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PID Controller Design for a Flexible-Link Manipulator

Ming-Tzu Ho and Yi-Wei Tu

Abstract— **This paper investigates the application of the H_∞ proportional-integral-derivative (PID) control synthesis method to tip position control of a flexible-link manipulator. To achieve high performance of PID control, this particular control design problem is cast into the H_∞ framework. Based on the recently proposed H_∞ PID control synthesis method, a set of admissible controllers is then obtained to be robust against uncertainty introduced by neglecting the higher-order modes of the link and to achieve the desired time-response specifications. The most important feature of the H_∞ PID control synthesis method is the ability to provide the knowledge of the entire admissible PID controller gain space which can facilitate controller fine tuning. Finally, experimental results are given to demonstrate the effectiveness of H_∞ PID control.**

I. INTRODUCTION

Due to distributed flexibility, the flexible-link manipulator is an inherently infinite-dimensional system with a large number of low-damped oscillatory modes. Moreover, the system has the nonminimum-phase behavior arising from the noncolocated actuator and sensor structure. The nonminimum-phase zeros impose intrinsic limitations on system performance and robustness. In general, to design a feedback controller for such an infinite-dimensional system, it is necessary to require a suitable reduced-order model by neglecting high-frequency modes. A controller designed on the basis of the reduced-order model could result in spillover instability [1] caused by the high-frequency modes neglected at the controller design phase. Therefore, the process of controller design must account for the high-frequency unmodelled dynamics. It has been shown that H_∞-based control [2], [3], [4] can systematically deal with various formats of model uncertainty. In the past, H_∞-based control has been extensively applied in the area of flexible structure control [5]-[11] to design high performance controllers such that the closed-loop systems are robust to model uncertainty and disturbance. However, the order of the resulting controller is at least as high as the model order and often much higher in the case where the plant must be augmented by dynamical scalings or weights in order to achieve the desired robustness or performance requirements. The high-order controllers may not be feasible for real-time implementation because of hardware and computational limitations. Unfortunately, the fixed-order H_∞ controller design is computationally intractable [12] using those H_∞-based control synthesis methods. Although the high-order controller can be approximated by a reduced-order controller, it is usually at the cost of closed-loop robustness and performance degradation.

Despite the advent of many sophisticated control theories and techniques, proportional-integral-derivative (PID) control is still one of the widely used control structures in industrial applications. The popularity of PID control is mainly due to its structural simplicity, demonstrated reliability, and broad applicability. Since the 1940's, many approaches, see [13] and the references therein, have been proposed for tuning PID controllers. Most of the existing PID tuning methods are developed in an *ad hoc* fashion with little theoretical guarantee on stability, robustness, and performance. With rigorous theoretical justification, recently several PID control synthesis methods [14]-[23] have been proposed. These results are applicable to a given but arbitrary single-input single-output linear time-invariant plant. The PID stabilization problems were solved in [14]-[19]. By converting the H_∞ design problem into simultaneous complex polynomial stabilization and using the complex PID stabilization results, [20]-[22] provided a linear-programming-based characterization of all admissible H_∞ PID controllers for a given plant. This characterization besides being computationally efficient revealed important structural properties of H_∞ PID controllers. It was shown that for a fixed proportional gain, the set of admissible integral and derivative gains lie in a union of convex sets. Based on a frequency gridding approach, [23] provided an alternative H_∞ PID control synthesis technique. The objective of this paper is to investigate the application of the H_∞ PID control synthesis method proposed in [20]-[22] to tip position control of a flexible-link manipulator. To cast this control design problem into the H_∞ framework, a finite-dimensional approximate model which presents only the first two modes is obtained by carrying out system identification. The neglected higher-frequency dynamics are treated as the frequency-weighted additive uncertainty. The H_∞ PID control synthesis method of [20]-[22] is then used to provide a set of admissible controllers to be robust against uncertainty introduced by neglecting the higher-order modes and to meet the desired time-response specifications.

The paper is organized as follows. In Section II, a brief description of the experimental setup is given. The results of system identification of the plant are given in Section III. In Section IV, the H_∞ PID control synthesis procedure is briefly stated and a set of admissible PID gain values is obtained for the flexible-link system. The effectiveness of the designed PID control is verified through the experimental results given in Section V. The issues of controller fine-tuning are also addressed. Finally, Section VI contains some concluding remarks.

