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Practical Novel Tests for Ensuring Safe Adaptive Control

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Abstract— **In this paper, we further illustrate the versatility and effectiveness of our novel tests for ensuring safe adaptive control in practice. The tests utilize a limited amount of experimental and possibly noisy data obtained from a closed-loop—consisting of an existing known stabilizing controller connected to an unknown plant—to infer if the introduction of a prospective controller will stabilize the unknown plant. The need and importance of this arise in iterative identification and control algorithms, multiple-model adaptive control (MMAC), and multi-controller adaptive switching.**

Index Terms*— *Adaptive Control, Multiple Model Adaptive Control, Robust Control, Iterative Identification and Control.

I. INTRODUCTION

SAFE adaptive control methods would generally refer to an assurance that a destabilizing controller is never switched in the closed-loop, even temporarily. The literature reports evidence of connecting a destabilizing controller at some point in the adaptive process, which although not remaining permanently in the loop, can do a lot of damage in the meantime [1], [2]. We discuss novel tests for verifying that the introduction of a new controller will stabilize an unknown plant using a *limited* amount of noisy experimental data obtained from the plant connected to an existing known stabilizing controller. These tests exploit phase information of the current closed-loop data to assess stability conditions, analogously to the Nyquist stability criterion, to ascertain closed-loop stability with the new controller.

There exist iterative control design methods which utilize the closed-loop data collected from an existing feedback interconnection in order to replace the current controller with a better performing controller, see e.g. [3] and the references therein. However, the existing stability tests to ascertain internal stability with the new controller before its implementation in the closed-loop are based either on the identification of a parametric 'full order model' of the current closed-loop transfer functions, or on an identification of a parametric 'full order model of the plant' from the current closed-loop transfer functions and the current controller or on the 'full estimation' of frequency bounds on the magnitude of the current closed-loop transfer functions [4], [5], [6]. The existing validation tests use the identification of the full dynamics of the current closed-loop system, and hence the amount of experimental effort required for validation purposes can be much larger than that required for the design of the controller update.

In contrast, we shall show that the validation tests of Section IV require gathering of information only on a limited known frequency region whose size depends on the size of the controller change. This promises to address, in retrospect, the so-called transient instability problem [7] in the context of multiple model adaptive control (MMAC) [8], [9] and iterative identification and control [10] methodologies. Here there is the possibility that the controller connected to the unknown plant at any particular time and frozen thereafter in combination with the plant provides an unstable closed loop. This happens partly because it is not always straightforward to accurately predict the new closed-loop transfer function that will result from changing a controller from one known controller to another known controller, when the first closed-loop transfer function is approximately known.

Furthermore, it is shown in [2], [11] that the data-driven Unfalsified Adaptive Control approach of [12], and the related references therein, can engender the worst problem of inserting a destabilizing controller in the closed-loop; moreover, such a destabilizing controller can remain in the loop for a long period of time resulting in very large closed-loop signals. It is reported in [2] that for a simple academic example, a maximum value of 1.228×10^6 was recorded for the plant input signal $u(t)$ when the the reference signal $r(t)$ was a sinusoid of magnitude 1. Indeed, one cannot even put a global upper bound on the time during which the destabilizing controller is attached.

This paper aims to highlight the effectiveness and versatility of our novel validation results for ensuring safe adaptive control in practice by presenting two benchmark examples. Section II collects the required and necessary definitions and notations, and elucidates the problem of concern by citing the internal stability results from the relevant literature. Section III is built on our earlier results [13], [14] and discusses the experimental setting which leads to the development of the novel validation tests of Section IV for SISO/MIMO systems. Two simulation examples are presented in Section V, and Section VI contains concluding remarks and future research directions.

II. NOTATIONS AND BACKGROUND

We shall denote by $\mathscr{H}_\infty$ the space of functions bounded and analytic in the open right-half complex plane, and the same function spaces with prefix $\mathscr{R}$ their real-rational proper subspaces. The plant is assumed to be a MIMO linear time-invariant system $P \in \mathscr{R}^{n\times n}$ (although at times we will restrict attention to scalar systems) and the controller is

