}(t+1) \ .$$

According to (15), for such control signal the cancellation error can be approximately expressed in the form[2]

$$\begin{aligned} c(t) &= K_o(q^{-1})u(t-1) + d(t) \\ &\cong d(t) - \widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t|t-1)\mathbf{K}_o^{-1}\mathbf{K}_o\mathbf{f}(t) \\ &= d(t) - \widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t|t-1)\mathbf{f}(t) = [\boldsymbol{\alpha}(t) - \widehat{\boldsymbol{\alpha}}(t|t-1)]^{\mathrm{T}}\mathbf{f}(t) \end{aligned}$$

which is identical with an analogous expression derived in the open-loop case. Since the transfer function $K_o(q^{-1})$ is unknown, the actual control rule will have the form

$$u(t) = -\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t+1|t)\mathbf{K}_n^{-1}\mathbf{f}(t+1) \tag{16}$$

where

$$\begin{aligned} \mathbf{K}_n &= \begin{bmatrix} \mathrm{Re}\{K_n(e^{-j\omega_o})\} & \mathrm{Im}\{K_n(e^{-j\omega_o})\} \\ -\mathrm{Im}\{K_n(e^{-j\omega_o})\} & \mathrm{Re}\{K_n(e^{-j\omega_o})\} \end{bmatrix} \\ &= k_n \begin{bmatrix} \cos\phi_n & \sin\phi_n \\ -\sin\phi_n & \cos\phi_n \end{bmatrix} \\ k_n &= |K_n(e^{-j\omega_o})| , \quad \phi_n = \mathrm{Arg}[K_n(e^{-j\omega_o})] \end{aligned}$$

and $K_n(q^{-1})$ denotes the nominal (assumed) transfer function of the plant. Similarly as in [7], we will design the one-step-ahead predictor $\widehat{d}(t+1|t) = \widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t+1|t)\mathbf{f}(t+1)$ in such a way that will guarantee automatic compensation of modeling errors. For this reason the nominal gain k_n and nominal phase ϕ_n will be considered nothing more than a convenient starting point for the adaptive control algorithm. Under (16) the output of the system can be approximately written down in the form

$$y(t) \cong c(t) + v(t) \tag{17}$$

where

$$c(t) = [\boldsymbol{\alpha}(t) - \mathbf{B}^{\mathrm{T}}\widehat{\boldsymbol{\alpha}}(t|t-1)]^{\mathrm{T}}\mathbf{f}(t)$$

and

$$\mathbf{B} = \mathbf{K}_n^{-1}\mathbf{K}_o = \begin{bmatrix} \mathrm{Re}\{\beta\} & \mathrm{Im}\{\beta\} \\ -\mathrm{Im}\{\beta\} & \mathrm{Re}\{\beta\} \end{bmatrix}, \ \beta = \frac{K_o(e^{-j\omega_o})}{K_n(e^{-j\omega_o})} .$$

Note that the quantity $|\beta| = k_o/k_n$ is the gain modeling error, and the quantity $\mathrm{Arg}\beta = \phi_o - \phi_n = \Delta\phi$ constitutes the phase error.

The one-step-ahead predictor of $\boldsymbol{\alpha}(t)$ will be computed recursively using

$$\widehat{\boldsymbol{\alpha}}(t+1|t) = \widehat{\boldsymbol{\alpha}}(t|t-1) + \mathbf{M}\mathbf{f}(t)y(t) \tag{18}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathrm{Re}\{\mu\} & -\mathrm{Im}\{\mu\} \\ \mathrm{Im}\{\mu\} & \mathrm{Re}\{\mu\} \end{bmatrix}$$

and μ denotes a complex-valued adaptation gain. For the real-valued adaptation gain ($\mathrm{Im}\{\mu\} = 0$) the second term on the right-hand side of (18) takes the form $\mu\mathbf{f}(t)y(t)$ and resembles the analogous term in (13). Later on we will show that application of a complex-valued gain is crucial as it allows one to compensate phase errors.