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II. Description of Experimental Setup

An experimental single-link flexible manipulator has been constructed as shown by the schematic diagram in Fig. 1. The flexible link is a rectangular stainless-steel bar (304) with 34.5-cm length, 3-cm width, and 0.045-cm thickness. The link is coupled to a permanent magnet DC motor by a hub. The transmitter of an ultrasonic sensor is mounted at the free end tip of the link and its receiver is fixed on top of the hub. This ultrasonic sensor is used to measure the tip deflection of the link. The transmitter of the ultrasonic sensor also acts as a payload. An optical encoder with resolution 1000 pulses/rev attached to the shaft of the DC motor is used to measure the angular position of the shaft. The controller is implemented on a DSP board, EVM320F240 Evaluation Module manufactured by Spectrum Digital, Inc. This board is based on the Texas Instruments TMS320F240 digital signal processor (20 MHz/16-bit). A voltage signal is generated according to the designed control law and is also supplied to a motor driver which drives the DC motor. A voltage amplifier is used to interface with the DSP board and the ultrasonic sensor. The flexible link has a circular motion in the horizontal plane. Due to flexibility of the link, the open-loop response of the tip motion has significant oscillation. The aim of this paper is to design a PID controller to position the tip of the link to a set point as fast as possible with the desired level of vibration suppression.

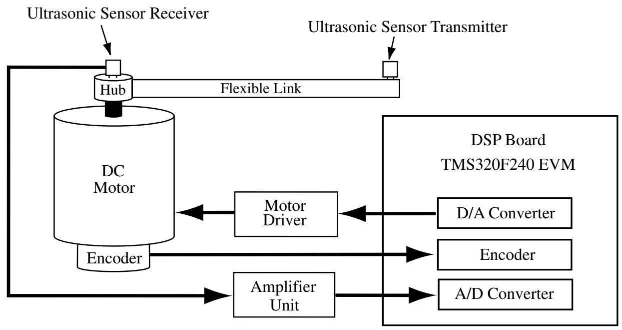

Fig. 1. Schematic diagram of the experimental setup.

III. System Identification and Uncertainty Description

The flexible link is a distributed-parameter system that can be described by an infinite-dimensional mathematical model. Control design methods often require excessive computational time, if they are applied to such a high-complexity model. In practice, the reduced-order model is used to conform to computational limitations. In this section, system identification is exploited to construct a reduced-order model of the system with an associated bound on the model mismatch from the actual system by means of the measurements of the response from the actual system. This reduced-order model and its uncertainty bound can then be used for controller synthesis.

A theoretical model of the link has been first derived based on the finite element method. Combining the derived model of the link and the theoretical model of the DC motor, an infinite-dimensional theoretical model of the voltage input of the motor to the tip angular position of the link is obtained. Although the theoretical model is impossible to exactly characterize the dynamical behavior of the system, it provides valuable *a priori* knowledge about this system. It is shown that the energy of the vibration modes is essentially dominated by the first two modes. Thus, for control purpose, it is adequate to use a model covering the first two modes. From the theoretical model, we know that the second vibration mode is located at 175.69013 rad/sec. A sampling rate at 500 Hz is adequate for the identification experiments. The input signal for system identification is a 0.1-200 Hz pseudo-random noise. To avoid aliasing, an eighth-order low-pass Butterworth filter is used with a cut-off frequency of 200 Hz. The time-domain input and output pairs are collected and then transformed into the frequency domain by using the fast Fourier transform. The leakage problem is minimized by using the Hamming window. The parameters of the transfer function model are obtained from least-squares estimation. The whiteness test on the residual error has been used as a validation test. A ninth-order identified model $G(s)$ that has a good match with the first two modes is obtained:

$$G(s) = \frac{n_6 s^6 + \cdots + n_1 s + n_0}{d_9 s^9 + \cdots + d_1 s + d_0}$$

where the values of the parameters of the identified model are given in Table I. The frequency response of the identified model and the measured frequency response are given in Fig. 2. The first four modes of the system have been shown in the measured frequency response. To be robust against any dissimilarities between the identified model and the actual system, a set of models needs to be established. This set of models allows one to capture the actual system in robust control design. Particularly, in this study, the model error between the identified model and the actual system is considered as additive uncertainty. Hence, the set of models is assumed to be of the perturbed form:

TABLE I

Parameters of the identified model.

n_6	-14340.4953	d_9	1
n_5	0.4446×10^7	d_8	486.7
n_4	0.5697×10^9	d_7	69317.7
n_3	-0.1908×10^{11}	d_6	0.1616×10^8
n_2	-0.9354×10^{12}	d_5	0.1062×10^{10}
n_1	0.6919×10^{13}	d_4	0.6167×10^{11}
n_0	0.2839×10^{15}	d_3	0.2624×10^{13}
		d_2	0.3595×10^{14}
		d_1	0.142×10^{15}
		d_0	0

$$\tilde{G}(s) = G(s) + W_2(s)\Delta(s) \quad (1)$$

where $W_2(s)$ is a weighting function and $\Delta(s)$ is any stable and proper transfer function satisfying the H_∞-norm bound

$$\|\Delta(s)\|_\infty := \sup_\omega |\Delta(j\omega)| \le 1. \quad (2)$$

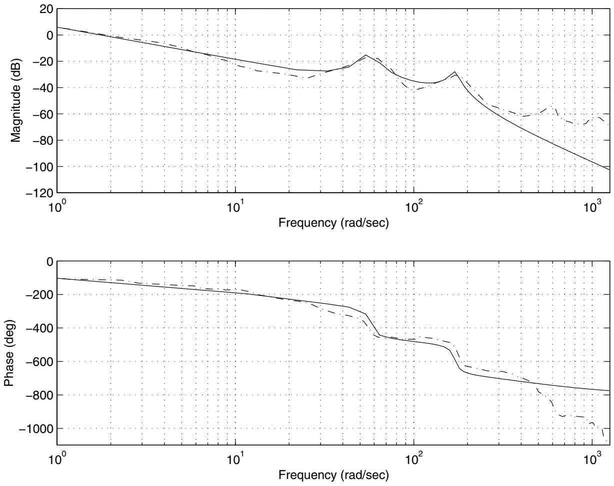

Fig. 2. Bode plots of the measured frequency domain data $(-\cdot)$ and $G(s)$ $(-)$.

The actual system is assumed to reside in $\tilde{G}(s)$. From (1) and (2), it gives that

$$|\tilde{G}(j\omega) - G(j\omega)| \quad \leq \quad |W_2(j\omega)|.$$

Frequency response variation is obtained by subtracting the measured frequency response and the frequency response of the identified model. Hence, the magnitude of the additive uncertainty weight is selected to envelope the magnitude of frequency response variation at low frequencies and the peaks of the third mode as well as the higher-frequency ones as shown in Fig. 3. The additive uncertainty weight $W_2(s)$ is chosen to be

$$W_2(s) \quad = \quad 0.11\frac{(\frac{1}{11}s+1)(\frac{1}{720}s+1)^2}{(\frac{1}{70}s+1)^3}.$$

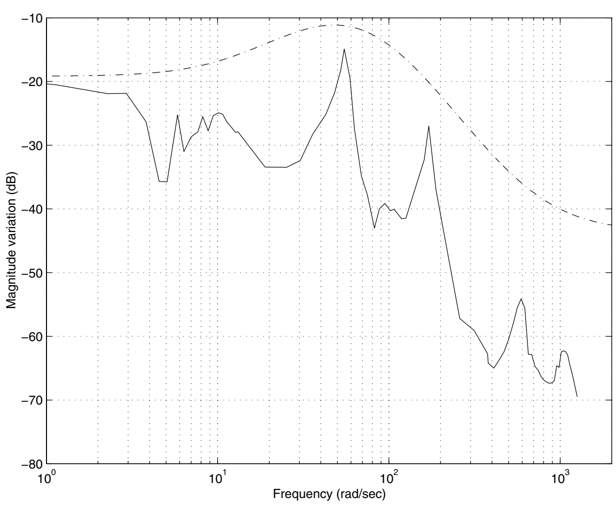

Fig. 3. Frequency response variation $(-)$ and additive uncertainty weight $(-\cdot)$.