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denoted by C; both assumed to be always proper transfer functions. The eigenvalues of $A \in \mathbb{C}^{m\times m}$ are denoted by $\lambda_1, \cdots, \lambda_m$ and its spectral radius $\rho(A) = \max_{1\leq i\leq m} |\lambda_i|$. The determinant of a matrix is denoted by $\det$ and its singular values by $\sigma_i(\cdot)$ with the largest singular value $\overline{\sigma}(\cdot)$ and the smallest singular value $\underline{\sigma}(\cdot)$. The number $\mathrm{wno}(\cdot)$ denotes the winding number of the Nyquist diagram of a scalar transfer function, evaluated on a contour along the imaginary axis and indented to the right around any pure imaginary pole. The nearest integer function $\mathrm{nint}[\cdot]$ returns the integer closet to $[\cdot]$ with the additional rule that half-integers are always rounded to even numbers. We denote $G(j\omega)^*$ as the complex conjugate transpose of frequency-response function $G(j\omega)$ at each ω, i.e. $G(j\omega)^* = G(-j\omega)^T$.

Definition 1: The *unwrapped* phase of a transfer function is denoted by unwarg and refers to the phase of the frequency response when it is in the form of a continuous function of the frequency. The unwrapped phase is derived from the phase frequency response by removing the discontinuities of 2π, and ensures that all appropriate multiples of 2π are included in the phase-frequency response [15].

Definition 2: The interconnection $[P, C]$ is "well-posed" if the transfer function matrix mapping $\left[\begin{smallmatrix} r \\ d \end{smallmatrix}\right]$ to $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ exists. Put another way, $[P, C]$ is well-posed if $(I - CP)^{-1} \in \mathscr{R}$.

Given such well-posedness, the four transfer functions of Fig. 1 can be written as

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix} = H(P, C) \begin{bmatrix} r \\ d \end{bmatrix}.$$

Definition 3: The interconnection $[P, C]$ is said to be "internally stable" if it is well-posed and $H(P, C) \in \mathscr{RH}_\infty$; i.e., each of the four transfer functions in $\left[\begin{smallmatrix} r \\ d \end{smallmatrix}\right] \mapsto \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right] \in \mathscr{RH}_\infty$.

Definition 4: The ordered pair $\{\tilde{U}, \tilde{V}\}$, with $\tilde{U}, \tilde{V} \in \mathscr{RH}_\infty$, is a left-coprime factorization (*lcf*) of $C \in \mathscr{R}$ if $\tilde{V}$ is invertible in $\mathscr{R}$, $C = \tilde{V}^{-1}\tilde{U}$, and $\tilde{U}$ and $\tilde{V}$ are left-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{\tilde{U}, \tilde{V}\}$ is a normalized *lcf* of C if $\{\tilde{U}, \tilde{V}\}$ is a *lcf* and $\tilde{V}\tilde{V}^* + \tilde{U}\tilde{U}^* = I$.

Definition 5:

$$G := \begin{bmatrix} N \\ M \end{bmatrix} \quad , \quad \tilde{K} := \begin{bmatrix} -\tilde{U} & \tilde{V} \end{bmatrix} \tag{1}$$

where G will be referred to as the graph symbol of P, and $\tilde{K}$ will be referred to as the inverse graph symbol of C.

Theorem 6: [16, Proposition 1.9] Let G and $\tilde{K}$ be defined as in (1). Then the following are equivalent:

i. $[P, C]$ is internally stable;
ii. $(\tilde{K}G)^{-1} \in \mathscr{RH}_\infty$;
iii. $\det(\tilde{K}G)(j\omega) \neq 0\ \forall\omega$ and $\mathrm{wno}\det(\tilde{K}G) = 0$.

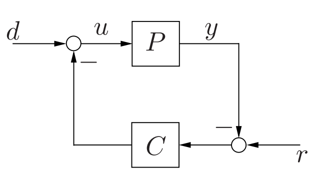

Fig. 1. Standard Feedback Configuration

III. Problem Set-up and Experimental Setting

The considered problem is that given an unknown plant, which is stabilized by a known controller C_0, and a limited amount of experimental data obtained with C_0, how can one verify—without actual insertion in the closed-loop—if the introduction of the new controller C_1 will stabilize the plant? The following theorem defines the *experimental setting* for the stability tests discussed the sequel.

Theorem 7: Let $[P, C_0]$ be internally stable. Let $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ be left coprime factorizations over $\mathscr{RH}_\infty$. Consider Fig. 2 and define $T : r \mapsto z$ to be

$$T = \begin{bmatrix} -\tilde{U}_1 & \tilde{V}_1 \end{bmatrix} \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \tag{2}$$

Then the following are equivalent:

a) $[P, C_1]$ is internally stable;
b) $T^{-1} \in \mathscr{RH}_\infty$;
c) $\det T(j\omega) \neq 0\ \forall\omega$ AND $\mathrm{wno}\det T = 0$;
d) $\det T(j\omega) \neq 0\ \forall\omega$ AND $\mathrm{unwarg}\det T(j\infty) = \mathrm{unwarg}\det T(j0)$.

