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Approximate trajectory tracking control of a velocity-senlorless VTOL aircraft with measurement delays

Shanwei Su and Yan Lin

Abstract— **In this paper, we develop a nonlinear controller to achieve trajectory tracking for a velocity-sensorless vertical take-off and landing (VTOL) aircraft with measurement delays. By applying Pade approximation technique, the original controlled system is approximately transformed into an augmented dimension system without any time delay. After constructing full-order observer, error coordinate transformation, and system decomposition, the tracking problem of the newly transformed system is changed into the stabilization problem of two non-minimum phase subsystems and one minimum phase subsystem. The resulting controller not only forces the VTOL aircraft to asymptotically track the desired trajectories, but also drives the unstable internal dynamics which stands for the non-minimum property of VTOL aircraft to follow the causal ideal internal dynamics (IID) solved via stable system center (SSC) method. Numerical simulation results illustrate the effectiveness of the proposed controller.**

I. INTRODUCTION

Over the last few years, controlling of VTOL aircraft has been paid considerable attention by the control community. The main difficulty of VTOL aircraft control lies in its non-minimum phase nature which limits the straightforward application of some powerful nonlinear control techniques such as feedback linearization and sliding mode control (SMC)(Shkolnikov and Shtessel 2002).

At present, the bulk of the existing work with respect to VTOL aircraft control covers two main branches: the stabilization control (Olfati-Saber 2002; Zavala, Fantoni, and Lozano 2003) and the trajectory tracking control (Hauser, Sastry and Meyer 1992; Huang and Yuan 2002; Al-Hiddabi and McClamroch 2002; Do, Jiang and Pan 2003). It should be pointed out that the aforementioned papers are all based on the assumption that there is no any time delay in the VTOL aircraft system. Actually, the unavoidable presence of time delay is a common feature in many systems (Liu, Zinober and Shtessel 2009), including flight control systems (FCSs). Time delays have significant effects on the flying qualities of FCSs (Smith 1986). If not dealt with properly, these delays may cause a significant degradation in controlling performance or even lead to loss of stability. So the time delays must be taken into account and dealt with properly when designing controllers for aircrafts (Zhu, Wang and Cai 2011).

Usually, the position and the attitude of unmanned aircrafts are measured by GPS and inertial measurement units (IMUs), respectively. However, the transmission of position information from the satellites to the aircraft and the measurements of attitudes by IMUs all exist time delays (Wang, Liu and Cai 2009). In [12] and [13], Pade approximation technique was originally introduced to deal with the attitude measurement time delays of VTOL aircraft. The problem of output tracking control for velocity-sensorless VTOL aircrafts with delayed outputs was addressed in [11]. However, in the papers (Wang, Liu and Cai 2009; Zhu, Wang and Cai 2010b) based on output feedback control, the simulation results demonstrated that VTOL aircrafts were merely forced to track a desired vertical trajectory without considering the lateral movement. As a result, the roll attitude is kept horizontal, *i.e.*, the asymptotically unstable internal dynamics is stabilized to be zero. Under this circumstance, the tracking problem is greatly simplified, as it is only during lateral movement that the non-minimum phase property reflected by the coupling between lateral and vertical thrusts is problematic (Consolini, Maggiore, Nielsen, and Tosques 2010).

This paper focuses on the problem of approximate trajectory tracking control for a velocity-sensorless VTOL aircraft in the presence of measurement delays. Compared with the control schemes in [11] and [13], which only considered the vertical movement of the VTOL aircraft in the presence of time delay, the lateral movement is taken into account in our scheme, during which, the unstable internal dynamics of the aircraft is forced to follow its causal and bounded IID (Gopalswamy and Hedrick 1993) solved via stable system center (SSC) method (Shkolnikov and Shtessel 2002) rather than directly stabilized to zero as in vertical movement. By virtue of the causality of the solved IID, the proposed scheme guarantees the output tracking of the VTOL aircraft even in the presence of unexpected changes of desired trajectories.

The paper is organized as follows. In section 2, the control problem of the VTOL aircraft is formulated. In section 3, the design procedure of controller is presented. In section 4, the stability analysis is provided. In section 5, the numerical simulation is given to show the effectiveness of the proposed design method. Section 6 draws the conclusions.

II. PROBLEM FORMULATION

The nominal mathematical model of the VTOL aircraft moving in the vertical-lateral plane is described as (Hauser,

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S.W. Su is Ph.D candidate of School of Automation, Beijing University of Aeronautics and Astronautics, Beijing, China, 100191 sushanwei@yahoo.com.cn

Y. Lin is with the School of Automation, Beijing University of Aeronautics and Astronautics, Beijing, China, 100191 linyanee2@yahoo.com.cn

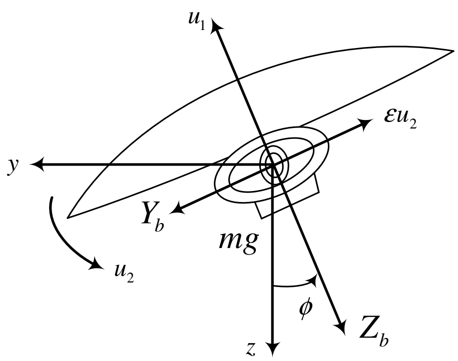

Fig. 1. Vertical take-off and landing aircraft.

