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Multi-period Optimal Energy Procurement and Demand Response in Smart Grid with Uncertain Supply

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Abstract— **We propose a simple model that integrates two-period electricity markets, uncertainty in renewable generation, and real-time dynamic demand response. A load-serving entity decides its day-ahead procurement to optimize expected social welfare a day before energy delivery. At delivery time when renewable generation is realized, it sets prices to manage demand and purchase additional power on the real-time market, if necessary, to balance supply and demand. We derive the optimal day-ahead decision, propose real-time demand response algorithm, and study the effect of volume and variability of renewable generation on the social welfare.**

I. INTRODUCTION

A. Motivation

There is a large literature on various forms of load side management in the electricity grid from the classical direct load control to the more recent real-time pricing [9], [1]. Direct load control in particular has been practised for a long time and optimization methods have been proposed to minimize generation cost e.g. [8], [11], [5], maximize utility's profit e.g. [19], or minimize deviation from users' desired consumptions e.g. [7], [21]. Almost all demand response programs today target large industrial or commercial users, or, in the case of residential users, a small number of them, for two, among others, important reasons. First, demand side management is invoked rarely to mostly cope with a large correlated demand spike due to weather or a supply shortfall due to faults, e.g., during a few hottest days in summer. Second, the lack of ubiquitous two-way communication in the current infrastructure prevents the participation of a large number of diverse users with heterogeneous and time-varying consumption requirements. Both reasons favor a simple and static mechanism involving a few large users that is sufficient to deal with the occasional need for load control, but both reasons are changing.

Renewable sources can fluctuate rapidly and by large amounts. As their penetration continues to grow, the need for regulation services and operating reserves will increase, e.g., [17], [4], [18]. This can be provided by additional peaker units, at a higher cost, or supplemented by real-time demand response [15], [23], [22], [4], [25]. We believe that demand response will not only be invoked to shave peaks, but will increasingly be called upon to improve security and reduce reserves by adapting elastic loads to intermittent and random renewable generation [20]. Indeed, [4], [2], [24] advocates the creation of a distribution/retail market to encourage greater load side participation as an alternative source for fast reserves. Such application however will require a much faster and more dynamic demand response than practised today. This will be enabled in the coming decades by the large-scale deployment of a sensing, control, and two-way communication infrastructure, including the flexible AC transmission systems, the GPS-synchronized phasor measurement units, and the advanced metering infrastructure, that is currently underway around the world [12].

Demand response in such context must allow the participation of a large number of users, and be dynamic and distributed. Such dynamic adaptation is being practised everyday on the Internet in the form of congestion control. Although the grid and the Internet are different in their engineering, economic, and regulatory structures, the precedence on the Internet lends hope to a much bigger scale and more dynamic and distributed demand response architecture and its benefit to grid operation. *Ultimately it will be cheaper to use photons than electrons to deal with a power shortage.* Our goal is to design algorithms for such a system.

B. Summary

Specifically we consider a set of users that are served by a single load-serving entity (LSE). The LSE may represent a regulated monopoly like most utility companies in the United States today, or a non-profit cooperative that serves a community of end users. Its purpose is (possibly regulated) to promote the overall system welfare. The LSE procures electricity on the wholesale electricity markets (e.g., day-ahead, real-time balancing, and ancillary services) and renewable sources and sells it on the retail market to end users. Each user, on the other hand, has a set of appliances (electric vehicle, air conditioner, lighting, battery, etc.) which can adapt their demand. The user's energy management system is to decide how much power to consume in each period of a day (i.e., demand response). The LSE is to make energy procurement decisions, including how much capacity it should procure a day ahead and, when the random renewable energy is realized at real time, how much balancing power to purchase on the spot market to meet the aggregate demand. The overall goal is to maximize the social welfare.

Our model captures three important features:

- **Uncertainty.** Part of the electricity supply is from renewable sources such as wind and solar, and thus uncertain.
- **Supply and demand.** LSE's supply decisions and the users' consumption decisions must be jointly optimized.
- **Two timescale.** The LSE must procure capacity on the day-ahead wholesale market while user consumptions

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should be adapted in real time to mitigate supply uncertainty.