[2]For some further comments on this approximation see Remark 2 at the end of this section

Substituting the right-hand side of (17) into (18) one obtains

$$\widehat{\boldsymbol{\alpha}}(t+1|t) = \widehat{\boldsymbol{\alpha}}(t|t-1) + \mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)[\boldsymbol{\alpha}(t) - \mathbf{B}^{\mathrm{T}}\widehat{\boldsymbol{\alpha}}(t|t-1)] + \mathbf{M}\mathbf{f}(t)v(t) \ . \quad (19)$$

Let

$$\Delta\widehat{\boldsymbol{\alpha}}(t) = \boldsymbol{\alpha}(t) - \mathbf{B}^{\mathrm{T}}\widehat{\boldsymbol{\alpha}}(t|t-1) \ .$$

Combining (5) and (19) one arrives at Combining (5) and (19) one arrives at

$$\begin{aligned}\Delta\widehat{\boldsymbol{\alpha}}(t+1) &= (\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t))\Delta\widehat{\boldsymbol{\alpha}}(t) \\ &\quad - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)v(t) + \mathbf{w}(t+1) \qquad (20)\\ &\cong (\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}/2)\Delta\widehat{\boldsymbol{\alpha}}(t) - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)v(t) + \mathbf{w}(t+1)\end{aligned}$$

where, similarly as in Section II, the approximation stems from the averaging theory. This leads to

$$\mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}(t+1)] = (\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}/2)\mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}(t)] \ . \quad (21)$$

Note that the matrix $\mathbf{B}^{\mathrm{T}}\mathbf{M}$ can be expressed in the form

$$\mathbf{B}^{\mathrm{T}}\mathbf{M} = \begin{bmatrix} \mathrm{Re}\{\beta\mu\} & -\mathrm{Im}\{\beta\mu\} \\ \mathrm{Im}\{\beta\mu\} & \mathrm{Re}\{\beta\mu\} \end{bmatrix} .$$

When

$$(1 - \mathrm{Re}\{\beta\mu\}/2)^2 + (\mathrm{Im}\{\beta\mu\}/2)^2 < 1 \quad (22)$$

both eigenvalues of the matrix $\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}/2$ lie inside the unit circle in complex plane, leading to $\mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}(t)] \underset{t\mapsto\infty}{\longmapsto} 0$, which entails $\mathrm{E}_\infty[c(t)] = 0$. This means that when the asymptotic stability condition (22) is fulfilled, the steady-state mean value of the cancellation error is zero even if $\beta \neq 1$, i.e., even if the assumed values of the gain and phase shift differ from the true values.

We will derive expression for the mean-squared cancelling error. Observe that

$$\begin{aligned}\mathrm{E}_\infty[\langle c^2(t)\rangle_\infty] &= \mathrm{E}_\infty[\Delta\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t)\left\langle \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\right\rangle_\infty \Delta\widehat{\boldsymbol{\alpha}}(t)] \\ &= \mathrm{E}_\infty[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2]/2 \ . \qquad (23)\end{aligned}$$

Due to mutual orthogonality of $\Delta\widehat{\boldsymbol{\alpha}}(t)$, $v(t)$ and $\mathbf{w}(t+1)$, after squaring both sides of (20) and taking expectations one obtains

$$\begin{aligned}\mathrm{E}[||\Delta\widehat{\boldsymbol{\alpha}}(t+1)||^2 &= \mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t)(\mathbf{I} - \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{\mathrm{T}}\mathbf{B})\times \\ &\quad\times (\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t))\Delta\widehat{\boldsymbol{\alpha}}(t)] \\ &\quad + \mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{\mathrm{T}}\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)\mathrm{E}[v^2(t)] + \mathrm{E}[||\mathbf{w}(t+1)||^2] \ . \qquad (24)\end{aligned}$$

Since $\mathbf{B}\mathbf{B}^{\mathrm{T}} = |\beta|^2\mathbf{I}$, $\mathbf{M}\mathbf{M}^{\mathrm{T}} = |\mu|^2\mathbf{I}$, $\mathbf{f}^{\mathrm{T}}(t)\mathbf{f}(t) \equiv 1$ and $\mathbf{M}^{\mathrm{T}}\mathbf{B} + \mathbf{B}^{\mathrm{T}}\mathbf{M} = (\beta\mu + \beta^*\mu^*)\mathbf{I} = 2\mathrm{Re}[\beta\mu]\mathbf{I}$, where $*$ denotes complex conjugation, one obtains

$$\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{\mathrm{T}}\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t) = |\beta\mu|^2$$

$$\begin{aligned}\boldsymbol{\Sigma}(t) &= (\mathbf{I} - \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{\mathrm{T}}\mathbf{B})(\mathbf{I} - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)) \\ &= \mathbf{I} - \mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{\mathrm{T}}\mathbf{B} - \mathbf{B}^{\mathrm{T}}\mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t) \\ &\quad + |\beta\mu|^2\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\end{aligned}$$

and, using averaging

$$\begin{aligned}\mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t)\boldsymbol{\Sigma}(t)\Delta\widehat{\boldsymbol{\alpha}}(t)] &\cong \mathrm{E}[\Delta\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t) < \boldsymbol{\Sigma}(t) >_\infty \Delta\widehat{\boldsymbol{\alpha}}(t)] \\ &\cong \{1 - \mathrm{Re}[\beta\mu] + |\beta\mu|^2/2\}\,\mathrm{E}[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2] \ .\end{aligned}$$