Sastry and Meyer 1992)

$$
\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= -u_1\sin x_5 + \varepsilon u_2 \cos x_5,\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= u_1\cos x_5 + \varepsilon u_2 \sin x_5 - 1,\\
\dot{x}_5 &= x_6,\\
\dot{x}_6 &= u_2,
\end{aligned} \tag{1}
$$

where x_1 and x_3 denote, respectively, the horizontal position y and vertical position z of the aircraft center of mass in the body-fixed reference frame shown in Fig.1, x_5 is the roll angle ϕ of the aircraft, x_2, x_4 and x_6 are the corresponding velocities, respectively, and ε is a small coefficient that characterizes the coupling between the rolling moment and the lateral force. '-1' denotes the acceleration of gravity; The control inputs are the thrust (directed out the bottom of the aircraft), u_1, and the rolling moment about the aircraft center of mass, u_2. The two outputs are $y_1 = x_1$, $y_2 = x_3$. By setting $x_i = 0$ $(i = 1, \ldots, 4)$ in (1), it can be seen that the resulting zero dynamics $\ddot{x}_5 = \frac{1}{\varepsilon}\sin x_5$ is asymptotically unstable for $\varepsilon \neq 0$, which means that the VTOL aircraft is non-minimum phase.

Under the assumption that without velocity measurements, say, x_2, x_4 and x_6 are not available for measurements and the signals x_1, x_3, x_5 are measured with known time delays τ_1, τ_2, τ_3 respectively, where τ_1, τ_2 are the time delays brought from the transmitting delays by GPS, and τ_3 is the time delay brought from measurement delay by IMU. The control objective is to design a feedback control law using the delayed signals $x_{2i-1}(t - \tau_i), i = 1, 2, 3$, so that the outputs $y_1 = x_1(t - \tau_1)$ and $y_2 = x_3(t - \tau_2)$ can track the smooth reference trajectories y_{1d} and y_{2d}, respectively.

We use $\bar{x}_{(2i-1)td}, i = 1, 2, 3$, to denote the actual delayed signal of $x_{2i-1}(t - \tau_i)$ and then have

$$\bar{x}_{(2i-1)td} = x_{2i-1}(t - \tau_i). \tag{2}$$

By replacing the time-delay function (2) with its first-order Pade approximation (Shtessel, Zinober and Shkolnikov 2003), we have

$$\frac{\bar{x}_{(2i-1)td}(s)}{x_{2i-1}(s)} = \exp(-\tau_i s) \approx \frac{2 - \tau_i s}{2 + \tau_i s}, \tag{3}$$

where s is the Laplace transform variable. Define an auxiliary variable $x_{(2i-1)td}$ as

$$\frac{x_{(2i-1)td}(s)}{x_{2i-1}(s)} = \frac{2 - \tau_i s}{2 + \tau_i s}. \tag{4}$$

It can be seen from (3) and (4) that the $x_{(2i-1)td}$ is the approximation of the delayed signal $\bar{x}_{(2i-1)td}$. From (1) and the equation (4), we obtain

$$\dot{x}_{(2i-1)td} = -\frac{2}{\tau_i}x_{(2i-1)td} + \frac{2}{\tau_i}x_{2i-1} - x_{2i}, i = 1, 2, 3. \tag{5}$$

Denote

$$
\begin{aligned}
X_i &= \begin{pmatrix} x_{(2i-1)td} \\ x_{2i-1} \\ x_{2i} \end{pmatrix},\ A_i = \begin{pmatrix} -\frac{2}{\tau_i} & \frac{2}{\tau_i} & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},\\
i &= 1, 2, 3,\\
g &= \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix},\ u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix},\\
B_1(x_5) &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\sin x_5 & \varepsilon\cos x_5 \end{pmatrix},\\
B_2(x_5) &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos x_5 & \varepsilon\sin x_5 \end{pmatrix},\ B_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}.
\end{aligned} \tag{6}
$$

Under the coordinate transformation of (X_1, X_2, X_3), the system (1) with measurement delays can be transformed into the following form without any time delay:

$$
\begin{aligned}
\dot{X}_i &= A_i X_i + B_i(x_5)u + (i-1)g,\ i = 1, 2,\\
\dot{X}_3 &= A_3 X_3 + B_3 u.
\end{aligned} \tag{7}
$$

The approximate measurement outputs are

$$y_i = CX_i,\ i = 1, 2, \tag{8}$$

where $C = (1, 0, 0)$.

