Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Robust H-Infinity Control for Uncertain Time-Delay TCP/AQM Network System

Yuanwei Jing, Hongwei Wang, Georgi M. Dimirovski, *Senior Member, IEEE* and Xiaoping Liu

***Abstract*—A robust active queue management (AQM) scheme based on H-infinty theory is presented for the problem of congestion control in TCP communication networks. In TCP/IP networks, the packet-dropping probability function is considered as a control input, and network parameters are time-varying. Thus a TCP/AQM controller is modeled as a uncertain time-delay system with a saturated input. The corresponding existence condition of the observer and controller are obtained by applying Lyapunov-Krasovskii function approach and the linear matrix inequality technique. Simulation results show that the proposed scheme can track the desired queue length very quickly under various network conditions, and avoid the problem of dynamic network congestion.**

I. Introduction

In the past few years, communication networks have become an essential part of many applications in engineering. Congestion control is a major problem because the quality of service cannot be guaranteed. The control mechanism often used to prevent the congestion phenomenon is the transmission control protocol (TCP). But as amount of the traffic over the Internet increases and the demand for quality of service (QoS) become stronger, it is no longer possible to exclusively rely on end hosts to perform end-to-end congestion control. There has been a growing recognition within the Internet community that the network itself must participate in congestion control. Active Queue Management (AQM) schemes have been proposed to complement the TCP network congestion control. AQM is router-based control mechanism which aims to reduce packet drops and improve network utilization. So the combination of TCP and AQM is the main approach to solve the problems of current Internet congestion control.

The random early detection (RED)[1], the most famous AQM algorithm, has obtained great success in network congestion control. It can eliminate the flow synchronization problem and attenuates the traffic load by monitoring the average queue length. However, it is difficult to tune RED parameters under different network environments, and the RED parameters are sensitive to the network load. In order to solve these problems existed in RED algorithm, some modified RED schemes, such as ARED[2], BLUE[3] and Random Exponential Marking (REM)[4] have been proposed in the literature. These AQM algorithms, however, show the weakness to detect and control congestion under large-delay network situations such as intercontinental TCP connection with hundreds of milliseconds' round trip time.

Recently, some AQM algorithms have been proposed based on control-theoretic analysis and design. In [5], the theory of stochastic equations is applied to develop a fluid-based model of the dynamics of the TCP. Several congestion control schemes based on this TCP model have been proposed to improve the performance of communication networks. For example, a PI controller is developed for a linearized system[6]. Compared with RED, the PI controller is more stable. However, the PI controller is sluggish with taking too long time to settle down to the referenced queue length. In [7], the differential component in the controller structure is introduced to avoid the overshoot and improve the damping and rise time of the controller in order to overcome the drawbacks of PI controller. Proportional-Integral-Differential (PID) controller can improve the dynamic response, reduce or eliminate the steady-state error. However, its performance gets bad based on the simplified TCP/AQM linearized model in large-delay networks. The further investigation shows that the previous simplified model ignores the delay term in some cases. So a robust sliding mode control scheme is introduced in [8] that possesses good performance and robustness for linear TCP/AQM model with the time delay and the uncertainties of the network parameters. In these papers they analyze the stability without considering the problem of input saturation. We know that the packet-dropping probability function is considered as a control input in TCP network, so the effect of a saturated actuator must be taken into account when designing a control scheme. Also communication networks are large-scale complex systems, it is impossible to measure the all state locally. So observer-based controllers are designed in [9], [10], but uncertainties and disturbance are not considered.

In this paper, we design a robust H_∞ controller for uncertain time-delay network system with input saturation, at the same time, mismatched uncertainties are considered in TCP mathematic model. For the practical TCP network,

Manuscript received March 10, 2008. This work is supported by the National Natural Science Foundation of China, under grant 60274009, and Specialized Research Fund for the Doctoral Program of Higher Education, under grant 20020145007, and also by Dogus University Fund for Science and Ministry of Education and Science of the Republic of Macedonia.

Hongwei Wang, Yuanwei Jing and Xiaoping Liu are with Faculty of Information Science and Engineering, Northeastern University, 110004, Shenyang, Liaoning, P.R. of China. (email: wanghw0819@163.com).