Hence the key is the coordination of day-ahead procurement and real-time demand response over two timescales in the presence of supply uncertainty. Moreover, the optimal decisions must be computed jointly by the LSE and the users as the necessary information is distributed among them. This paper focuses on the design of such distributed algorithms to maximize social welfare, and the impact of uncertainty on the optimal welfare.

C. Other related work

A large literature exists on demand response. Besides those cited above, more recent works are surveyed in [6]. The models that are closest to ours, developed independently, are [4], [10]. All our models include random renewable generation, consider both day-ahead and real-time markets, and allow demand response, but our objectives and system operations are quite different. [4] advocates the establishment of a retail market where users (e.g., PHEVs) can buy power from or sell reserves, in the form of demand response capability, to their LSE. The model in [10] includes non-elastic users that are price non-responsive, and elastic users that can either leave the system or defer their consumptions when the electricity price is high. The goal is to maximize LSE's profit over day-ahead procurement, day-ahead prices for non-elastic users, and real-time prices for elastic users.

II. MODEL AND PROBLEM FORMULATION

Consider a set $\mathcal{N}$ of N users that are served by a single load-serving entity (LSE). We use a discrete-time model with a finite horizon that models a day. Each day is divided into T periods, indexed by $t \in \mathcal{T} = \{1, 2, \cdots, T\}$. The duration of a period can be 5, 15, or 60 mins, corresponding to the time resolution at which energy dispatch or demand response decisions are made.

A. Model

Without loss of generality, we assume each user $i \in \mathcal{N}$ operates a single appliance (or i may index appliances rather than users) so we don't need another subscript to index appliances. Let $q_i(t)$ denote its energy consumption in period $t \in \mathcal{T}$, and q_i the vector $(q_i(t), \forall t)$ over the whole day. An appliance i is characterized by:

- a utility function $U_i(q_i(t); t)$ that quantifies the utility that user i obtains from using appliance i and consuming $q_i(t)$ amount of energy in period t;
- a set of consumption constraints. For example,

$$\underline{q}_i(t) \leq q_i(t) \leq \overline{q}_i(t), \forall t \quad (1)$$

$$\sum_t q_i(t) \geq \overline{Q}_i \quad (2)$$

  i.e., the consumption in each period is in a certain range, and the total consumption must exceed $\overline{Q}_i$. If the appliance cannot use electricity in some period t, then we define $\underline{q}_i(t) = \overline{q}_i(t) = 0$.

Note that we can define different utility functions and consumption constraints to model a wide range of appliances. This is further discussed in Section II-B and [6]. This paper adopts the above version only to simplify exposition.

The LSE procures power for delivery in each period t, in two steps. First, one day in advance, it procures "day-ahead" capacities $P_d(t)$ for each period t, which incur capacity costs $c_d(P_d(t); t)$. This allows the LSE to use up to $P_d(t)$ amount of energy in period t of the following day, from the day-ahead capacity it has reserved. Let $P_o(t) \leq P_d(t)$ denote the amount of the day-ahead energy that the LSE actually uses the following day and $c_o(P_o(t); t)$ denote its cost (in addition to the capacity cost c_d). The renewable energy in each period t is a non-negative random variable $P_r(t)$ and it costs $c_r(P_r(t); t)$. Note that we do NOT assume that $P_r(t)$'s are independent across t. Instead, temporal correlation of $P_r(t)$'s is allowed. It is desirable to use as much renewable power as possible. For notational simplicity only, we assume $c_r(P_r; t) \equiv 0$ for all $P_r \geq 0$ and all t. At time t^- (real time), the random variable $P_r(t)$ is realized and used to satisfy demand. (If $P_r(t) > \sum_i q_i(t)$, we assume that the extra renewable energy can be "dumped". For example, the blade angle of the wind turbines can be adjusted to reduce generation.) The LSE satisfies any excess demand $[\sum_i q_i(t) - P_r(t)]_+$ by using energy $P_o(t)$, up to the day-ahead capacity $P_d(t)$ procured in advance. If there is still excess demand, the LSE procures the balance $P_b(t)$ from the real-time energy market, which incurs a cost $c_b(P_b(t); t)$. Hence the demands $q_i(t) \geq 0$ and the supplies $(P_d(t), P_r(t), P_o(t), P_b(t)) \geq 0$ must satisfy:

$$\sum_i q_i(t) \leq P_r(t) + P_o(t) + P_b(t)$$

$$P_o(t) \leq P_d(t)$$

We make the following assumptions:

A1: For each t, the utility functions $U_i(q_i)$ are concave increasing and continuously differentiable, and the cost functions $c_d(\cdot; t)$, $c_o(\cdot; t)$ and $c_b(\cdot; t)$ are convex increasing and continuously differentiable, with $c_d(0; t) = c_o(0; t) = c_b(0; t) = 0$.

A2: For each t, $c'_b(0; t) > c'_o(P_o; t), \forall P_o \geq 0$, i.e., the marginal cost of balancing energy is strictly higher than the marginal cost of day-ahead energy.

For example, if the constant unit price of $P_b(t)$ is higher than that of $P_o(t)$, then assumption A2 is satisfied. We also assume that $\underline{q}_i \geq 0$ for all i and $\overline{Q} \geq 0$.

The real-time decisions $(P_o(t), P_b(t))$ are made by the LSE so as to minimize the total cost, as follows. Given the demand vector $q(t) := (q_i(t), \forall i)$, let $Q(t) := \sum_i q_i(t)$ be the total demand and $\Delta(Q(t)) := Q(t) - P_r(t)$ the excess demand, in excess of the renewable generation $P_r(t)$. Note that $\Delta(Q(t))$ is a random variable in and before period $t-1$, but its realization is known to the LSE at time t^-. Assumption A2 implies that the LSE will use the balancing power only after the day-ahead power is exhausted, i.e., $P_b(t) > 0$ only if $\Delta(Q(t)) > P_d(t)$. Hence, given the excess demand $\Delta(Q(t))$ and the day-ahead capacity $P_d(t)$, the LSE's decision in period t that minimizes its total energy cost is:

$$\begin{aligned} P_o^*(t) &= [\Delta(Q(t))]_0^{P_d(t)} \\ P_b^*(t) &= [\Delta(Q(t)) - P_d(t)]_+ \end{aligned}$$

The total supply cost that the LSE incurs is then a function only of $P_d(t)$ and $Q(t)$:

$$c(Q(t), P_d(t); P_r(t), t) = c_d(P_d(t); t) + \\ c_o\left([\Delta(Q(t))]_0^{P_d(t)}; t\right) + c_b\left([\Delta(Q(t)) - P_d(t)]_+; t\right) \quad (3)$$

i.e., the total cost consists of the capacity cost c_d, the cost c_o of day-ahead energy, and the cost c_b of the real-time balancing energy.

B. Generalizations

Both the user and the supply models can be generalized [6]. Our results here can be extended to these more general models but we adopt the version above to simplify exposition. For example, instead of (1) and (2), the consumption constraints can take the form of a general linear inequality:

$$A_i q_i \le \eta_i$$

The utility functions U_i can be functions of the vectors $q_i := (q_i(t), \forall t)$ instead of $q_i(t)$ for each t, i.e., they are not necessarily separable in t. This generalized user model is quite flexible. In [6], by unifying several models in the literature, we show how various types of appliances [such as HVAC (heat, ventilation, air conditioner), refrigerator, and plug-in hybrid electric vehicles, batteries] can be modeled by these general utility functions and linear inequalities. For batteries the lower bound $\underline{q}_i$ may be negative. Furthermore each user can have multiple appliances, not just one.

For example, for the class of appliances that control the temperature, such as the air conditioner and refrigerator, the utility function should depend on how far is the controlled temperature differs from the desired temperature. The controlled temperature in slot t, in turn, could depend on the temperature in the previous slot, the electricity consumption in slot t, and the outside temperature. Reference [6] shows that with this formulation, the utility function is not separable in t, and the consumption constraints are still linear, but different from (1) and (2). As another example, the utility of an electric vehicle may only depend on the total consumption (i.e., total charge), but not the consumption in each slot as in section II-A.