Since $x_{2i-1}, x_{2i},\ i = 1, 2, 3$, are unaccessible, and to avoid noise-sensitivity caused by reduced-order observer, the following *full-order observer* is designed for the purpose of the tracking control:

$$
\begin{aligned}
\dot{\hat{X}}_i &= A_i\hat{X}_i + B_i(\hat{x}_5)u + (i-1)g + G_iC(X_i - \hat{X}_i),\\
i &= 1, 2,\\
\dot{\hat{X}}_3 &= A_3\hat{X}_3 + B_3u + G_3C(X_3 - \hat{X}_3).
\end{aligned} \tag{9}
$$

where $G_i = (g_{i1}, g_{i2}, g_{i3})^T$, $1 \leq i \leq 3$, are selected to make $\tilde{A}_{0i} = A_i - G_iC$ be Hurwitz. Let the observer errors be defined as

$$\tilde{X}_i = X_i - \hat{X}_i, i = 1, 2, 3. \tag{10}$$

And subtracting (9) from (7), we have

$$
\begin{aligned}
\dot{\tilde{X}}_i &= \tilde{A}_{0i}\tilde{X}_i + [B_i(x_5) - B_i(\hat{x}_5)]u,\ i = 1, 2,\\
\dot{\tilde{X}}_3 &= \tilde{A}_{03}\tilde{X}_3,
\end{aligned} \tag{11}
$$

where

$$\tilde{X}_i = \left(\tilde{x}_{(2i-1)td}, \quad \tilde{x}_{2i-1}, \quad \tilde{x}_{2i} \right)^T,$$
$$\tilde{A}_{0i} = \begin{pmatrix} -\frac{2}{\tau_i} - g_{i1} & \frac{2}{\tau_i} & -1 \\ -g_{i2} & 0 & 1 \\ -g_{i3} & 0 & 0 \end{pmatrix}, \ i = 1, 2, 3.$$

Define

$$\phi_i = x_{(2i-1)td} + x_{2i-1}, \ i = 1, 2, \quad (12)$$

then

$$\dot{\phi}_i = \frac{2}{\tau_i}\phi_i - \frac{4}{\tau_i}x_{(2i-1)td}. \quad (13)$$

The control aim is to let output $y_i = x_{(2i-1)td} \to y_{id}$, so ϕ_{id}, the desired value of ϕ_i, satisfies

$$\dot{\phi}_{id} = \frac{2}{\tau_i}\phi_{id} - \frac{4}{\tau_i}y_{id}. \quad (14)$$

Since $\frac{2}{\tau_i} > 0$, the system in Eq.(14) is non-minimum phase, a stable numerical solution can be obtained via output regulation theory (Isidori and Byrnes 1990), detailed solution will be given in Section 5. By noticing (12), the desired value of x_{2i-1} is $\phi_{id} - y_{id}$, thus the corresponding desired value of x_{2i} is $\dot{\phi}_{id} - \dot{y}_{id}$.

Further, define the following error coordinate transformation

$$\begin{aligned} e_{(2i-1)td} &= \hat{x}_{(2i-1)td} - y_{id}, \\ e_{2i-1} &= \hat{x}_{2i-1} - (\phi_{id} - y_{id}), \\ e_{2i} &= \hat{x}_{2i} - (\dot{\phi}_{id} - \dot{y}_{id}), \ i = 1, 2, \\ \eta_0 &= \hat{x}_{5td} + \hat{x}_5, \\ \eta_1 &= \hat{x}_5, \\ \eta_2 &= \varepsilon\hat{x}_6 - e_2\cos\hat{x}_5 - e_4\sin\hat{x}_5, \end{aligned} \quad (15)$$

whose time derivative yields

$$\begin{aligned} \dot{e}_{(2i-1)td} &= -\frac{2}{\tau_i}e_{(2i-1)td} + \frac{2}{\tau_i}e_{2i-1} - e_{2i} + \varphi_{id}, \\ \dot{e}_{2i-1} &= e_{2i} + g_{i2}\tilde{x}_{(2i-1)td}, \\ \dot{e}_{2i} &= v_i + g_{i3}\tilde{x}_{(2i-1)td}, \qquad i = 1, 2, \\ \dot{\eta}_0 &= -\frac{2}{\tau_3}\eta_0 + \frac{4}{\tau_3}\eta_1 + (g_{31} + g_{32})\tilde{x}_{5td}, \\ \dot{\eta}_1 &= \frac{1}{\varepsilon}(\eta_2 + e_2\cos\eta_1 + e_4\sin\eta_1) + g_{32}\tilde{x}_{5td}, \\ \dot{\eta}_2 &= \frac{1}{\varepsilon}(\eta_2 + e_2\cos\eta_1 + e_4\sin\eta_1) \\ &\quad (e_2\sin\eta_1 - e_4\cos\eta_1) + \\ &\quad (\ddot{\phi}_{1d} - \ddot{y}_{1d} - g_{13}\tilde{x}_{1td})\cos\eta_1 + \\ &\quad (1 + \ddot{\phi}_{2d} - \ddot{y}_{2d} - g_{23}\tilde{x}_{3td})\sin\eta_1 + \\ &\quad [(e_2\sin\eta_1 - e_4\cos\eta_1)g_{32} + \varepsilon g_{33}]\tilde{x}_{5td}. \end{aligned} \quad (16)$$