Proof: Note that $T = (\tilde{K}_1G)(\tilde{K}_0G)^{-1}$, and

- (b) and Theorem 6.ii are equivalent since $(\tilde{K}_0G)$, $(\tilde{K}_0G)^{-1} \in \mathscr{RH}_\infty$;
- (c) and Theorem 6.iii are equivalent since $\{[P, C_0]$ internally stable$\} \Leftrightarrow \{\det(\tilde{K}_0G)(j\omega) \neq 0\,\forall\omega$ AND $\mathrm{wno}\det(\tilde{K}_0G) = 0\}$ for which $\mathrm{wno}\det(T) = \mathrm{wno}\det(\tilde{K}_1G) - \mathrm{wno}\det(\tilde{K}_0G)$;
- (d) and (c) are equivalent because $T \in \mathscr{RH}_\infty$ and is bi-proper and therefore

$$\begin{aligned}\mathrm{wno}\det(T) &= \mathscr{Z}(T) \\ &= \frac{1}{\pi}[\mathrm{unwarg}\det T(j\infty) - \mathrm{unwarg}\det T(j0)]\end{aligned}$$

where $\mathscr{Z}(T)$ is the number of open RHP zeros of T.

■

Note that one cannot explicitly construct the transfer function T as the plant P is unknown. However, the stable mapping $T : r \rightarrow z$ in Fig. 2 can be studied in a safe experiment—where no instability can occur—to infer the required properties of T via the reference signal r and the constructed output signal z (computed as a filtered version of the measured signals $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ via $\tilde{K}_1$).

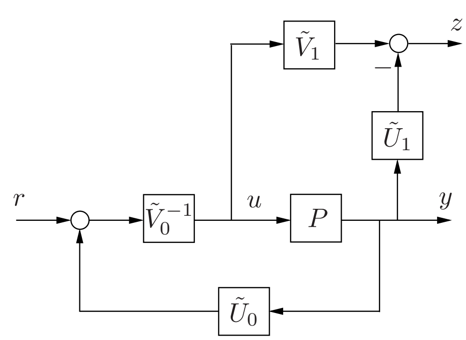

Fig. 2. Experimental setting: $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$.

IV. Novel Stability Verification Tests

The data-based stability tests of this section are built on the experimental setting defined in Section III with the aim of verifying condition (d) in Theorem 7. For the development of these stability tests, following assumptions are introduced.

***Assumption** 8:* The factors $\tilde{V}_0$ and $\tilde{V}_1$ are such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$.

***Assumption** 9:* The transfer functions PC_0 and PC_1 are strictly proper.

It is evident that Assumption 8 is without loss of generality and Assumption 9 captures a typical situation. Notice that the transfer function T can be written as

$$T = \tilde{V}_1(I - C_1P)(I - C_0P)^{-1}\tilde{V}_0^{-1} \tag{3}$$

for which under Assumptions 8 and 9 we have

$$\det T(j\infty) = \frac{\det \tilde{V}_1(j\infty)}{\det \tilde{V}_0(j\infty)} \frac{\det(I - C_1P)(j\infty)}{\det(I - C_0P)(j\infty)} = 1\,. \tag{4}$$

Thus, $\det T(j\infty)$ is strictly positive and known and will be used as a datum for the verification of condition (d) in Theorem 7 as shown in the following *falsification* test.

***Theorem** 10:* Let the suppositions of Theorem 7 and Assumptions 8 and 9 hold. Let e_i denote a reference signal where a step function is applied at the $i-$th input while the other inputs are kept at 0. Perform n experiments with reference signal $r(t) = e_i(t)$, $i = 1, \dots, n$ and let $\bar{z}_i$ be the steady state output of T recorded in each experiment. Define $\bar{Z} = [\bar{z}_1, \dots, \bar{z}_n]$. Then

$$[P, C_1] \text{ is internally stable } \Rightarrow \det \bar{Z} > 0\,.$$

Thus if $\det \bar{Z} \leq 0$, stability of $[P, C_1]$ is falsified.

Proof: See [13]. ■

It readily follows that the aforementioned result on steady-state step response measurements in Theorem 10 is robust and in fact unaffected by all disturbances with finite energy.