On the supply side, assumption A2 can be relaxed. In that case the LSE chooses $(P_o^*(t), P_b^*(t))$ at time $t-$ that solves the problem:

$$c_s(\Delta(Q(t)), P_d(t); t) := \min_{P_o(t), P_b(t)} \{c_o(P_o(t); t) + c_b(P_b(t); t) \\ \mid P_b(t) \ge 0, P_o(t) + P_b(t) \ge \Delta(Q(t)), P_d(t) \ge P_o(t) \ge 0\}$$

with $\Delta(Q(t)) = Q(t) - P_r(t)$. The total cost is then

$$c(Q(t), P_d(t); P_r(t), t) := c_d(P_d(t); t) + c_s(\Delta(Q(t)), P_d(t); t).$$

C. Objective: welfare maximization

Recall that $q := (q(t),\ t \in \mathcal{T})$ and $Q(t) := \sum_i q_i(t)$. The social welfare of the day is the standard user utility minus supply cost:

$$\begin{aligned} W(q, P_d; P_r) &:= \sum_i \sum_{t=1}^{T} U_i(q_i(t); t) - \\ &\quad \sum_{t=1}^{T} c(Q(t), P_d(t); P_r(t), t) \end{aligned} \quad (4)$$

where $P_d := (P_d(t),\ t \in \mathcal{T})$ and $P_r := (P_r(t),\ t \in \mathcal{T})$. Recall that the LSE's objective is not to maximize its profit through selling electricity, but rather to maximize the expected social welfare. Note that the maximization of $E[W(q, P_d; P_r)]$ is over day-ahead procurement P_d and real-time consumption q in the presence of random renewable generation $P_r(t)$. It is critical that, in the presence of uncertainty, $q(t)$ should be decided after $P_r(t)$ have been realized at times t^- (i.e., real-time demand response). P_d however must be decided a day ahead before $P_r(t)$ are realized. Therefore, the day-ahead procurement and the real-time demand response must be coordinated over two timescales to maximize the expected welfare. In this paper, we will propose distributed algorithms to achieve or approximate the maximal expected welfare.

III. The case without time correlation

To gain some intuition, we first consider the simpler case without the constraint (2) that couples the consumption decisions $q(t)$ across time. In this case, maximizing the expected social welfare of the day reduces to separately maximizing the expected social welfare for each time period. This is equivalent to the case of $T = 1$. Welfare maximization for each period is (we drop t from the notation):

$$\max_{P_d \ge 0} \left\{ -c_d(P_d) + E \max_{q \in [\underline{q}, \overline{q}]} W_1(q; P_d, P_r) \right\} \quad (5)$$

where the real-time welfare, given decision P_d and realization of P_r, is:

$$\begin{aligned} W_1(q; P_d, P_r) &:= \sum_i U_i(q_i) - c_o\left([\Delta(Q)]_0^{P_d}\right) \\ &\quad -c_b\left([\Delta(Q) - P_d]_+\right) \end{aligned} \quad (6)$$

The expectation E in (5) is taken with respect to P_r. The order of maximizations and expectation in (5) reflects the fact that the decision P_d must be made a day ahead based on the distribution of P_r, but the consumption decisions q should be made in real time after P_r is realized. Given P_d and a realization of P_r, $W_1(q; P_d, P_r)$ is a deterministic function of q. Hence our problem decomposes into two subproblems:

1) *Real-time demand response:* optimize real-time welfare W_1 over consumptions q given P_d, P_r:

$$\tilde{W}(P_d; P_r) := \max_{q \in [\underline{q}, \overline{q}]} W_1(q; P_d, P_r). \quad (7)$$