where $\varphi_{id} = -\frac{4}{\tau_i}y_{id} + \frac{2}{\tau_i}\phi_{id} - \dot{\phi}_{id} + g_{i1}\tilde{x}_{(2i-1)td}$, and the new inputs v_i is defined as

$$\begin{aligned} v_1 &= -u_1\sin\hat{x}_5 + \varepsilon u_2\cos\hat{x}_5 - (\ddot{\phi}_{1d} - \ddot{y}_{1d}), \\ v_2 &= u_1\cos\hat{x}_5 + \varepsilon u_2\sin\hat{x}_5 - (\ddot{\phi}_{2d} - \ddot{y}_{2d}) - 1. \end{aligned} \quad (17)$$

The internal dynamics expressed by the last three equations in (16) can be described as

$$\dot{\eta} = \Phi(\eta, e, \tilde{X}, Y_d), \quad (18)$$

where

$$\begin{aligned} \eta &= (\eta_0, \eta_1, \eta_2)^T, \ e = (e_{1td}, e_1, e_2, e_{3td}, e_3, e_4)^T, \\ \tilde{X} &= (\tilde{x}_{1td}, \tilde{x}_{3td}, \tilde{x}_{5td})^T, \ Y_d = (\ddot{\phi}_{1d}, \ddot{y}_{1d}, \ddot{\phi}_{2d}, \ddot{y}_{2d})^T. \end{aligned}$$

Rewriting (18) by separating its linear part from its non-linear part yields

$$\dot{\eta} = A_\eta\eta + A_e e + q(\eta, e, \tilde{X}, Y_d), \quad (19)$$

where

$$A_\eta = \frac{\partial\Phi(\eta, e, \tilde{X}, Y_d)}{\partial\eta}(0,0,0,0) = \begin{pmatrix} -\frac{2}{\tau_3} & \frac{4}{\tau_3} & 0 \\ 0 & 0 & \frac{1}{\varepsilon} \\ 0 & 1 & 0 \end{pmatrix},$$
$$A_e = \frac{\partial\Phi(\eta, e, \tilde{X}, Y_d)}{\partial e}(0,0,0,0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\varepsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$q(\eta, e, \tilde{X}, Y_d) = \Phi(\eta, e, \tilde{X}, Y_d) - A_e e - A_\eta\eta = (q_0, q_1, q_2)^T,$$

the specific form of q shows that

$$\begin{aligned} q_0 &= (g_{31} + g_{32})\tilde{x}_{5td}, \\ q_1 &= \frac{1}{\varepsilon}(e_2\cos\eta_1 + e_4\sin\eta_1 - e_2) + g_{31}\tilde{x}_5, \\ q_2 &= \frac{1}{\varepsilon}(\eta_2 + e_2\cos\eta_1 + e_4\sin\eta_1)(e_2\sin\eta_1 - e_4\cos\eta_1) \\ &\quad + (\ddot{\phi}_{1d} - \ddot{y}_{1d} - g_{13}\tilde{x}_{1td})\cos\eta_1 \\ &\quad + (1 + \ddot{\phi}_{2d} - \ddot{y}_{2d} - g_{23}\tilde{x}_{3td})\sin\eta_1 \\ &\quad + [(e_2\sin\eta_1 - e_4\cos\eta_1)g_{32} + \varepsilon g_{33}]\tilde{x}_{5td} - \eta_1. \end{aligned} \quad (20)$$