Georgi. M. Dimirovski is with Dogus University, Faculty of Engineering, Istanbul, TR-34722, R. of Turkey; and SS Cyril and Methodius University, Faculty of FEIT, Skopje, MK-1000, R. of Macedonia (email: gdimirovski@dogus.edu.tr).

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

constructing a Lyapunov-Krasovskii function, an observed-based controller is developed for AQM to support the TCP on the basis of the LMI technique. We show that the proposed control strategy has reliable asymptotic stability and robust against variations in the RTT, the number of TCP sessions, the bursting and the unresponsive flows, etc.

The remainders of this paper are organized as follows. The TCP model and the control objective are discussed in Section II. Section III presents the observer-based H_∞ controller for AQM, considering the effect of time-delay and uncertainties. Simulation results of the proposed scheme for various network conditions are shown in section IV. Finally, we conclude our work in section V.

II. TCP NETWORK DYNAMIC MODEL

In [7], a nonlinear dynamic model of the TCP behavior is developed using fluid-flow and stochastic differential equation analysis, which is as follows.

$$\dot{W}(t)=\frac{1}{R(t)}-\frac{W(t)}{2}\frac{W(t-R(t))}{R(t-R(t))}p(t-R(t)) \tag{1}$$

$$\dot{q}(t)=\begin{cases}-C+\dfrac{N(t)}{R(t)}W(t) & q(t)>0\\ \max\left\{0,-C+\dfrac{N(t)}{R(t)}W(t)\right\} & q(t)=0\end{cases} \tag{2}$$

where W is the average TCP window size (in packets), q is the queue length at a router (in packets), T_p is the propagation delay (in seconds), R is the transmission RTT, equal to $q/C+T_p$, C is the link capacity (in packets/s), N is the number of TCP sessions and p is the packet-dropping probability function $(0\le p\le 1)$. All variables are assumed non-negative.

Assume $R(t)=R_0$, $N(t)=N$, $C(t)=C$ be the nominal values of $R(t)$, $N(t)$ and $C(t)$. Let $\dot{W}(t)=0$ and $\dot{q}(t)=0$. The equilibrium point (W_0,q_0,p_0) could be obtained, which satisfies $W_0^2p_0=2$ and $W_0=R_0C/N$. Furthermore, Eq.(1) is linearized at the operating point such that the nonlinear model could be expressed in the form of the following linear time-delay model [2].

$$\begin{cases}\delta\dot{W}(t)=-\dfrac{N}{R_0^2C}\left(\delta W(t)+\delta W(t-R_0)\right)\\ \quad-\dfrac{1}{R_0^2C}\left(\delta q(t)-\delta q(t-R_0)\right)-\dfrac{R_0C^2}{2N^2}\delta p(t-R_0)\\ \delta\dot{q}(t)=\dfrac{N}{R_0}\delta W(t)-\dfrac{1}{R_0}\delta q(t)\end{cases} \tag{3a}$$

Output equation is as follows.

$$z(t)=\delta q(t)=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\delta W(t)\\ \delta q(t)\end{bmatrix} \tag{3b}$$

where $\delta W\doteq W-W_0$, $\delta q\doteq q-q_0$, $\delta p\doteq p-p_0$.

Let $x_1=\delta W(t)$, $x_2=\delta q(t)$, $x(t)=\begin{bmatrix}x_1 & x_2\end{bmatrix}^{\mathrm{T}}$, $u(t)=\delta p(t)$ $(-p_0\le u(t)\le 1-p_0)$, $\tau=R_0$. The plant (3) can be described as

$$\begin{cases}\dot{x}(t)=Ax(t)+A_{\mathrm{d}}x(t-\tau)+Bu(t-\tau)\\ z(t)=Cx(t)\end{cases} \tag{4}$$

where $x(t)\in\mathrm{R}^2$, $u(t)\in\mathrm{R}^1$ and $y(t)\in\mathrm{R}^1$ represent the state, the control input and the system output, respectively. A, A_{d}, B and C are constant matrices of appropriate dimensions expressed in the following forms:

$$A=\begin{bmatrix}-\dfrac{N}{R_0^2C} & -\dfrac{1}{R_0^2C}\\ \dfrac{N}{R_0} & -\dfrac{1}{R_0}\end{bmatrix},\ A_{\mathrm{d}}=\begin{bmatrix}-\dfrac{N}{R_0^2C} & \dfrac{1}{R_0^2C}\\ 0 & 0\end{bmatrix},\ B=\begin{bmatrix}-\dfrac{R_0C^2}{2N^2}\\ 0\end{bmatrix},$$

$C=\begin{bmatrix}0 & 1\end{bmatrix}$.