Condition (d) in Theorem 7 can be verified in both its necessary and sufficient parts by using more sophisticated identification techniques. However, this is not desirable due to the complexity involving a full sine sweep. In the sequel, a novel test is proposed which proposes a mechanism to measure the frequency response of T up to a finite frequency ω_0. The measurement can tolerate error as its purpose is simply to facilitate computation of a certain phase change.

***Lemma** 11:* Let the suppositions of Theorem 7 hold. Then

$$T = I + T', \text{ where} \tag{5}$$

$$T' = [-(\tilde{U}_1 - \tilde{U}_0) \quad (\tilde{V}_1 - \tilde{V}_0)] \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \tag{6}$$

Proof: See [13]. ■

Note that T' in (6) is strictly proper under Assumptions 8 and 9. Hence it can be expected that measuring the frequency response of $T = I + T'$ up to a frequency, say ω_0, where the response of T' has nearly vanished is enough to characterize the full frequency response of T. This fact is utilized next.

***Theorem** 12:* Suppose the hypothesis of Theorem 7 and Assumption 8 and 9 hold. Define $T' \in \mathscr{RH}_\infty^{n\times n}$ by $T' = T - I$ via (5). Let $\omega_0 \in [0, \infty)$ be a frequency such that

$$\begin{cases} \rho(T'(j\omega)) < 1 & n = 1 \\ \rho(T'(j\omega)) < \sin(\frac{\pi}{n}) & n \geq 2 \end{cases} \quad \forall \omega \geq \omega_0 \tag{7}$$

Then the condition

$$\det T(j\omega) \neq 0 \ \forall \omega \in [0, \omega_0) \quad \text{AND}$$
$$2\pi \times \text{nint}\left[\frac{\text{unwarg} \det T(j\omega_0)}{2\pi}\right] = \text{unwarg} \det T(j0) \tag{8}$$

is equivalent to condition (d) in Theorem 7.

Proof: Lemma 11 shows that $T(j\infty) = I$ under Assumption 8 and 9. For the case $n = 1$, the inequality in (7) becomes $|T'(j\omega)| < 1 \quad \forall \omega \geq \omega_0$, which implies

$$2\pi \times \text{nint}\left[\frac{\text{unwarg}\, T(j\omega_0)}{2\pi}\right] = \text{unwarg}\, T(j\infty)\,.$$

For $n \geq 2$ in (7), observe that $|\lambda_i(T')| \leq \rho(T')$ and that

$$\det T = \prod_{i=1}^{n} \lambda_i(I + T') = \prod_{i=1}^{n} [1 + \lambda_i(T')]\ .$$

Since inequality (7) holds, $[1 + \lambda_i(T')](j\omega)$ lies in the interior of a circle of center 1 and radius $\xi = \sin(\pi/n)$ $\forall \omega \geq \omega_0$ and $\det T(j\omega) \neq 0$ $\forall \omega \in [\omega_0, \infty)$. Hence the angle θ depicted in Fig. 3 is precisely π/n. Consequently, the angle of the complex number $1 + \lambda_i(T')$ for each i and each $\omega \in [\omega_0, \infty)$ lies in $\left(-\frac{\pi}{n}, \frac{\pi}{n}\right)$. Thus the angle of the complex number $\det T(j\omega)$ is in $(-\pi, \pi)$ for each $\omega \in [\omega_0, \infty)$ and $\det T$ can never complete a contour around the origin $\forall \omega \in [\omega_0, \infty)$, because the contour can never cross the negative real axis. Since $\det T(j\omega)$ is a continuous function of frequency and is equal to unity at infinity frequency,

$$2\pi \times \text{nint}\left[\frac{\text{unwarg} \det T(j\omega_0)}{2\pi}\right] = \text{unwarg} \det T(j\infty)\,.$$

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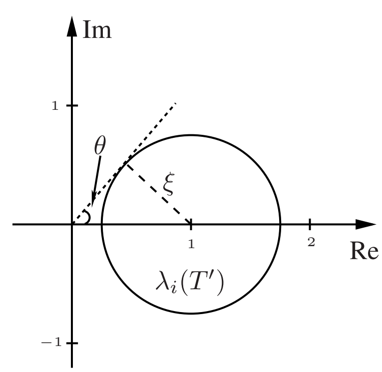


Fig. 3. Graphical representation of the condition on $\lambda_i(T')$. The angle θ is of the form π/n.