If $\varepsilon > 0$, the matrix A_η is non-Hurwitz, the internal dynamics is asymptotically unstable. To curb this phenomenon, a proper control is absolutely necessary, the way out is to force the unstable internal dynamics η to track its bounded IID η_d, which can be obtained by setting $e, \tilde{X}$ to their desired states in (21). Under ideal conditions, $\tilde{X} = 0$, and $e = 0$. Consequently, the differential equation about η_d appears

$$\begin{pmatrix} \dot{\eta}_{0d} \\ \dot{\eta}_{1d} \\ \dot{\eta}_{2d} \end{pmatrix} = \begin{pmatrix} -\frac{2}{\tau_3} & \frac{4}{\tau_3} & 0 \\ 0 & 0 & \frac{1}{\varepsilon} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{0d} \\ \eta_{1d} \\ \eta_{2d} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\psi, \quad (21)$$

where $\psi = (\ddot{\phi}_{1d} - \ddot{y}_{1d})\cos\eta_{1d} + (1 + \ddot{\phi}_{2d} - \ddot{y}_{2d})\sin\eta_{1d} - \eta_{1d}$. Note that (21) has no stable numerical solutions due to the non-Hurwitz property of A_η for $\varepsilon > 0$. However, this does not mean that a bounded solution cannot be found for such an unstable system (Al-Hiddabi and McClamroch 2002). In fact, under suitable assumptions, via the noncausal stable inversion approach in (Devasia, Chen and Paden 1996) or the SSC method in (Shkolnikov and Shtessel 2002), a bounded solution can be computed. It is worth noting that stable inversion approach is of limited practical use due to the fact that the desired trajectories and their any changes must be exactly known in advance, the offline pre-computing procedure is conducted backward in time, and the resulting IID is noncausal. On the contrary, the SSC method does not necessarily require the future information of the desired trajectories, the online solving procedure for bounded IID is performed forward in time. Therefore, we turn to the SSC method to get the causal IID of (21) that will be given in Section 5 in detail.

Let

$$\tilde{\eta}_i = \eta_i - \eta_{id},\ i = 0, 1, 2. \tag{22}$$

And subtracting (21) from (19) results in

$$\begin{aligned}
\dot{\tilde{\eta}}_0 &= -\frac{2}{\tau_3}\tilde{\eta}_0 + \frac{4}{\tau_3}\tilde{\eta}_1 + \tilde{q}_0, \\
\dot{\tilde{\eta}}_1 &= \frac{1}{\varepsilon}e_2 + \frac{1}{\varepsilon}\tilde{\eta}_2 + \tilde{q}_1, \\
\dot{\tilde{\eta}}_2 &= \tilde{\eta}_1 + \tilde{q}_2,
\end{aligned} \tag{23}$$

where $\tilde{q}_0 = q_0$, $\tilde{q}_1 = q_1$, $\tilde{q}_2 = q_2 - \psi$.

Then the system (16) whose last three equations are replaced by (23) can be rewritten as

$$\begin{aligned}
\dot{e}_{(2i-1)td} &= \dot{\hat{x}}_{(2i-1)td} - \dot{y}_{id} \\
&= -\frac{2}{\tau_i}e_{(2i-1)td} + \frac{2}{\tau_i}e_{2i-1} - e_{2i} + \varphi_{id}, \\
\dot{e}_{2i-1} &= \dot{\hat{x}}_{2i-1} - (\dot{\phi}_{id} - \dot{y}_{id}) \\
&= e_{2i} + g_{i2}\tilde{x}_{(2i-1)td}, \\
\dot{e}_{2i} &= \dot{\hat{x}}_{2i} - (\ddot{\phi}_{id} - \ddot{y}_{id}) \\
&= v_i + g_{i3}\tilde{x}_{(2i-1)td}, \qquad i = 1, 2,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{\tilde{\eta}}_0 &= -\frac{2}{\tau_3}\tilde{\eta}_0 + \frac{4}{\tau_3}\tilde{\eta}_1 + \tilde{q}_0, \\
\dot{\tilde{\eta}}_1 &= \frac{1}{\varepsilon}e_2 + \frac{1}{\varepsilon}\tilde{\eta}_2 + \tilde{q}_1, \\
\dot{\tilde{\eta}}_2 &= \tilde{\eta}_1 + \tilde{q}_2.
\end{aligned} \tag{25}$$

By defining $\eta_{e_{2i-1}} = e_{(2i-1)td} + e_{2i-1}$, $i = 1, 2$. The time derivative of $\eta_{e_{2i-1}}$

$$\dot{\eta}_{e_{2i-1}} = \frac{2}{\tau_i}\eta_{e_{2i-1}} - \frac{4}{\tau_i}e_{(2i-1)td} + \varphi_{id} + g_{i2}\tilde{x}_{(2i-1)td},$$

which means that the subsystem (24) is non-minimum phase.