This leads to the following steady-state relationship

$$\begin{aligned}\mathrm{E}_\infty[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2] &= \{1 - \mathrm{Re}[\beta\mu] + |\beta\mu|^2/2\}\,\mathrm{E}_\infty[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2] \\ &\quad + |\beta\mu|^2\sigma_v^2 + 2\sigma_w^2 \ .\end{aligned}$$

Finally, solving the above equation with respect to $\mathrm{E}_\infty[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2]$, one arrives at [cf. (23)]

$$\begin{aligned}\mathrm{E}_\infty[\langle c^2(t)\rangle_\infty] &= \mathrm{E}_\infty[||\Delta\widehat{\boldsymbol{\alpha}}(t)||^2]/2 \\ &= \frac{\sigma_w^2 + |\beta\mu|^2\sigma_v^2/2}{\mathrm{Re}[\beta\mu] - |\beta\mu|^2/2} \ . \qquad (25)\end{aligned}$$

Denote by μ_{opt} the gain that minimizes the mean-squared cancellation error. Straightforward calculations lead to

$$\begin{aligned}\mu_{\mathrm{opt}} &= \arg\min_{\mu\in\mathrm{C}} \mathrm{E}_\infty[\langle c^2(t)\rangle_\infty] \\ &= \frac{1}{\beta}\left[-\xi + \sqrt{\xi^2 + 2\xi}\right] \ . \qquad (26)\end{aligned}$$

When the slow variation condition (8) is fulfilled one obtains $\mu_{\mathrm{opt}} \cong \sqrt{2\xi}/\beta = h_\infty/\beta$ and

$$\mathrm{E}_\infty[\langle c^2(t)\rangle_\infty\,|\mu = \mu_{\mathrm{opt}}] \cong \sqrt{2}\,\sigma_v\sigma_w \ . \quad (27)$$

Note that the right-hand side of (27) coincides with the right-hand side of (14). This means that, no matter how large the gain and phase mismatch, one can always choose such value of the adaptation gain μ that will make the disturbance rejection scheme statistically efficient. In next section we will propose a method for automatic adjustment of the adaptation gain μ.

Remark 1

Suppose that, analogously as in the Kalman filter algorithm (13), a scalar, real-valued gain $\mu > 0$ is used in (18) instead of the matrix gain $\mathbf{M}$, i.e. that $\mathbf{M} = \mu\mathbf{I}$. Then, under (8), it holds that

$$\mu'_{\mathrm{opt}} = \arg\min_{\mu\in\mathrm{R}_+} \mathrm{E}_\infty[\langle c^2(t)\rangle_\infty] \cong \sqrt{2\xi}\,/|\beta|$$

and

$$\mathrm{E}_\infty[\langle c^2(t)\rangle_\infty\,|\mu = \mu'_{\mathrm{opt}}] \cong \frac{\sqrt{2}\,\sigma_v\sigma_w}{\cos\Delta\phi} \quad (28)$$

where $\Delta\phi = \mathrm{Arg}\beta$, which means that even if μ is chosen in the optimal way, for large phase errors one may face substantial loses in rejection efficiency. Application of a matrix gain is therefore a *necessary* condition for compensation of phase modeling errors. It allows one to avoid performance degradation.