In practical network systems, all parameters will vary for external condition. So the controller design should take into account the time delay and uncertain nature of the linearized TCP fluid model of (4). Also, the control input $u(t)$ is a saturated function. The following uncertain time-delay system with input saturation can be derived

$$\begin{cases}\dot{x}(t)=(A+\Delta A)x(t)+(A_{\mathrm{d}}+\Delta A_{\mathrm{d}})x(t-\tau)\\ \qquad+(B+\Delta B)sat(u(t-\tau))+G\omega(t)\\ z(t)=Cx(t)\end{cases} \tag{5}$$

where ΔA, ΔA_{d} and ΔB are uncertainties depending on network parameters, $\omega(t)$ is external disturbance.

In the process of designing controller, the following assumptions are taken.

Assumption 1: the pairs (A,B) and (A,C) are controllable and observable, respectively.

Assumption 2 : The matrices $\Delta A(t)$, $\Delta A_{\mathrm{d}}(t)$ and ΔB satisfy $\Delta A=D_1F_1(t)E_1$, $\Delta A_{\mathrm{d}}=D_2F_2(t)E_2$, $\Delta B=D_3F_3(t)E_3$, where D_i and E_i are constant matrices of appropriate dimensions, $F_i(t)$ satisfies $\|F_i(t)\|\le I$, $i=1,2,3$.

The saturated input is expressed by the following nonlinearity.

$$sat(u(t-\tau))=\begin{cases}u_{\max} & u(t-\tau)\ge u_{\max}\\ u(t-\tau) & u_{\min}\le u(t-\tau)\le u_{\max}\\ u_{\min} & u(t-\tau)<u_{\min}\end{cases} \tag{6}$$

where $u_{\min}=-p_0$ and $u_{\max}=1-p_0$, which implies

$u_{\min} < 0 < u_{\max}$. From Eq.(6), the saturation term in Eq.(5) can be rewritten as

$$\mathrm{sat}\left(u(t-\tau)\right) = \beta\left(u(t-\tau)\right)u(t-\tau) \tag{7}$$

where

$$\beta\left(u(t-\tau)\right) = \begin{cases} u_{\max}/u(t-\tau) & \text{if } u(t-\tau) \ge u_{\max} \\ 1 & \text{if } u_{\min} \le u(t-\tau) < u_{\max} \\ u_{\min}/u(t-\tau) & \text{if } u(t-\tau) < u_{\min} \end{cases} \tag{8}$$

and

$$0 \le \beta\left(u(t-\tau)\right) \le 1, \text{ for all } t \ge 0 \tag{9}$$

Therefore, based on Eqs.(6)~(9), the system in Eq.(5) can be rewritten in an equivalent form as

$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + (A_{\mathrm{d}} + \Delta A_{\mathrm{d}})x(t-\tau) \\ \qquad + (B+\Delta B)\beta\left(u(t-\tau)\right)\left(u(t-\tau)\right) + G\omega(t) \\ z(t) = Cx(t) \end{cases} \tag{10}$$

The AQM scheme will be proposed in the above-mentioned model. The designed robust H_∞ controller could not only reduce the sensitivity to network parameters but also eliminate inaccuracies due to the use of the linear model with a saturated input.

III. Design Of H_∞ Controller For AQM

Since communication networks are large-scale complex systems, it is impossible to measure the size of the state variable window locally. A more practical approach is to design an observer-based H_∞ controller, which is capable of achieving asymptotic stability of robust performance.