From (19), we see that the internal dynamics (25) is linear relative with e_{1td}, e_1, e_2, and linear irrelative with e_{3td}, e_3, e_4. Thus the system (27) can be decomposed into the following three parts: non-minimum phase part 1

$$\begin{aligned}
\dot{e}_{1td} &= -\frac{2}{\tau_1}e_{1td} + \frac{2}{\tau_1}e_1 - e_2 + \varphi_{1d}, \\
\dot{e}_1 &= e_2 + g_{12}\tilde{x}_{1td}, \\
\dot{e}_2 &= v_1 + g_{13}\tilde{x}_{1td}, \\
\dot{\tilde{\eta}}_1 &= \frac{1}{\varepsilon}e_2 + \frac{1}{\varepsilon}\tilde{\eta}_2 + \tilde{q}_1, \\
\dot{\tilde{\eta}}_2 &= \tilde{\eta}_1 + \tilde{q}_2.
\end{aligned} \tag{26}$$

non-minimum phase part 2

$$\begin{aligned}
\dot{e}_{3td} &= -\frac{2}{\tau_2}e_{3td} + \frac{2}{\tau_2}e_3 - e_4 + \varphi_{2d}, \\
\dot{e}_3 &= e_4 + g_{22}\tilde{x}_{3td}, \\
\dot{e}_4 &= v_2 + g_{23}\tilde{x}_{3td},
\end{aligned} \tag{27}$$

and minimum phase part

$$\dot{\tilde{\eta}}_0 = -\frac{2}{\tau_3}\tilde{\eta}_0 + \frac{4}{\tau_3}\tilde{\eta}_1 + \tilde{q}_0. \tag{28}$$

Now, the tracking problem for the original system (1) with measurement delays has been approximately converted into a stabilization problem for the new subsystems (26) and (27) without any time delay. The decomposition technique (Al-Hiddabi and McClamroch 2002) enables us to design control laws for the two non-minimum phase subsystems, respectively, meanwhile the minimum phase part (28) which is irrelevant to v_i can be left alone when designing control laws.

III. Control law design

By defining $z_1 = (e_{1td}, e_1, e_2, \tilde{\eta}_1, \tilde{\eta}_2)^T$, the part (26) can be rewritten as

$$\dot{z}_1 = A_{z_1}z_1 + b_1 v_1 + W_1, \tag{29}$$

where

$$\begin{aligned}
A_{z_1} &= \begin{pmatrix} -\frac{2}{\tau_1} & \frac{2}{\tau_1} & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\varepsilon} & 0 & \frac{1}{\varepsilon} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix},\ b_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \\
W_1 &= \left(\varphi_{1d}, g_{12}\tilde{x}_{1td}, g_{13}\tilde{x}_{1td}, \tilde{q}_1, \tilde{q}_2\right)^T.
\end{aligned}$$

By the same way, define $z_2 = (e_{3td}, e_3, e_4)^T$, the part (27) appears as

$$\dot{z}_2 = A_{z_2}z_2 + b_2 v_2 + W_2, \tag{30}$$

where

$$\begin{aligned}
A_{z_2} &= \begin{pmatrix} -\frac{2}{\tau_2} & \frac{2}{\tau_2} & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, b_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\
W_2 &= \left(\varphi_{2d}, g_{22}\tilde{x}_{3td}, g_{23}\tilde{x}_{3td}\right)^T.
\end{aligned}$$

Replacing $v_i = v_{nli} - K_i z_i$, $i = 1, 2$, in (29) and (30) yields

$$\dot{z}_i = A_{0i}z_i + b_i v_{nli} + W_i, \tag{31}$$

where K_i is selected so that $A_{0i} = A_{z_i} - b_i K_i$ is a Hurwitz matrix; hence, for any given symmetric positive definite matrix Q_i, there exists a unique symmetric positive definite matrix P_i satisfying the Lyapunov equation

$$A_{0i}^T P_i + P_i A_{0i} = -Q_i. \tag{32}$$

Pre-multiplying the vector $z_i^T P_i$ on both sides of (31) results in

$$z_i^T P_i \dot{z}_i = z_i^T P_i A_{0i} z_i + z_i^T P_i b_i v_{nli} + z_i^T P_i W_i. \tag{33}$$

Because $z_i^T P_i b_i v_{nli}$ and $z_i^T P_i W_i$ are all scalars, we select

$$z_i^T P_i b_i v_{nli} = -z_i^T P_i W_i. \tag{34}$$

The minimum-norm solution can be obtained as

$$v_{nli} = -\frac{(z_i^T P_i b_i)^T z_i^T P_i W_i}{(z_i^T P_i b_i)(z_i^T P_i b_i)^T} = -\frac{\delta_i}{\sigma_i}, \tag{35}$$

where for convenience, we use δ_i and σ_i to denote the numerator and denominator of the above equation, respectively. In order to avoid control singularity problem, we redesign the control law v_{nli} as

$$v_{nli}=\begin{cases}-\frac{\delta_i}{\sigma_i}, & \sigma_i>\epsilon,\\ -\frac{\delta_i}{\sigma_i}\tanh^2(\frac{\lambda\sigma_i}{\epsilon}), & \sigma_i\le\epsilon,\end{cases} \tag{36}$$

where ϵ is a small positive constant.