Remark 2

When deriving the expression (25), describing dependence of the mean-squared cancellation error on μ, we have exploited the steady-state approximation (15), stemming from the fact that linear systems basically scale and shift in phase sinusoidal inputs. Another source of approximation errors

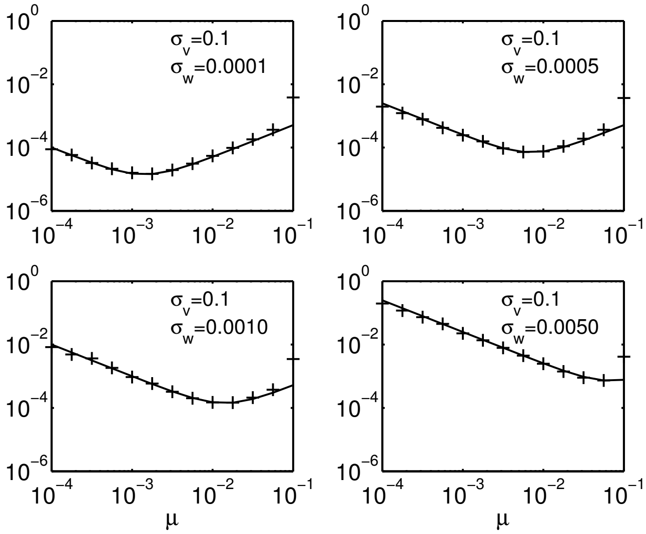

Fig. 2. Comparison of theoretical values of the mean-squared cancellation error, obtained using the steady-state plant approximation (solid line), with the experimental values (crosses).

is due to averaging. A special simulation experiment was arranged to check how well the resulting theoretical formula fits the true error values. The simulated discrete-time plant

$$K_o(z) = \frac{0.0952}{1 - 0.9048z^{-1}} \tag{29}$$

was adopted from [4] and corresponds to a continuous-time plant with transfer function $P(s) = 1/(1 + 0.01s)$ sampled at the rate of 1 kHz. Simulations were carried for $\sigma_v = 0.1$ and for 4 different rates of amplitude variation $\sigma_w \in \{0.0001, 0.0005, 0.001, 0.005\}$, in the absence of modeling errors ($\beta = 1$). For each (σ_w, μ) pair the experiment, covering 20000 time-steps, was repeated 500 times for different realizations of $\{v(t)\}$ and $\{\mathbf{w}(t)\}$. In all cases $\boldsymbol{\alpha}(0)$ was set to $[1, 1]^{\mathrm{T}}$ and $\widehat{\boldsymbol{\alpha}}(0)$ was set to $[0, 0]^{\mathrm{T}}$. The results, summarized in Fig. 2, were obtained by means of combined ensemble and time averaging, after discarding the first 10000 samples (to ensure that the steady-state conditions are reached). Note the good agreement of experimental values with theoretical expectations for the considered (and practically meaningful) range of adaptation gains.

IV. Self-optimizing controller

In this section we will design an adaptive algorithm for on-line tuning of a complex-valued adaptation gain μ. We will adjust μ recursively by minimizing the following local measure of fit, made up of exponentially weighted system outputs

$$V(t;\mu) = \frac{1}{2}\sum_{i=1}^{t} \rho^{t-i} y^2(i;\mu) \ .$$

The forgetting constant ρ ($0 < \rho < 1$) decides upon the effective averaging range. To evaluate the estimate $\widehat{\mu}(t) = \arg\min_{\mu\in\mathbb{C}} V(t;\mu)$ we will use the recursive prediction error (RPE) approach [14]

$$\widehat{\mu}(t) = \widehat{\mu}(t-1) - \left[V''(t;\widehat{\mu}(t-1))\right]^{-1} V'(t;\widehat{\mu}(t-1))$$

where

$$V'(t;\widehat{\mu}(t-1)) \cong \left(\frac{\partial y(t;\widehat{\mu}(t-1))}{\partial \mu}\right)^{*} y(t;\widehat{\mu}(t-1))$$

$$V''(t;\widehat{\mu}(t-1)) \cong \rho V''(t-1;\widehat{\mu}(t-2)) + \left|\frac{\partial y(t;\widehat{\mu}(t-1))}{\partial \mu}\right|^2$$

and symbolic differentiation with respect to a complex variable is defined as [15]

$$\frac{\partial}{\partial \mu} = \frac{1}{2}\left[\frac{\partial}{\partial \mathrm{Re}[\mu]} - j\frac{\partial}{\partial \mathrm{Im}[\mu]}\right] \ .$$

and usually referred to as Wirtinger (or CR) calculus. Using Wirtinger calculus one obtains

$$\begin{aligned}
\frac{\partial y(t)}{\partial \mu} &= -\,\mathbf{f}^{\mathrm{T}}(t)\mathbf{B}^{\mathrm{T}}\,\frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} \\
\frac{\partial \widehat{\boldsymbol{\alpha}}(t+1|t)}{\partial \mu} &= \frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} + \mathbf{M}\mathbf{f}(t)\frac{\partial y(t)}{\partial \mu} \\
&\quad + \frac{\partial \mathbf{M}}{\partial \mu}\,\mathbf{f}(t)y(t) \ .
\end{aligned} \tag{30}$$