For the system (10), state observer and memoryless feedback controller are given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\beta\left(u(t-\tau)\right)u(t-\tau) + L\left(y(t) - C\hat{x}(t)\right) \tag{11}$$

$$u(t) = -K\hat{x}(t) \tag{12}$$

where L is the gain matrix of the observer, K is the feedback gain matrix.

Define the state error

$$e(t) = x(t) - \hat{x}(t) \tag{13}$$

Then from (10)~(13), we can obtain the following augmented system

$$\begin{aligned} \dot{x}(t) = {} & (A+\Delta A)x(t) + (A_d + \Delta A_d - B\beta K - \Delta B\beta K) \\ & x(t-\tau) + (B\beta K + \Delta B\beta K)e(t-\tau) + G\omega(t) \end{aligned} \tag{14a}$$

$$\begin{aligned} \dot{e}(t) = {} & \Delta A x(t) + (A - LC)e(t) + (A_d + \Delta A_d - \Delta B\beta K) \\ & x(t-\tau) + \Delta B\beta K e(t-\tau) + G\omega(t) \end{aligned} \tag{14b}$$

The objective of this section is given as follows: (1) If disturbance input $\omega(t) = 0$, the closed loop system is asymptotically stable; (2) For any disturbance input $\omega(t) \in L_2[0,\infty]$, the output $z(t)$ satisfies H_∞ performance condition $\|z(t)\|_2^2 \le \gamma^2 \|\omega(t)\|_2^2$, where γ is a known positive constant.

The following lemma is useful in designing an expected robust observer-based H_∞ controller for the uncertain linear time-delay system (10).

Lemma 1 For any $z, y \in \mathrm{R}^{n\times n}$,

$$\pm 2z^{\mathrm{T}}y \le z^{\mathrm{T}}z + y^{\mathrm{T}}y.$$

Lemma 2 For any $x, y \in \mathrm{R}^{n\times n}$ and real matrix $F(t)$ with appropriate dimensions, where $\|F(t)\| \le 1$,

$$\pm 2x^{\mathrm{T}}Fy \le x^{\mathrm{T}}x + y^{\mathrm{T}}y.$$

Next, we choose the Lyapunov-Krasovskii function, using the linear matrix inequality technology to guarantee the stability of the observer and to reduce the effect of model uncertainties on the estimated state. The following theorem offers the theoretical basis for achieving the desired design goal.

Theorem: Consider the augmented system (14) along with the observer (11) and controller (12). If there exist symmetric positive definite matrices P and Q, which satisfy the following matrix inequality

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + 2E_1^{\mathrm{T}}E_1 + 2I + 2E_2^{\mathrm{T}}E_2 + C^{\mathrm{T}}C & P & K^{\mathrm{T}} \\ P & -S_1^{-1} & 0 \\ K & 0 & -S_2^{-1} \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} A^{\mathrm{T}}Q + QA & Q & K^{\mathrm{T}} \\ Q & -S_3^{-1} & 0 \\ K & 0 & -S_4^{-1} \end{bmatrix} < 0 \tag{16}$$

then the system (10) is asymptotically stable and $z(t)$ satisfy H_∞ performance condition. The observer gain is $L = Q^{-1}C^{\mathrm{T}}$, where

$$S_1 = D_1D_1^{\mathrm{T}} + D_2D_2^{\mathrm{T}} + D_3D_3^{\mathrm{T}} + A_dA_d^{\mathrm{T}} + 2BB^{\mathrm{T}} + 2\gamma^{-2}GG^{\mathrm{T}}$$

$$S_2 = S_4 = 2E_3^{\mathrm{T}}E_3 + I,$$

$$S_3 = D_1D_1^{\mathrm{T}} + D_2D_2^{\mathrm{T}} + 2D_3D_3^{\mathrm{T}} + A_{\mathrm{d}}A_{\mathrm{d}}^{\mathrm{T}} + 2\gamma^{-2}GG^{\mathrm{T}}.$$

Proof: If the disturbance input $\omega(t) = 0$, choose the following Lyapunov-Krasovskii function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{17}$$

where

$$V_1(t) = x^{\mathrm{T}}(t)Px(t)$$

$$V_2(t) = e^{\mathrm{T}}(t)Qe(t)$$

$$V_3(t) = \int_{t-\tau}^{t} x^{\mathrm{T}}(s)H_1x(s)\,\mathrm{d}s + \int_{t-\tau}^{t} e^{\mathrm{T}}(s)H_2e(s)\,\mathrm{d}s$$