The aforementioned theorems outline experimental tests to assess stability of $[P, C_1]$ before inserting C_1. For the application of Theorem 12, recall that $\rho(T'(j\omega)) \leq \overline{\sigma}(T'(j\omega))$ and $\overline{\sigma}(T'(j\omega)) \to 0$ as $\omega \to \infty$. One can obtain an estimate of ω_0 knowing a rough estimate of the bandwidth of $[P, C_0]$ and assuming that $\overline{\sigma}(T'(j\omega))$ remains below the right-hand side of inequality (7) over some known high-frequency region. Notice that since $\mathrm{nint}[\mathrm{unwarg}\det T(j\omega_0)/2\pi]$ is used in condition (8), a rough estimate of $\mathrm{unwarg}\det T(j\omega_0)/2\pi$ is enough and hence the test can tolerate estimation errors. Moreover, the structure of T' in (6) reveals that a small controller change certainly implies a smaller ω_0 and hence reduced experimental effort in estimating $\overline{\sigma}(T'(j\omega))$. In practice, to check the condition in (7), one can use

$$\overline{\sigma}(T') \leq \|T'\|_F = (\sum_{i,j} |T'_{ij}|^2)^{0.5} \tag{9}$$

along with $\rho(T'(j\omega)) \leq \overline{\sigma}[T'(j\omega)]$ to find an upper bound on $\lambda_i[T'(j\omega)]$ at each frequency. Alternatively $\overline{\sigma}[T'(j\omega)] \leq \sqrt{n}\,\|T'(j\omega)\|_1$ can be utilized at each frequency.

V. Simulation Examples

In this section, we consider a SISO system and a MIMO system to illustrate the advantages and effectiveness of our stability tests captured in Theorem 10 and Theorem 12.

A. Example 1: Applicability of our Results to SISO Systems

Consider the system used in [6] with

$$P = \frac{1.2(\frac{1}{2}s+1)(\frac{-1}{4}s+1)}{(\frac{2}{3}s+1)(\frac{1}{3}s+1)(\frac{1}{10}s+1)}$$

with a DC-gain of $K = 1.2$ and a non-minimum-phase zero at $z = 4$. In [6], a Multiple Model Adaptive Control scheme is used with the control objective of extending the bandwidth of the complementary sensitivity transfer function that exceeds that of the open-loop plant. This control objective was set with the consideration of practical limitations imposed by the existence of the non-minimum-phase zero. The multiple model set consisted of 441 plant models defined by

$$P_i = \frac{K_i(\frac{2}{3}s+1)(\frac{-1}{z_i}s+1)}{(\frac{1}{2}s+1)(\frac{1}{3}s+1)(\frac{1}{3}s+1)} \tag{10}$$

with the modeled DC-gain $K_i \in [0.2, 2]$ and the modeled non-minimum-phase zero $z_i \in [1, 10]$, both varying in 20 logarithmically equally spaced intervals. For each model in the set, controllers are designed to achieve the objective of expanding closed-loop bandwidth. To show the effectiveness of our tests, we consider only three controllers from the controller set designed to achieve closed-loop bandwidths of $2\,\mathrm{rad}\,.s^{-1}$ and $2.1\,\mathrm{rad}\,.s^{-1}$ using the models in (10) corresponding to $K_i \in \{0.6720, 0.5953, 1.0911\}$ and $z_i = 2.9764$, respectively. Let the stabilizing C_0 below

$$C_0 = \frac{-0.738(s+3)^2(s+2)}{(s+1.5)(s^2+2.976s+11.91)}$$

ensure a closed-loop bandwidth of $2\,\mathrm{rad}\,.s^{-1}$, $[P, C_0] \in \mathscr{RH}_\infty$ with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$,

$$\tilde{V}_0 = \frac{(s+1.5)(s^2+2.97s+11.91)}{(s+1.56)(s^2+4.47s+11.45)}$$

satisfying Assumption 8, $\tilde{V}_0(j\infty) = 1$, and

$$\tilde{U}_0 = \frac{-0.7382(s+3)^2(s+2)}{(s+1.56)(s^2+4.47s+11.45)}\ .$$

Suppose the adaptive control decision unit suggests the use of C_1, which achieves a bandwidth of $2.1\,\mathrm{rad}\,.s^{-1}$,

$$C_1 = \frac{-0.875(s+3)^2(s+2)}{(s+1.5)(s^2+2.976s+12.5)}$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$,