Note that

$$\frac{\partial \mathbf{M}}{\partial \mu} = \frac{1}{2}\begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} = \mathbf{H} \ . \tag{31}$$

Since the matrix $\mathbf{B}$ is not known, the obtained recursive formulas can't be used in their present form. To circumvent this problem we will use the coefficient fixing technique introduced in [7], namely we will set $\beta = |\mu|/\mu$, which leads to[3]

$$\mathbf{B}^{\mathrm{T}} = |\mu|\mathbf{M}^{-1} \tag{32}$$

and results in the following modified recursions

$$\begin{aligned}
\frac{\partial y(t)}{\partial \mu} &= -c_{\mu}\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{-1}\,\frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} \\
\frac{\partial \widehat{\boldsymbol{\alpha}}(t+1|t)}{\partial \mu} &= \frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} + \mathbf{M}\mathbf{f}(t)\frac{\partial y(t)}{\partial \mu} \\
&\quad + \mathbf{H}\mathbf{f}(t)y(t) \ .
\end{aligned} \tag{33}$$

Using averaging the second recursion in (33) can be rewritten in the following approximate form

$$\begin{aligned}
\frac{\partial \widehat{\boldsymbol{\alpha}}(t+1|t)}{\partial \mu} &= (\mathbf{I} - c_{\mu}\mathbf{M}\mathbf{f}(t)\mathbf{f}^{\mathrm{T}}(t)\mathbf{M}^{-1})\frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} \\
&\quad + \mathbf{H}\mathbf{f}(t)y(t) \\
&\cong (1 - |\mu|/2)\frac{\partial \widehat{\boldsymbol{\alpha}}(t|t-1)}{\partial \mu} + \mathbf{H}\mathbf{f}(t)y(t)
\end{aligned}$$

Hence, to guarantee stable operation of (33), one must request that $|1 - |\mu|/2| < 1$ which is equivalent to $|\mu| < 4$. Note that the stability condition does not put any constraint on the phase of μ.

[3]According to (26), for the optimal choice of μ the matrix gain $\mathbf{M}\mathbf{B}^{\mathrm{T}} = \mathbf{B}\mathbf{M}^{\mathrm{T}}$, determining properties of the closed-loop system, reduces to $h_{\infty}\mathbf{I}$. Note that (32), which entails $\mathbf{M}\mathbf{B}^{\mathrm{T}} = |\mu|\mathbf{I}$, preserves structure of the optimal solution.

Let $r(t) = V''(t;\widehat{\mu}(t-1))$, $z_y(t) = \partial y(t;\widehat{\mu}(t-1))/\partial\mu$ and $\mathbf{z}_\alpha(t) = \partial\widehat{\boldsymbol{\alpha}}(t+1|t;\widehat{\mu}(t-1))/\partial\mu$. The proposed disturbance rejection algorithm with automatic gain tuning can be summarized as follows

$$\begin{aligned}
z_y(t) &= -|\widehat{\mu}(t-1)|\mathbf{f}^{\mathrm{T}}(t)\hat{\mathbf{M}}^{-1}(t-1)\mathbf{z}_\alpha(t-1)\\
\mathbf{z}_\alpha(t) &= \mathbf{z}_\alpha(t-1) + \hat{\mathbf{M}}(t-1)\mathbf{f}(t)z_y(t) + \mathbf{H}\mathbf{f}(t)y(t)\\
r(t) &= \rho r(t-1) + |z_y(t)|^2\\
\widehat{\mu}(t) &= \widehat{\mu}(t-1) - \frac{z_y^*(t)y(t)}{r(t)}\\
\hat{\mathbf{M}}(t) &= \begin{bmatrix} \mathrm{Re}\{\widehat{\mu}(t)\} & -\mathrm{Im}\{\widehat{\mu}(t)\} \\ \mathrm{Im}\{\widehat{\mu}(t)\} & \mathrm{Re}\{\widehat{\mu}(t)\} \end{bmatrix}\\
\widehat{\boldsymbol{\alpha}}(t+1|t) &= \widehat{\boldsymbol{\alpha}}(t|t-1) + \hat{\mathbf{M}}(t)\mathbf{f}(t)y(t)\\
u(t) &= -\,\widehat{\boldsymbol{\alpha}}(t+1|t)\mathbf{K}_n^{-1}\mathbf{f}(t+1)
\end{aligned} \tag{34}$$