P and Q are symmetry positive matrices, H_1 and H_2 satisfy the following form

$$H_1 = 2I + 2E_2^{\mathrm{T}} E_2 + 2K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K + K^{\mathrm{T}} K,$$
$$H_2 = 2K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K + K^{\mathrm{T}} K.$$

Taking the time derivative of V_1, we have

$$\begin{aligned}\dot{V}_1 &= \dot{x}^{\mathrm{T}}(t) P x(t) + x^{\mathrm{T}}(t) P \dot{x}(t) \\ &= x^{\mathrm{T}}(t)\left(A^{\mathrm{T}}P + PA\right)x(t) + 2x^{\mathrm{T}}(t) P\Delta A x(t) + \\ &\quad 2x^{\mathrm{T}}(t) P\left(A_d + \Delta A_d - B\beta K - \Delta B\beta K\right)x(t-\tau) + \\ &\quad 2x^{\mathrm{T}}(t) P\left(B\beta K + \Delta B\beta k\right)e(t-\tau)\end{aligned} \tag{18}$$

Using the lemmas, Assumption 2 and Eq.(9), we have the following inequality

$$\begin{aligned}\dot{V}_1 \le\; & x^{\mathrm{T}}(t)\left(A^{\mathrm{T}}P + PA + PD_1 D_1^{\mathrm{T}} P + PD_2 D_2^{\mathrm{T}} P + 2PD_3 D_3^{\mathrm{T}} P\right. \\ & \left. + PA_d^{\mathrm{T}} A_d P + 2PBB^{\mathrm{T}} P + E_1^{\mathrm{T}} E_1\right)x(t) + x^{\mathrm{T}}(t\text{-}\tau)\left[I + K^{\mathrm{T}} K\right. \\ & \left. + E_2^{\mathrm{T}} E_2 + K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K\right]x(t-\tau) + e^{\mathrm{T}}(t-\tau)\left[K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K + \right. \\ & \left. K^{\mathrm{T}} K\right]e(t-\tau)\end{aligned} \tag{19}$$

Using the same method, it can be shown that

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\ &\le x^{\mathrm{T}}(t) H_3 x(t) + e^{\mathrm{T}}(t) H_4 e(t)\end{aligned} \tag{20}$$

where

$$\begin{aligned}H_3 = &A^{\mathrm{T}}P + PA + PD_1 D_1^{\mathrm{T}} P + PD_2 D_2^{\mathrm{T}} P + PD_3 D_3^{\mathrm{T}} P + \\ &PA_d A_d^{\mathrm{T}} P + 2PBB^{\mathrm{T}} P + 2E_1^{\mathrm{T}} E_1 + 2I + 2E_2^{\mathrm{T}} E_2 + \\ &2K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K + K^{\mathrm{T}} K\end{aligned}$$

$$\begin{aligned}H_4 = &A^{\mathrm{T}}Q + QA + QD_1 D_1^{\mathrm{T}} Q + QD_2 D_2^{\mathrm{T}} Q + 2QD_3 D_3^{\mathrm{T}} Q + \\ &QA_d A_d^{\mathrm{T}} Q + 2K^{\mathrm{T}} E_3^{\mathrm{T}} E_3 K + K^{\mathrm{T}} K\end{aligned}$$

We know that $H_3 < 0$, $H_4 < 0$, thus $\dot{V} < 0$. The closed-loop system is asymptotically stable.