$$\tilde{V}_1 = \frac{(s+1.5)(s^2+2.976s+12.5)}{(s+1.578)(s^2+4.747s+11.68)}$$

satisfying Assumption 8, $\tilde{V}_1(j\infty) = 1$, and

$$\tilde{U}_1 = \frac{-0.875(s+3)^2(s+2)}{(s+1.578)(s^2+4.747s+11.68)}\ .$$

Setting up the experimental configuration of Fig. 2 and utilizing Theorem 10 to check if C_1 is stabilizing, we perform experiments with reference signal $r(t) = step(t)$ and the step response is measured at the output z. The steady state value of $T : r \to z$ is $\bar{z} = -0.0711 < 0$ and hence the stability of $[P, C_1]$ is falsified. Indeed, computing $H(P, C_1)$ shows that it has one pole at $s = 0.0136$ which conforms with the results. Note that our validation test above does not include any identification and is very easy to carry out while the results in [6] require identification of closed-loop transfer functions to prevent switching to destabilizing controllers.

Let the next suggested controller to replace C_0 be C_2 from the set, achieving a bandwidth of $2.1\,\mathrm{rad}\,.s^{-1}$,

$$C_2 = \frac{-0.4773(s+3)^2(s+2)}{(s+1.5)(s^2+2.976s+12.5)}$$

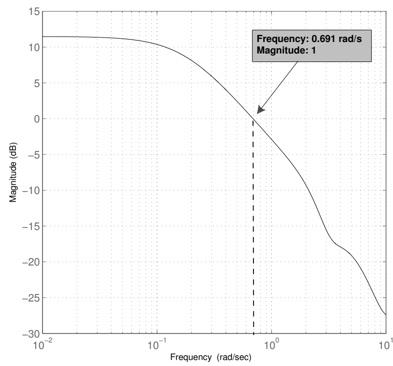
with a left coprime factorization, $C_2 = \tilde{V}_2^{-1}\tilde{U}_2$,

$$\tilde{V}_2 = \frac{(s+1.5)(s^2+2.976s+12.5)}{(s+1.528)(s^2+3.847s+12.18)}$$

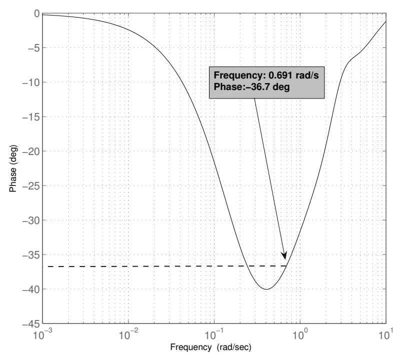
satisfying Assumption 8, $\tilde{V}_2(j\infty) = 1$, and

$$\tilde{U}_2 = \frac{-0.4774(s+3)^2(s+2)}{(s+1.528)(s^2+3.847s+12.18)}\ .$$

Setting up the experimental configuration of Fig. 2 for simulation and using the results of Theorem 10 to check if C_2 is stabilizing, we perform experiments with reference signal $r(t) = step(t)$ and the step response is measured at the output z. The steady state value of $T : r \to z$ is $\bar{z} = 4.74 > 3\,\forall t > 30$ which does not falsify the stability of $[P, C_2]$. Thus, we shall use the results of Theorem 12 to check if C_2 is stabilizing. As shown in Fig. 4a, the simulation results reveal that $|T - 1| \leq 1\ \forall\omega \geq 0.691\ \mathrm{rad}/s$. Given that $\mathrm{unwarg}\,T(j0) = 0$ and $\mathrm{unwarg}\,T(j\omega_0) = -0.2039\pi$ as shown in Fig. 4b, the condition in Theorem 12 holds



(a) Magnitude Response of $T' = T - 1$



(b) Phase Response of T

Fig. 4. Example 1: Magnitude and Phase responses

and hence C_2 is stabilizing. Indeed, computing $H(P, C_2)$ confirms that C_2 is stabilizing. The Nyquist diagram of T is shown in Fig. 5.