Remark

The control rule (16) was based on an implicit assumption that $\mathbf{K}_n = \mathbf{K}_o$, i.e. that $\beta = 1$. A similar technique, called coefficient fixing, is often used to "robustify" self-tuning minimum-variance regulators [16], [17]. In both cases, under certain conditions, the modeling biases are automatically compensated when estimation is carried in a closed loop. Substitution $\beta = |\mu|/\mu$ can be considered a modified version of the coefficient fixing technique. Even though the assumed value of β usually differes from the true one, the values of $\widehat{\mu}(t)$, computed using the algorithm (34), converge in the mean to the optimal value μ_{opt}. One can show that, similarly as in the case of complex-valued systems, the substitution $\beta = 1$ (which might look as a more "natural" choice) does not allow one to compensate phase modeling errors greater than $\pi/2$ – see [7] for more details.

V. Safeguards

Following [7] we will propose several modifications increasing robustness of the proposed disturbance rejection scheme.

First, to avoid erratic behavior of the algorithm during startup/transient periods, it is advisable to set the maximum allowable values for $|\widehat{\mu}(t)|$, $|\widehat{\mu}(t)-\widehat{\mu}(t-1)|$ and $r(t)$, further denoted by $\mu_{\max}$, $\Delta\mu_{\max}$ and $r_{\max}$, respectively. These are typical "safety valves" used in adaptive control.

Second, to improve numerical stability of the recursive algorithm used for computation of sensitivity derivatives $z_y(t) = \partial y(t)/\partial\mu$ and $z_\alpha(t) = \partial\widehat{\boldsymbol{\alpha}}(t+1|t)/\partial\mu$, one can replace the term $|\mu|$ on the right-hand side of (33) with c_μ, where c_μ is a small positive constant. Note that this is equivalent to setting $\beta\mu = c_\mu$, or equivalently $\mathbf{B}^{\mathrm{T}} = c_\mu\mathbf{M}^{-1}$. Due to normalization introduced by the RPE approach this modification has usually minor influence on the steady-state values of $\widehat{\mu}(t)$, but it may be quite helpful during all transient periods. Additionally, and quite importantly, tracking properties of the algorithm modified in the way described above *do not depend* on the modeling error β – for more details see [8].

Denote by $\mathrm{sat}(x, a)$, $x \in \mathrm{C}$, $a \in \mathrm{R}_+$, a complex-valued saturation function

$$\mathrm{sat}(x,a) = \begin{cases} x, & \text{if } \ |x| \le a \\ a\,\dfrac{x}{|x|}, & \text{if } \ |x| > a \end{cases} .$$

Then the modified disturbance rejection algorithm that combines all "fixes" described above can be summarized as follows

$$\begin{aligned}
z_y(t) &= -c_\mu\mathbf{f}^{\mathrm{T}}(t)\hat{\mathbf{M}}^{-1}(t-1)\mathbf{z}_\alpha(t-1)\\
\mathbf{z}_\alpha(t) &= \mathbf{z}_\alpha(t-1) + \hat{\mathbf{M}}(t-1)\mathbf{f}(t)z_y(t) + \mathbf{H}\mathbf{f}(t)y(t)\\
\rho(t) &= 1 - c_\rho|\widehat{\mu}(t-1)|\\
\widetilde{r}(t) &= \rho(t)r(t-1) + |z_y(t)|^2\\
r(t) &= \min(\widetilde{r}(t), r_{\max})\\
\Delta\mu(t) &= \mathrm{sat}\left(z_y^*(t)y(t)/r(t), \Delta\mu_{\max}\right)\\
\widetilde{\mu}(t) &= \widehat{\mu}(t-1) - \Delta\mu(t)\\
\widehat{\mu}(t) &= \mathrm{sat}(\widetilde{\mu}(t), \mu_{\max})\\
\hat{\mathbf{M}}(t) &= \begin{bmatrix} \mathrm{Re}\{\widehat{\mu}(t)\} & -\mathrm{Im}\{\widehat{\mu}(t)\} \\ \mathrm{Im}\{\widehat{\mu}(t)\} & \mathrm{Re}\{\widehat{\mu}(t)\} \end{bmatrix}\\
\widehat{\boldsymbol{\alpha}}(t+1|t) &= \widehat{\boldsymbol{\alpha}}(t|t-1) + \hat{\mathbf{M}}(t)\mathbf{f}(t)y(t)\\
u(t) &= -\,\widehat{\boldsymbol{\alpha}}^{\mathrm{T}}(t+1|t)\mathbf{K}_n^{-1}\mathbf{f}(t+1)
\end{aligned} \tag{35}$$