In the following, we will prove $z(t)$ satisfying H_∞ performance condition.

$$J = \int_0^\infty \left[z^{\mathrm{T}}(t) z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t)\right]\mathrm{d}t \tag{21}$$

For any disturbance $\omega(t)$, we have

$$\begin{aligned}J &= \int_0^\infty \left[z^{\mathrm{T}}(t) z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t) + \dot{V}\right]\mathrm{d}t - V(\infty) + V(0) \\ &\le \int_0^\infty \left[z^{\mathrm{T}}(t) z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t) + \dot{V}\right]\mathrm{d}t \\ &\le \int_0^\infty \left[x^{\mathrm{T}}(t) C^{\mathrm{T}} C x(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t) + x^{\mathrm{T}}(t) H_3 x(t) + \right. \\ &\qquad \left. e^{\mathrm{T}}(t) H_4 e(t) 2x^{\mathrm{T}}(t) PG\omega(t) + 2e^{\mathrm{T}}(t) QG\omega(t)\right]\mathrm{d}t\end{aligned}$$

where

$$\begin{aligned}&2x^{\mathrm{T}}(t) PG\omega(t) \\ &\le 2\gamma^{-2} x^{\mathrm{T}}(t) PGG^{\mathrm{T}} P x(t) + 0.5\gamma^2 \omega^{\mathrm{T}}(t)\omega(t) \\ &2e^{\mathrm{T}}(t) QG\omega(t) \\ &\le 2\gamma^{-2} e^{\mathrm{T}}(t) QGG^{\mathrm{T}} Q e(t) + 0.5\gamma^2 \omega^{\mathrm{T}}(t)\omega(t)\end{aligned}$$

Then we have

$$J \le x^{\mathrm{T}}(t) H_5 x(t) + e^{\mathrm{T}}(t) H_6 e(t)$$

where

$$H_5 = H_3 + C^{\mathrm{T}} C + 2\gamma^{-2} PGG^{\mathrm{T}} P,$$
$$H_6 = H_4 + 2\gamma^{-2} QGG^{\mathrm{T}} Q.$$

By matrix inequality (15) and (16), we get $H_5 < 0$, $H_6 < 0$, which imply $J < 0$. The output $z(t)$ satisfies H_∞ performance condition. The proof is completed.

Remark: Theorem 1 illustrates that even existing network uncertainties and time delay, the queue length can track the desired queue length if we can design suitable controller.

IV. Simulation Results

In this section, we validate the effectiveness and performance of the scheme of this paper by MATLAB/ SIMULINK. We will give simulation results for the proposed observer-base H_∞ controller under the variations of network parameters. At the same time, we will draw comparisons between traditional observer-based controller (OBC)[10] and the observer-based H_∞ controller about performance under disturbance.

The choosing of the parameters is based on [10]. The number of the active TCP sessions $N = 100$, the link capacity $C = 1250$ packets/s, the equilibrium point of round-trip time $R_0 = 0.2s$, the desire queue size is $q_{\mathrm{d}} = 150$ packets, the desired window size is $W_0 = 2.5$ packets, $P_0 = 2/2.5^2 = 0.32$. Therefore, $u_{\min} = -0.32$, $u_{\max} = 0.68$.

The following uncertainties parameters are considered: $D_1 = D_2 = D_3 = [0.2 \quad 0.1]^{\mathrm{T}}$, $E_1 = [0 \quad 0.5]$, $E_2 = [0.5 \quad 0]$, $E_3 = 0.5$, $F_1(t) = F_2(t) = F_3(t) = \sin t$, $G = [1 \quad 1]^{\mathrm{T}}$, $\gamma = 2$.

For the proposed controller, we use LMI toolbox in the Matlab from the theorem to solve matrices P and Q.

$$P = \begin{bmatrix} 13.0309 & 0.2444 \\ 0.2444 & 0.2410 \end{bmatrix},\ Q = \begin{bmatrix} 13.0181 & 0.1659 \\ 0.1659 & 0.2436 \end{bmatrix}.$$

It can be seen that matrices P and Q are symmetry positive, so they satisfy the demand of the theorem.

The following observer and feedback gains are obtained

$$K = [0.0521 \quad 0.0010],\ L = \begin{bmatrix} -0.0540 \\ 4.1423 \end{bmatrix}.$$

We can see that, in Fig.1, OBC and controller can obtain stability responses with disturbance, but the proposed controller enable the queue length to converge to its set value quickly and keep the queue oscillation small.

Fig.2-Fig.5 plot the simulation results of observer-based H_∞ controller of different network parameters. In Fig.2 we choose the parameters of network as above, we can see that the designed H_∞ controller can obtained fast and stable responses without disturbance.