By noticing (17), the practical inputs are obtained via the input transformation

$$\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}-\sin\hat{x}_5 & \varepsilon\cos\hat{x}_5\\ \cos\hat{x}_5 & \varepsilon\sin\hat{x}_5\end{pmatrix}^{-1}\times\begin{pmatrix}v_1+\ddot{\phi}_{1d}-\ddot{y}_{1d}\\ v_2+\ddot{\phi}_{2d}-\ddot{y}_{2d}+1\end{pmatrix}. \tag{37}$$

IV. STABILITY ANALYSIS

In this section, we present the boundness analysis of all the signals of the closed-loop system.

For the Hurwitz matrix $\tilde{A}_{0i}$ and any given symmetric positive definite matrix $\tilde{Q}_i$, there exists a unique symmetric positive definite matrix $\tilde{P}_i$ satisfying the Lyapunov equation

$$\tilde{A}_{0i}^T\tilde{P}_i+\tilde{P}_i\tilde{A}_{0i}=-\tilde{Q}_i,\ i=1,2,3. \tag{38}$$

Select the following Lyapunov function

$$V_i=\tilde{X}_i^T\tilde{P}_i\tilde{X}_i,\ i=1,2,3,\ V_o=\sum_{i=1}^{2}V_i+\gamma V_3. \tag{39}$$

where γ is a positive constant.

Due to space limitations, the detailed stability analysis is omitted.

V. SIMULATION RESULTS

A. Solving ϕ_d via output regulation theory

The desired output trajectories are $y_{1d}=R\cos(\omega t)$, $y_{2d}=R\sin(\omega t)$. The sine signals can be generated by the exosystem

$$\dot{w}=Sw, S=\begin{pmatrix}0 & \omega\\ -\omega & 0\end{pmatrix}, \tag{40}$$

where $w=(w_1,w_2)^T$, and thus $y_{1d}=RC_1w$, $y_{2d}=RC_2w$, where $C_1=(0,1)$, $C_2=(1,0)$.

Focusing on the linear equation (14), define $\phi_{id}=\Pi_iw$, the solution can be obtained via output regulation theory

$$\begin{aligned}\phi_{1d} &= R\begin{pmatrix}\frac{-4\tau_1\omega}{4+\tau_1^2\omega^2} & \frac{8}{4+\tau_1^2\omega^2}\end{pmatrix}w,\\ \phi_{2d} &= R\begin{pmatrix}\frac{8}{4+\tau_2^2\omega^2} & \frac{4\tau_2\omega}{4+\tau_2^2\omega^2}\end{pmatrix}w.\end{aligned} \tag{41}$$

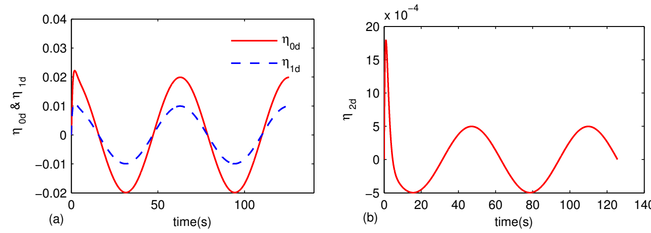

Fig. 2. (a) The desire IID η_{0d} and η_{1d}. (b) The desired IID η_{2d}.

B. VTOL IID ($\varepsilon=0.5$)

By (21), the IID η_d of the VTOL aircraft can be rewritten in the following form

$$\dot{\eta}_{0d}=-\frac{2}{\tau_3}\eta_{0d}+\frac{4}{\tau_3}\eta_{1d},\ \dot{\bar{\eta}}_d=A_{\bar{\eta}}\bar{\eta}_d+b\psi, \tag{42}$$

where

$$\bar{\eta}_d=\begin{pmatrix}\eta_{1d}\\ \eta_{2d}\end{pmatrix},\ A_{\bar{\eta}}=\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix},\ b=\begin{pmatrix}0\\1\end{pmatrix}.$$

The characteristic polynomial of the exosystem (40)

$$P(\lambda)=|\lambda I-S|=\lambda^2+\omega^2, \tag{43}$$

from which we can determine the polynomial order $k=2$, the coefficients $p_1=0, p_0=\omega^2$. By setting the desired eigenvalue $s_{1,2}=-1$, the corresponding characteristic equation $(s+1)^2=s^2+2s+1=0$; hence, the parameters $c_1=2, c_0=1$. According to the formula in [1], we get

$$\begin{aligned}P_1 &= (I+c_1Q_1^{-1}+c_0Q_1^{-2})(I+p_1Q_1^{-1}+p_0Q_1^{-2})^{-1}-I\\ &= \frac{1}{1+0.5\omega^2}\begin{pmatrix}0.5(1-\omega^2) & 2\\ 1 & 0.5(1-\omega^2)\end{pmatrix},\end{aligned} \tag{44}$$

$$\begin{aligned}P_0 &= c_0Q_1^{-1}-(P_1+I)p_0Q_1^{-1}\\ &= \frac{1}{1+0.5\omega^2}\begin{pmatrix}-\omega^2 & 1-\omega^2\\ 0.5(1-\omega^2) & -\omega^2\end{pmatrix},\end{aligned} \tag{45}$$

where $Q_1=A_{\bar{\eta}}$.