B. Example 2: Applicability of our Results to MIMO Systems

We shall now consider a five-state dynamic model of the distillation process obtained from [17], which includes liquid flow dynamics and composition dynamics as well as disturbances. Let $P \in \mathscr{R}^{2\times 2}$ be

$$P = \frac{1}{(s+0.183)(s+0.0736)(s+0.0051)(s^2+0.924s+1.193)} \begin{bmatrix} P^{11} & P^{12} \\ P^{21} & P^{22} \end{bmatrix}$$
$$P^{11} = 0.3012(s+0.1786)(s+0.0783)(s^2+1.263s+1.726)$$
$$P^{12} = 0.1906(s+0.1764)(s+0.0708)(s^2-3.251s+3.762)$$
$$P^{21} = -0.4023(s+0.1634)(s+0.0922)(s^2+0.8788s+1.18)$$
$$P^{22} = -0.8735(s+0.1592)(s+0.0640)(s^2+0.88s+1.019)$$

Here, we consider a set of controllers consisting of two controllers C_0 and C_1, which are designed via μ-optimal design based on DK-iteration using the following idealized model of a simplified distillation process

$$P_{simp}(s) = \frac{1}{75s+1} \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix} .$$

These controllers are extracted from the first two DK-iterations and then truncated by employing the closed-loop controller reduction method detailed in [18, Sec. 4.3]. It is assumed that the Hankel singular values of the graph symbol of the controller are decreasingly ordered ($\sigma_1 > \sigma_2 > \cdots$), balanced realization is performed, and the result is truncated to retain all Hankel singular values greater than $0.06\,\sigma_1$.

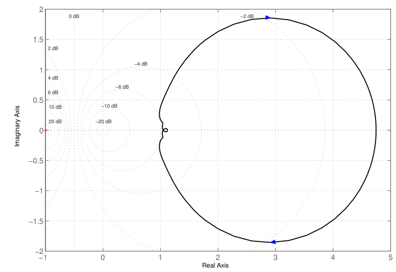


Fig. 5. Nyquist plot of $T(j\omega)$ in Example 1.

Let C_0 be a stabilizing controller, $[P, C_0] \in \mathscr{RH}_\infty$,

$$C_0 = \frac{1}{(s+5.846)(s+0.5047)(s-0.0007)(s+0.0002)} \begin{bmatrix} C_0^{11} & C_0^{12} \\ C_0^{21} & C_0^{22} \end{bmatrix}$$
$$C_0^{11} = -1.5477(s+1.974)(s+0.0231)(s-0.0004)$$
$$C_0^{12} = 0.6529(s-2.675)(s-0.0188)(s-0.0012)$$
$$C_0^{21} = -1.0881(s-0.9831)(s-0.0476)(s-0.001)$$
$$C_0^{22} = 1.669(s+1.626)(s+0.0296)(s-0.0005)$$

with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$,

$$\tilde{V}_0 = \frac{1}{(s+6.356)(s+0.9046)(s+0.0372)(s+0.0056)} \begin{bmatrix} \tilde{V}_0^{11} & \tilde{V}_0^{12} \\ \tilde{V}_0^{21} & \tilde{V}_0^{22} \end{bmatrix}$$
$$\tilde{U}_0 = \frac{1}{(s+6.356)(s+0.9046)(s+0.0372)(s+0.0056)} \begin{bmatrix} \tilde{U}_0^{11} & \tilde{U}_0^{12} \\ \tilde{U}_0^{21} & \tilde{U}_0^{22} \end{bmatrix}$$
$$\tilde{V}_0^{11} = -(s+6.095)(s+0.7012)(s+0.0178)(s-0.0005)$$
$$\tilde{V}_0^{12} = -0.07(s-13.13)(s-0.0456)(s-0.001)$$
$$\tilde{V}_0^{21} = -0.07(s-13.47)(s-0.0445)(s-0.0011)$$
$$\tilde{V}_0^{22} = -(s+6.125)(s+0.6982)(s+0.0178)(s-0.0005)$$
$$\tilde{U}_0^{11} = 1.5477(s+2.372)(s+0.0527)(s+0.0061)$$
$$\tilde{U}_0^{12} = -0.6529(s-2.381)(s^2+0.0102s+8.647e-5)$$
$$\tilde{U}_0^{21} = 1.0881(s-0.6265)(s-0.0535)(s+0.0037)$$
$$\tilde{U}_0^{22} = -1.669(s+2.036)(s+0.0581)(s+0.0060) .$$