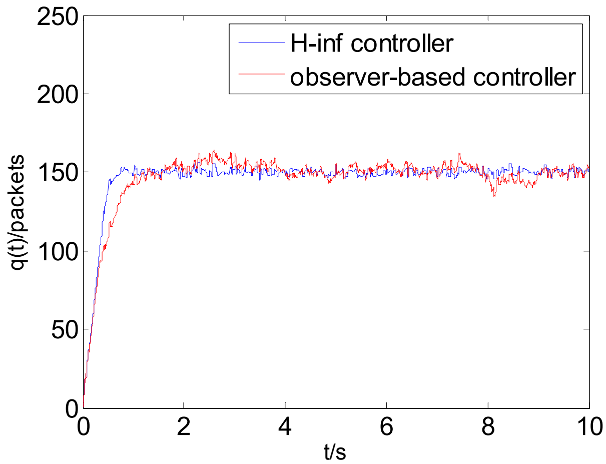

Fig.1. The instantaneous queue length responses with disturbance

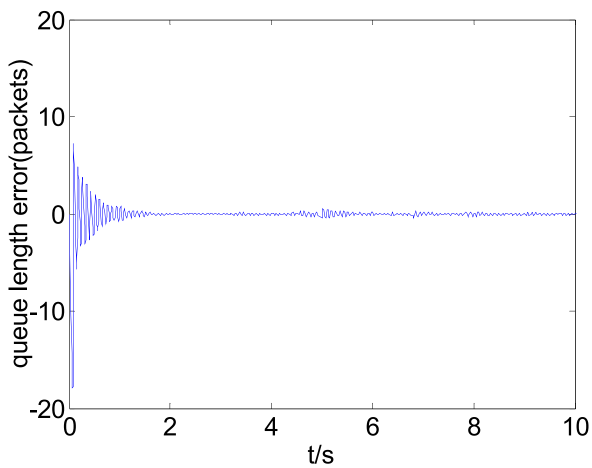

Fig. 2. Queue length error with fixed network parameter

In order to test the robust performance of H_∞ controller for varied parameters, we vary N from 50 to 80; the simulation results are given in Fig.3. The superior steady performance of H_∞ controller is observed when network parameters change.

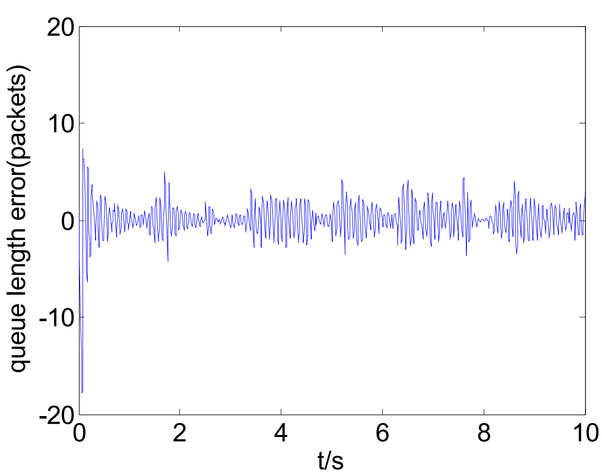

Fig. 3. Queue length error with varied network parameters

In Fig.4-5, we add UDP flows (transmitting on $1\text{Mbit}/\text{s}$) to the TCP flows at $5\,th$ second, choosing fixed and varied network parameters, respectively. We can see H_∞ controller shows better performance when UDP flows go down, with exhibiting faster responses and better regulation properties. From these simulations, it is obvious that the proposed scheme can be applied to a more complex network topology.