2) *Day-ahead capacity procurement:* maximize expected welfare over P_d:

$$\max_{P_d \geq 0} \{ -c_d(P_d) + E[\tilde{W}(P_d; P_r)] \}. \quad (8)$$

We now consider each subproblem in turn. For real-time demand response, problem (7) is equivalent to

$$\begin{aligned} \tilde{W}(P_d; P_r) \quad = \quad & \max_{q, y_o, y_b} \{\sum_i U_i(q_i) - c_o(y_o) - c_b(y_b)\} \\ s.t. \quad & \underline{q}_i \leq q_i \leq \bar{q}_i, \forall i; y_o, y_b \geq 0, \\ & y_o \leq P_d, \\ & P_r + y_o + y_b \geq \sum_i q_i. \end{aligned} \quad (9)$$

Associate dual variables μ_1 and μ_2 with the last two constraints. Then a partial Lagrangian is

$$\begin{aligned} & \mathcal{L}(q, y_o, y_b; \mu_1, \mu_2) \\ = \; & \sum_i U_i(q_i) - c_o(y_o) - c_b(y_b) + \mu_1(P_d - y_o) \\ & + \mu_2(P_r + y_o + y_b - \sum_i q_i). \end{aligned} \quad (10)$$

So, a primal-dual algorithm to solve problem (9) is Algorithm 1. We make the following technical assumption.

A3: First, all utility functions U_i satisfies $|U_i'(q_i(t); t)| < V < \infty, \forall q_i(t) \in [\underline{q}_i(t), \bar{q}_i(t)], \forall t$. Second, there exists $P_{max} \geq \sum_i \bar{q}_i(t)$ such that $c_d'(P_{max}), c_o'(P_{max}), c_b'(P_{max}) < B < \infty$. This implies that one can support the maximal possible demand $\sum_i \bar{q}_i(t)$ using only day-ahead energy or real-time energy with finite marginal cost.

Algorithm 1: Given P_d and P_r, compute real-time consumption

Initially, every user lets $q_i^0 \in [\underline{q}_i, \bar{q}_i]$. The LSE lets $\mu_1^0 = \mu_2^0 = 0$, and $y_o^0 = y_b^0 = 0$. In iteration $k = 0, 1, 2, \ldots$, do the following.

1) Each user i computes q_i^{k+1}, and report it to the LSE through a communication network:

$$q_i^{k+1} \quad = \quad \left(q_i^k + \beta^k \cdot [U_i'(q_i^k) - \mu_2^k]\right)_{\underline{q}_i}^{\bar{q}_i}$$

where $\beta^k > 0$ is the step size. That is, the "price" posed to the users is μ_2^k.

2) The LSE computes $\mu_1^{k+1}, \mu_2^{k+1}, y_o^{k+1}, y_b^{k+1}$:

$$\begin{aligned} \mu_1^{k+1} \quad &= \quad [\mu_1^k + \beta^k(y_o^k - P_d)]_+, \\ \mu_2^{k+1} \quad &= \quad [\mu_2^k + \beta^k(\sum_i q_i^k - P_r - y_o^k - y_b^k)]_+, \\ y_o^{k+1} \quad &= \quad [y_o^k + \beta^k(-c_o'(y_o^k) - \mu_1^k + \mu_2^k)]_0^{P_{max}}, \\ y_b^{k+1} \quad &= \quad [y_b^k + \beta^k(-c_b'(y_b^k) + \mu_2^k)]_0^{P_{max}} \end{aligned}$$

where B and P_{max} are defined in Assumption A3. The LSE reports μ_2^{k+1} to the users.

With proper step sizes (e.g., $\beta^k = 1/(k+1)$) and under assumptions A1˜A3, Algorithm 1 converges to the set of optimal solutions and dual variables. More formally, we have the following.