VI. Simulation results

A. Steady-state performance

The purpose of this simulation experiment was to examine the steady-state error compensation capabilities of the algorithm (34). None of the proposed safety jacketing measures was applied. The only user-dependent tuning "knob" ρ was set to 0.9999. Simulations were carried for the Guo & Bodson's plant (29) with the following measurement noise and sinusoidal disturbance settings: $\sigma_v = 0.1$, $\sigma_w = 0.001/\sqrt{2}$, $\omega_0 = 0.1$, $\alpha(0) = [0.5, 0.5]^T$. In the absence of modeling errors the optimal value of μ is under such conditions equal to $\mu_{\mathrm{opt}} = h_\infty = 0.01$.

Table I shows the mean-squared output errors observed for different values of β (12 selections, characterized in terms of magnitude and phase errors). All numbers were obtained by means of combined ensemble (100 realizations of $\{v(t)\}$ and $\{\mathbf{w}(t)\}$) and time ($t \in [10001, 40000]$) averaging, after the algorithm has reached its steady-state behavior.

Argβ[$^\circ$]	$\lvert\beta\rvert = 0.25$	$\lvert\beta\rvert = 1$	$\lvert\beta\rvert = 4$
0	1.0105	1.0106	1.0109
60	1.0105	1.0106	1.0109
120	1.0105	1.0106	1.0109
180	1.0105	1.0106	1.0109

TABLE I

Steady-state mean-squared output-error $\mathrm{E}_\infty[y^2(t)]\cdot 10^{-2}$ measured for different magnitude and phase modeling errors. The theoretical lower error bound is in this case equal to $1.01{\cdot}10^{-2}$

Note that the proposed control scheme is insensitive to phase errors and almost insensitive to magnitude errors. In all cases considered the mean-squared output errors are by less than 0.1% larger than the minimum error achievable when μ is set to its optimal value $\mu_{\rm opt} = h_\infty/\beta$ and not estimated. For the mean-squared cancellation errors the analogous degradation does not exceed 6%. This means that the proposed disturbance rejection scheme is doing a remarkably good job in compensating modeling errors and optimizing the closed-loop system performance.

B. Transient performance

The objective of this experiment was to demonstrate the ability of the proposed algorithm to cope with sudden plant changes. The Guo & Bodson's (29) plant was switched at the instant $t = 15000$ to second-order nonminimum phase plant with a pair of complex poles at $\omega = \omega_0$, described by

$$K_1(z) = \frac{0.1 - 0.14z^{-1}}{1 - 1.8507z^{-1} + 0.8649z^{-2}} \ .$$

Fig. 3 shows averaged results of 100 simulation runs, obtained for the algorithm (35) with the following settings: $c_\mu = 0.005$, $\rho = 0.999$, $\mu_{max} = 0.05$, $\Delta\mu_{max} = \widehat{\mu}(t-1)/50$, $r_{max} = 400$. The nominal plant gain was fixed at $k_n = 1$. The corresponding modeling errors were equal to: $|\beta| = 0.708$, $\mathrm{Arg}\beta = -42.2^\circ$ for $t < 15000$ and $|\beta| = 2.91$, $\mathrm{Arg}\beta = -84^\circ$ for $t \geq 15000$. The adaptation was started with zero initial conditions, except for $\mu(0) = 0.02$, $r(0) = 100$.

As expected, the algorithm converges in the mean to the optimal steady-state settings. Additionally, it deals favorably both with the initial convergence problem and with abrupt plant change. When the experiment was started or when a change to the plant dynamics occurred, the magnitude of the adaptation gain $\widehat{\mu}(t)$ temporarily increased to quickly compensate large initial modeling errors; later on it gradually decayed to settle down around its optimal steady-state value. Note very quick response to phase errors and (usually) much slower response to magnitude errors – the effect caused by diverse sensitivity of system output to two types of modeling errors.

VII. Conclusion

The problem of elimination of a sinusoidal disturbance of known frequency, acting at the output of an unknown linear stable plant was considered. It was not assumed that a reference signal, correlated with disturbance, is available. The proposed solution is based on coefficient fixing – the technique originally developed for the purpose of adaptive minimum-variance control – combined with automatic adjustment of the adaptation gain. Computer simulations confirm that when the amplitude of the disturbance evolves according to the random-walk model, the resulting regulator converges locally in the mean to the optimal (minimum-variance) regulator.