The IID $\bar{\eta}_d$ can be solved from the following matrix differential equation

$$\ddot{\bar{\eta}}_d+c_1\dot{\bar{\eta}}_d+c_0\bar{\eta}_d=-(P_1\dot{\theta}_d+P_0\theta_d), \tag{46}$$

where $\theta_d=(0,1)^T\psi$. When $R=1$, $\omega=0.1$, the IID η_d of the VTOL aircraft solved via the SSC method is given in Fig.2.

C. Simulation results

The coupling coefficient ε is selected to be 0.5, which means that the VTOL aircraft is a strongly non-minimum phase system. The desired output trajectory $y_{1d}=R\cos(\omega t)$, $y_{2d}=R\sin(\omega t)$, where the amplitude R and the frequency ω switch, respectively from 1 to 1.2 and 0.1 to 0.2 at random time 62.8+5·rand(1). Such a situation may occur in the case of obstacle avoidance. The initial conditions are chosen as $x(0)=(1.5,0,-0.5,0.2,0.28,0)^T$.The observer initial conditions are chosen as $\hat{x}(0)=(0,1.3,0.1,0,-0.3,0.1,0.2,0.1,0.1)^T$. The observer gain

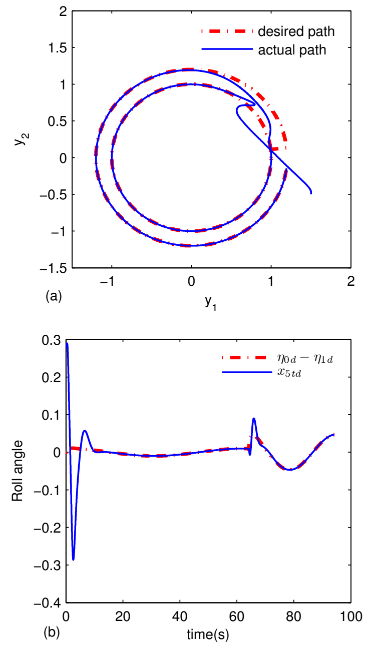

Fig. 3. (a) The desired path and the actual path. (b) The desired roll angle $\eta_{0d}-\eta_{1d}$, the actual roll angle x_{5d}.

$G_1 = G_2 = (-1, 0.8125, 0.25)^T$, such that the eigenvalues of $\tilde{A}_{01}$ and $\tilde{A}_{02}$ are all placed at -1; $G_3 = (-1, 2.97, 2.7)^T$, which set the eigenvalues of $\tilde{A}_{03}$ at -3. The controller gains of the non-minimum phase subsystems are $K_1 = (-0.2857, -3.7143, 6, 10.7143, 15.1429)^T$ and $K_2 = (-3.375, 3.625, -1)^T$, such that the eigenvalues of A_{01} and A_{02} are all placed at -2 and -1. The corresponding matrix $\tilde{P}_i$ is computed from (32) with $\tilde{Q}_i = I$ and $\epsilon = 0.1$, $\lambda = 6$.

From the simulation results, we can conclude that although the measurement delays exist in the feedback channels, not only the perfect output tracking is achieved, but also the unstable internal dynamics, the delayed roll angle of VTOL aircraft in physical meaning, is forced to follow its corresponding IID which is subject to the desired outputs and stands for the desired roll angle. The simulation also shows that owing to the causality of the IID solved via the SSC method adopted by our scheme, the roll angle is able to adapt the change of the desired roll angle when some unexpected changes of desired trajectories occur. To the end, it is illustrated that the proposed control scheme based on the application of Pade approximation technique to tackle delays is valid and feasible.

VI. CONCLUSION

We have presented a nonlinear controller to achieve approximate trajectory tracking for a velocity-sensorless VTOL aircraft in the presence of measurement delays. The control development was based on Pade approximation technique to deal with time delay, the construction of full-order observer, error coordinate transformation, and system decomposition. The resulting controller not only forced the VTOL aircraft to track the desired trajectories, but also drived the unstable internal dynamics to follow the causal IID. It has been proved that the overall closed-loop system is asymptotically stable. Simulation results have verified the validity of the proposed controller. Finally, we point out that in this paper, the time delay τ is exactly known. However, in actual control systems, the time delay τ may include delay perturbation, i.e., $\tau = \tau_0 + \Delta\tau$, where τ_0 is the known nominal time delay, $\Delta\tau$ is the unknown delay perturbation. Current work is under way to consider this case of time delay, which is still an open problem.

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