Proposition 1: Let $\mathcal{B}$ be the set of saddle points $(q^*, y_o^*, y_b^*; \mu_1^*, \mu_2^*)$ of $\mathcal{L}(q, y_o, y_b; \mu_1, \mu_2)$. If the step sizes satisfy $\sum_k \beta^k = \infty$ and $\sum_k (\beta^k)^2 < \infty$ (e.g., $\beta^k = 1/(k+1)$) and assumptions A1˜A3 hold, then $(q^k, y_o^k, y_b^k; \mu_1^k, \mu_2^k)$ converges to the set $\mathcal{B}$ (i.e., $\lim_{k \to \infty} \min_{w \in \mathcal{B}} ||(q^k, y_o^k, y_b^k; \mu_1^k, \mu_2^k) - \mathbf{w}||_2 = 0$). By continuity , we have $W_1(q^k; P_d, P_r) \to \max_{q \in [\underline{q}, \bar{q}]} W_1(q; P_d, P_r)$.

Proof: The proof is similar to Theorem 3.1 in [26]. ■

Proposition 2: (i) $\tilde{W}(P_d; P_r)$ is concave in P_d.

(ii) Given P_d and P_r, let μ_1^* be an optimal dual variable associated with the constraint $y_o \leq P_d$ in (9). Then, with P_r fixed, μ_1^* is a subgradient of $\tilde{W}(P_d; P_r)$ at the point P_d.

Proof: For (i), since $\tilde{W}(P_d; P_r)$ is the optimal value of the convex optimization problem (9), it is concave in P_d [3]. For (ii), note that P_d is associated with the dual variable μ_1 only. So, the result follows from the standard sensitivity analysis [3] in convex optimization (see Eq. (5.57) in [3]). ■

Now, for day-ahead capacity procurement, the LSE decides P_d to maximize social welfare (i.e., solves problem (8)). A subgradient of the objective function in (8) is $E(\mu_1^*) - c_d'(P_d)$ (where μ_1^* depends on P_d, P_r). So, a stochastic subgradient algorithm is as follows. Algorithm 2 is run one day in advance by simulating the system (i.e., drawing samples of P_r).

Algorithm 2: Day-ahead energy

1) Initially, let $P_d^0 = 0$.
2) For $m = 0, 1, 2, \ldots$, independently generate a sample of P_r (denoted by P_r^m), and run Algorithm 1 to find μ_1^*, and denote it by μ_1^{*m}. Then, compute

$$P_d^{m+1} = \{P_d^m + \alpha^m[\mu_1^{*m} - c_d'(P_d^m)]\}_0^{P_{max}}$$

where $\alpha^m = 1/(m+1)$ is the step size.

Similar to Theorem 3.1 in [26], one can show that P_d^m converges to the set of optimal solutions with probability 1.

IV. THE CASE WITH TIME CORRELATION

Now we consider the case with the time correlation constraint (2). Other constraints in, for example, [6], can be treated similarly. With time correlations, the optimal real-time demand response policy is the solution of a dynamic program [13]. In general, the dynamic program is hard to solve explicitly. We propose the following online algorithm. The algorithm is optimal in certain cases and is an approximate algorithm otherwise [13].

Algorithm 3: Real-time demand response with uncertain renewable energy

We use P_d^* to denote the choice of day-ahead energy and $q^*(t) := (q_i^*(t), \forall i), t = 1, 2, \ldots, T$ to denote the choice of demand in slot t under Algorithm 3.

1) One day ahead, determine the day-ahead energy $P_d^*(t), t = 1, 2, \ldots, T$ as follows. Use a distributed

algorithm (similar to Algorithm 1) to solve the (deterministic) optimization problem

$$\begin{array}{ll} \max_{q,P_d\geq 0} & W(q,P_d;\bar{P}_r) \\ s.t. & (1),(2) \end{array} \quad (11)$$

where W is the welfare function defined in (4), $q = (q_i(t), \forall i, t)$, and $\bar{P}_r = E(P_r)$ with $P_r = (P_r(t), \forall t \in \mathcal{T})$. In other words, we maximize the social welfare assuming that the renewable energy is fixed at $\bar{P}_r$. Let the solution of (11) be $(\tilde{q}, \tilde{P}_d)$. Use $\tilde{P}_d$ as the day-ahead energy, i.e., let $P_d^* = \tilde{P}_d$.