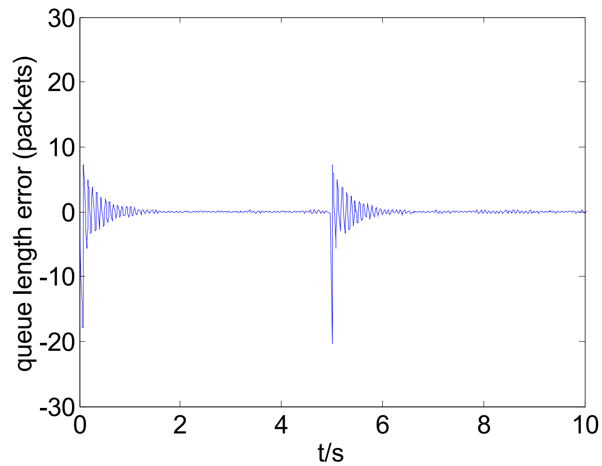

Fig. 4 Queue length error with fixed network and bursting flows

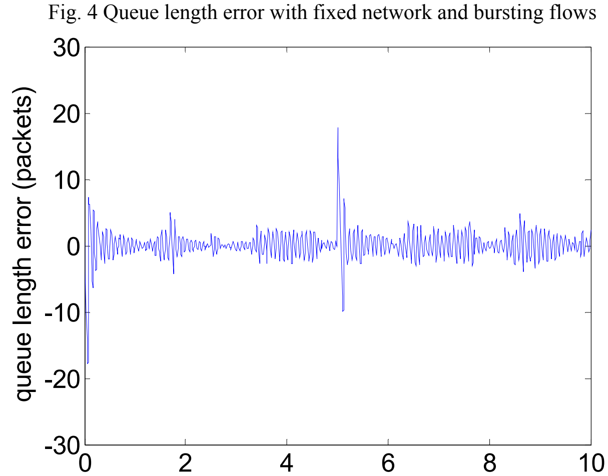

Fig. 5. Queue length error with varied network parameters and bursting flows

V. Conclusion

In this paper, we use linearization method to analyze a previously developed nonlinear model of TCP. For TCP network systems with the uncertain time-delay and external disturbance, an observer-based H-infinity controller is designed. On the basis of the Lyapunov-Krasovskii functional approach, by solving two linear matrices inequalities, the corresponding congestion control law is developed to achieve asymptotic stability. The simulation results demonstrate that the proposed AQM congestion control schemes possess well performance in various network conditions.

REFERENCES

[1] S. Floyd and V. Jacobson, "Random Early Detection Gateway for Congestion Avoidance," *IEEE/ACM Transaction on Networking*, vol. 1, no. 4, pp. 397-413, 1993.

[2] S. Floyd, R. Gummadi, and S. Shenker, "Adaptive RED: an algorithm for increasing the robustness of RED's active queue management," Technical Report, August 2001. *Available from: <http://www.icir. org/floyd/papers/adaptive Red.pdf>*.

[3] W. C. Feng, K. G. Shin, D. D. Kandlur, and D. Saha, "The blue active queue management algorithms," *IEEE Trans. Networking*, vol. 10, no.4, pp. 513–528, 2002.

[4] S. Athuraliya, V. H. Li, S. H. Low, et al , "REM: active queue management," *IEEE Network*, vol. 15, no. 3, pp. 48-53, 2001.

[5] V. Misra, W. B. Gong, D. Towsley, "Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED," *in: Proceedings of the ACM/SIGCOM, Stockholm*, pp. 151–160, 2000.

[6] C. Hollot, V. Misra, D. Towsley, and W.-B. Gong, "On designing improved controllers for AQM routers supporting TCP flows," in *Proceedings of the IEEE INFOCOM*, *Alaska,USA,* pp. 1726–1734, 2001.

[7] C. Hollot, V. Misra, D. Towsley, et al, "A control theoretic analysis of RED," *in Proceedings of IEEE INFOCOM, Anchorage, Alaska,USA* pp. 1510-1519,2001.

[8] H. W. Wang, B. Meng, and Y. W. Jing, "An adaptive fuzzy sliding mode control for AQM systems," *in Proceedings of the 2008 American Control Conference Westlin Seattle Hotel, Washington, USA*. pp. 4445-4450, 2008.

[9] M. Sabato, D. B. Mario, G. Franco, " Reduction-based robust active queue management control," *Control Engineering Practice,*vol.15, pp.177-186,2007.

[10] C. K. Chen, Y. C. Hung, T. L. Liao,et al, "Design of robust active queue management controllers for a class of TCP communication networks," *Information Sciences*, vol. 177, pp. 4059-4071,2007.