Theorem 10 puts forward a solution to the problem of checking in advance using collected closed-loop data if the controller C_1 given here by

$$C_1 = \frac{1}{(s+22.71)(s+11.35)(s+1.838)(s-0.0313)} \begin{bmatrix} C_1^{11} & C_1^{12} \\ C_1^{21} & C_1^{22} \end{bmatrix}$$
$$C_1^{11} = -43.305(s+19.69)(s+1.77)(s+0.0231)$$
$$C_1^{12} = -21.1218(s+28.17)(s+1.875)(s-0.0987)$$
$$C_1^{21} = 18.3868(s+27.68)(s+2.171)(s-0.1072)$$
$$C_1^{22} = 32.0272(s+20.24)(s+1.717)(s+0.0372)$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$,

$$\tilde{V}_1 = \frac{1}{(s+59.41)(s+28.31)(s+1.464)(s+0.3251)} \begin{bmatrix} \tilde{V}_1^{11} & \tilde{V}_1^{12} \\ \tilde{V}_1^{21} & \tilde{V}_1^{22} \end{bmatrix}$$
$$\tilde{U}_1 = \frac{1}{(s+59.41)(s+28.31)(s+1.464)(s+0.3251)} \begin{bmatrix} \tilde{U}_1^{11} & \tilde{U}_1^{12} \\ \tilde{U}_1^{21} & \tilde{U}_1^{22} \end{bmatrix}$$
$$\tilde{V}_1^{11} = -(s+35.04)(s+21.56)(s+1.77)(s+0.0279)$$
$$\tilde{V}_1^{12} = 20.603(s+25.23)(s+1.845)(s-0.1174)$$
$$\tilde{V}_1^{21} = 20.603(s+23.5)(s+2.057)(s-0.1187)$$
$$\tilde{V}_1^{22} = -(s+45.07)(s+20.07)(s+1.827)(s+0.0233)$$
$$\tilde{U}_1^{11} = 43.305(s+32.34)(s+1.125)(s+0.5017)$$
$$\tilde{U}_1^{12} = 21.122(s+16.38)(s+2.421)(s+0.0247)$$
$$\tilde{U}_1^{21} = -18.387(s+13.22)(s+3.196)(s-0.04082)$$
$$\tilde{U}_1^{22} = -32.027s+39.71)(s^2+1.577s+0.6223) .$$

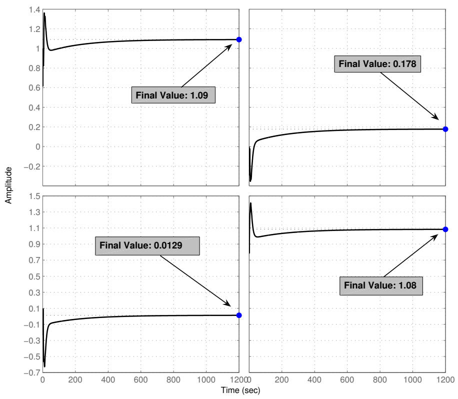


Fig. 6. Step Responses: $r(t) = step(t) \cdot e_1$ and $r(t) = step(t) \cdot e_2$

We setup the experimental configuration of Fig. 2 and perform two experiments with reference signals $r(t) = step(t) \cdot e_1$ and $r(t) = step(t) \cdot e_2$. The step responses are shown in Fig. 6 and the steady state values of $T : r \rightarrow z$ are

$$\bar{Z} = \begin{bmatrix} 1.09 & 0.178 \\ 0.0129 & 1.08 \end{bmatrix}$$

with $\det(\bar{Z}) = 1.1749 > 0$ which does not falsify the stability of $[P, C_1]$. Thus, we shall use the results of Theorem 12 to check if C_1 is stabilizing.

Given that $T' = T - I \in \mathscr{RH}_\infty^{2\times 2}$, we shall follow the results of Theorem 12 for $n = 2$, which requires us to find the frequency ω_0 such that $\rho(T'(j\omega)) < \sin(\pi/2)\ \forall\omega \geq \omega_0$. As discussed in Section IV, one can use (9) along with $\rho(T'(j\omega)) \leq \overline{\sigma}(T'(j\omega))$ to find an upper bound on the eigenvalues of $T'(j\omega)$ in order to check the condition above. The simulation results reveal that $\overline{\sigma}(T'(j\omega)) < 1\,\forall\omega \geq 1.8401\ \mathrm{rad}/s$. Given that $\mathrm{unwarg}\det T(j0) = 0$ and $\mathrm{unwarg}\det T(j\omega_0) = -0.0079\pi$, the condition in (8) in Theorem 12 holds and hence C_1 is stabilizing. Indeed, computing $H(P, C_1)$ confirms that C_1 is stabilizing.

VI. Conclusions

The simulations of Section V clearly showed the effectiveness and applicability of the proposed tests. The novel validation tests aim to protect internal stability with the introduction of a new controller C_1 by utilizing a limited amount of experimental data obtained from the stable closed-loop interconnection $[P, C_0]$. Our current research focuses on extending our results to include nonlinear controllers and possibly nonlinear plants.

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