2) Let $t = 1$.
3) In period t, determine the consumption of each user in this period as follows. Note that at this time $\{P_r(\tau), 0 \leq \tau \leq t\}$ have been observed by the LSE. So, the conditional distribution of $\{P_r(\tau), \tau > t\}$ is known. Denote $\bar{P}_r^t := E(P_r | P_r(\tau), \forall \tau \leq t)$.
Use a distributed algorithm (similar to Algorithm 1) to solve the following problem:

$$\begin{array}{ll} \max_{q} & W(q,P_d^*;\bar{P}_r^t) \\ s.t. & (1),(2) \\ & q_i(\tau) = q_i^*(\tau), \forall \tau < t, \forall i \end{array} \quad (12)$$

where $q_i^*(\tau), \tau < t$ is the consumption of user i in slot $\tau < t$ that is already decided in the earlier slot τ. That is, we maximize the social welfare, given the decisions already made before slot t (i.e., P_d^* and $q_i^*(\tau), \forall \tau < t, \forall i$) and the current $P_r(t)$, and assuming that future renewable energy is fixed at $\bar{P}_r^t(\tau), \tau > t$.
Let the solution of (12) be $\tilde{q}^t$. Use $\tilde{q}^t(t)$ as the consumption in slot t, i.e., let $q^*(t) = \tilde{q}^t(t)$. Finally, choose $P_o^*(t) = [\sum_i q_i^*(t) - P_r(t)]_0^{P_d^*(t)}$, and the real-time energy as $P_b^*(t) = [\sum_i q_i^*(t) - P_r(t) - P_d^*(t)]_+$.
4) If $t < T$, increment t and repeat step 3.

V. Impact of Renewable Energy on the Social Welfare

An important element in our model is the uncertain renewable energy. In the future, the penetration of renewable energy and its impact are expected to increase. In this section, we investigate how the statistics of the renewable energy affects the achievable social welfare in our model. For simplicity, we consider the case without time correlation, so we can focus on one time slot. (The results can be extended to the case with time correlation [13].) Assume that the renewable energy in one time slot is parametrized by $a \geq 0$ and $b \geq 0$ as follows.

$$P_r(a,b) := a \cdot \mu_r + b \cdot V_r \geq 0.$$

where $\mu_r > 0$ is a constant, and V_r is a zero-mean random variable. So, a and b indicate the mean and variance, respectively, of the renewable energy. In particular, $E[P_r(a,b)] = a\mu_r$, and $var[P_r(a,b)] = b^2 E[V_r^2]$.

Let $J^*(a,b)$ be the maximal expected welfare when the renewable energy is $P_r(a,b)$. We have the following results.

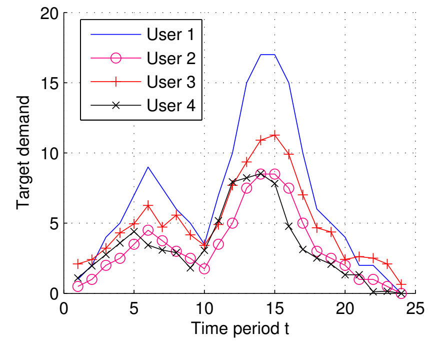

Fig. 1: Target demand profiles of the users

Proposition 3: (i) If $b \geq 0$ is fixed, $J^*(a,b)$ is non-decreasing with $a \geq 0$.
(ii) If $a \geq 0$ is fixed, $J^*(a,b)$ is non-increasing with $b \geq 0$.
(iii) Assume that $\mu_r + V_r \geq 0$, so that $P_r(s,s) = s \cdot (\mu_r + V_r) \geq 0, \forall s \geq 0$. Then $J^*(s,s)$ is non-decreasing with $s \geq 0$.

Remark: In other words, the maximal expected welfare increases with the mean, decreases with the variance, and increases with the scale of P_r (e.g., the farm size).