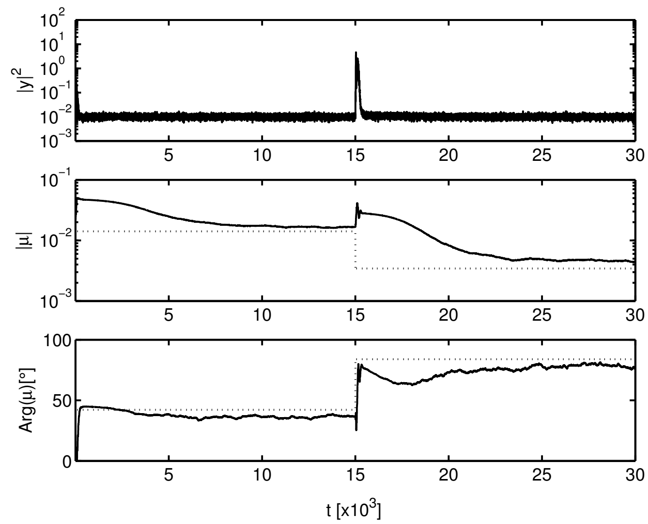

Fig. 3. Mean transient behavior of the disturbance rejection algorithm (average results of 100 simulation runs). Solid lines – ensemble averages of the estimated values, dotted lines – optimal steady-state values.

References

[1] Bodson, M. and S.C. Douglas (1997). Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency. *Automatica*, vol. 33, 2213–2221.
[2] Bodson, M. (2001). Performance of an adaptive algorithm of sinusoidal disturbance rejection in high noise. *Automatica*, vol. 37, 1133–1140.
[3] Wu, B. and M. Bodson (2004). Multi-channel active noise control for periodic sources – indirect approach. *Automatica*, vol. 40, 203–212.
[4] Guo, X. and M. Bodson (2007). Adaptive rejection of multiple sinusoids of unknown frequency. *Proc. European Control Conference*, Kos, Greece, 121–128.
[5] Landau I.D., A. Constantinescu & D. Rey (2005). Adaptive narrow band disturbance rejection applied to an active suspension – an internal model principle approach. *Automatica*, vol. 41, 563–574.
[6] Landau I.D. & D.I. Patrascu (2007). Rejection of unknown multiple narrow band disturbances – a direct adaptive control approach. *Proc. European Control Conference*, Kos, Greece, 135–141.
[7] Niedźwiecki, M. and M. Meller (2008). A study in vibration control: the unknown plant – known frequency case. Gdańsk University of Technology, Technical Report # ETI-1/2008, http://www.eti.pg.gda.pl/katedry/ksa/pracownicy/Maciej.Niedzwiecki.
[8] Niedźwiecki, M. and M. Meller (2008). Tracking analysis of an adaptive vibration controller. *Proc. 47th CDC*, Cancun, Mexico.
[9] Bai, E.-W., L.-C. Fu and S. Sastry (1988). Averaging analysis for discrete time and sampled data adaptive systems. *IEEE Trans. Circuits and Systems*, vol. 35, 137–148.
[10] Benveniste, A. and G. Ruget (1982). A measure of the tracking capability of recursive stochastic algorithms with constant gains. *IEEE Trans. Automat. Contr.*, vol. 25, 788–794.
[11] Lewis, F.L. (1986). *Optimal Estimation*. Wiley. New York.
[12] van Trees, H.L. and K.L. Bell, Eds. (2007). *Bayesian Bounds for Parameter Estimation and Nonlinear Filtering/Tracking*. Wiley, New York.
[13] Niedźwiecki, M. (2000). *Identification of Time-varying Processes*. Wiley. New York.
[14] Söderström, T. and P. Stoica (1988). *System Identification*. Prentice Hall. Englewood Cliffs NJ.
[15] Haykin, S. (1996). *Adaptive Filter Theory*. Prentice Hall, Englewood Cliffs NJ.
[16] Åström, K.J. *et al.* (1977). Theory and applications of self-tuning regulators. *Automatica*, vol. 13, 457–476.
[17] Niedźwiecki, M. (1989). Steady-state and parameter tracking properties of self-tuning minimum variance regulators. *Automatica*, vol. 25, 597–602.