Proof: See [14]. ■

VI. Numerical Results

The setup is as follows. Let $T = 24$, representing 24 hours. The first time period is 8-9am, the second is 9-10am, and so on. The utility function of user i is $U_i(q_i) = \sum_{t=1}^T U_i(q_i(t);t) = -\sum_{t=1}^T [q_i(t) - y_i(t)]^2$ where $y_i(t)$ is user i's target consumption in slot t. That is, the further his actual demand profile $\{q_i(t)\}$ deviates from the target, the less is his utility. Fig. 1 shows the target demand profiles of $N = 4$ users in our simulation. The unit of energy is kWh. For each time period, assume that the cost functions are $c_d(P) = (P^2 + P)/2$, $c_o(P) = P/2$, and $c_b(P) = P^2/2 + 5P$.[1]

Assume that $P_r(t)$ is uniformly distributed between 0 and $2\bar{P}_r(t) > 0$, so that its mean is $E(P_r(t)) = \bar{P}_r(t)$. Also, $P_r(t)$'s are independent across t. The values of $(\bar{P}_r(t), \forall t)$ are (2, 3, 4, 5, 5, 6, 6, 7, 6, 5, 4, 3, 2, 2, 3, 4, 4, 4, 4, 3, 3, 2, 2, 2).

A. Without time correlation

In the case without time correlation, we impose no constraint on the total consumption of each user i. (The only constraint is that $q_i(t) \geq 0$.) This can model the scenario where users are willing to shed load instead of shifting load. We run Algorithm 2 for each time period. Fig. 2 (a) shows that the computed value of $P_d(6)$, the day-ahead energy of period 6, converges. For other t's, $P_d(t)$'s converges similarly. The expected social welfare under the converged $P_d(t)$ is given in Fig. 2 (b), compared to the case without demand response (where the users consume the target demands and the LSE optimizes $P_d(t)$.)

[1]Reference [16] developed a model of electricity generation with piecewise quadratic cost functions.

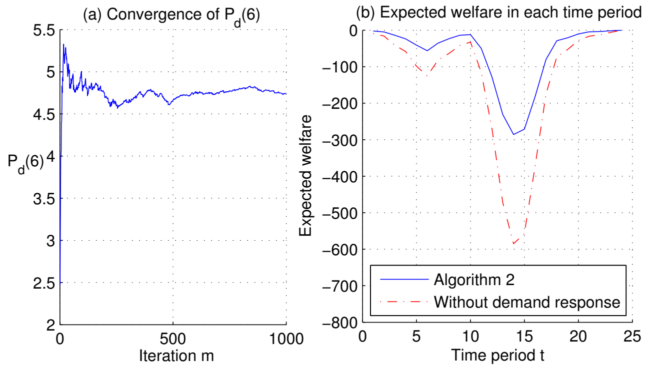

Fig. 2: Results of Algorithm 2

B. With time correlation

In this case, we further impose the constraint that $\sum_t q_i(t) \geq \sum_i y_i(t)$. That is, user i can shift his demand from one time period to another, but his total consumption $\sum_t q_i(t)$ must be at least $\sum_i y_i(t)$.

Due to the limit of space, the simulation results are presented in the companion paper [13]. The results show the features of the solution given by Algorithm 3. Specifically, the users tend to opportunistically use the available renewable energy, and at the same time flatten their consumption over time. Also, Algorithm 3 performs well (in terms of the expected social welfare achieved) compared to optimal solution, even when the penetration of renewable energy is high.

VII. Conclusion

This paper has investigated multi-period energy procurement and demand responses in the presence of uncertain supply of renewable energy. Specifically, we have provided decentralized algorithms with two-way communication for the load-serving entity and the users, aiming to maximize social welfare. We have studied the performance of the algorithms through both analysis and simulations. We have provided insight on the effect of clean, but random renewable energy on the social welfare.

This paper has focused on one type of utility functions and consumption constraints. In the future, we will incorporate other types of appliances as well, such as those modeled in [6]. Our algorithms can be easily extended to that case. The challenges lie in understanding the performance of Algorithm 3 in more general settings, and possibly designing more efficient algorithms. Also, we are interested in considering the case with distributed renewable generations on the user side, which will become more common in the future, and investigate how that changes the structure of optimal energy procurement and demand response strategies.

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