IV. H_∞ PID DESIGN FOR ROBUST PERFORMANCE

Controllers provide robust performance for a system if they guarantee the closed-loop stability and achieve the desired performance specifications in the presence of model uncertainty. In this section, based on the results of [20]-[22], we present a design procedure to determine PID controller gain settings for achieving robust performance. Consider the single-input and single-output feedback control system shown in Fig. 4.

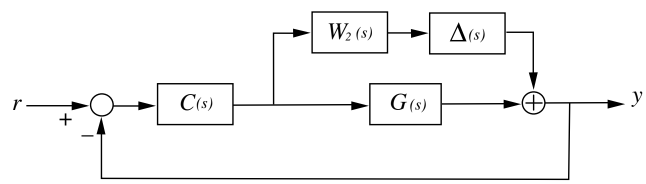

Fig. 4. Feedback control system with additive uncertainty.

Here r is the command signal, y is the output. $G(s) = \frac{N(s)}{D(s)}$ is the nominal plant, where $N(s)$ and $D(s)$ are coprime polynomials. $\Delta(s)$ is any stable and proper transfer function with $\|\Delta(s)\|_\infty \leq 1$. The weight $W_2(s)$ describes the frequency-domain characteristic of model uncertainty. $C(s)$ is the PID controller which is designed for robustly stabilizing the closed-loop system and achieving the desired performance specifications. To prevent noise amplification and internal instability induced by the pure derivative term of the ideal PID controller structure, the PID controller is modified in the following form:

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s(1+0.01s)}. \tag{3}$$

A necessary and sufficient condition for robust performance [25] is

$$\||W_1(s)S(s)| + |W_2(s)C(s)S(s)|\|_\infty \quad < \quad 1 \tag{4}$$

where

$$S(s) = \frac{1}{1+C(s)G(s)}$$

is the sensitivity function and $W_1(s)$ is the weighting function that describes the frequency-domain characteristic of the performance specifications. To convert the robust performance condition (4) into simultaneous polynomial stabilization, we first consider the following lemma:

Lemma 4.1: [21] Let

$$\frac{A(s)}{B(s)} = \frac{a_0 + a_1 s + \cdots + a_x s^x}{b_0 + b_1 s + \cdots + b_x s^x}$$

and

$$\frac{E(s)}{F(s)} = \frac{e_0 + e_1 s + \cdots + e_y s^y}{f_0 + f_1 s + \cdots + f_y s^y}$$

be stable and proper rational functions with $b_x \neq 0$ and $f_y \neq 0$. Then

$$\left\|\left|\frac{A(s)}{B(s)}\right| + \left|\frac{E(s)}{F(s)}\right|\right\|_\infty \quad < \quad 1 \tag{5}$$

if and only if

(a) $\left|\frac{a_0}{b_0}\right| + \left|\frac{e_0}{f_0}\right| < 1$;

(b) $B(s)F(s) + e^{j\theta}A(s)F(s) + e^{j\phi}E(s)B(s)$ is Hurwitz for all θ and $\phi \in [0,\ 2\pi)$.

Consider the stable weighting functions $W_1(s) = \frac{N_{W1}(s)}{D_{W1}(s)}$ and $W_2(s) = \frac{N_{W2}(s)}{D_{W2}(s)}$, where $N_{W1}(s)$, $D_{W1}(s)$, $N_{W2}(s)$, and $D_{W2}(s)$ are some real polynomials. Also, we denote the closed-loop characteristic polynomial to be

$$\begin{aligned}\rho(s,\ k_p,\ k_i,\ k_d) \ \ &\triangleq\ \ s(1+0.01s)D(s) \\ &\quad +(k_i + k_p s + k_d s^2)N(s).\end{aligned}$$

For notational simplicity, we define the complex polynomial

$$\begin{aligned}&\psi(s,\ k_p,\ k_i,\ k_d,\ \theta,\ \phi) \triangleq \\ &\qquad s(1+0.01s)D_{W1}(s)D_{W2}(s)D(s) \\ &\qquad +e^{j\theta}s(1+0.01s)N_{W1}(s)D_{W2}(s)D(s) \\ &\qquad +(k_d s^2 + k_p s + k_i)[D_{W1}(s)D_{W2}(s)N(s) \\ &\qquad +e^{j\phi}D_{W1}(s)N_{W2}(s)D(s)].\end{aligned}$$

Based on Lemma 4.1, the problem of synthesizing PID controllers for robust performance can be converted into the problem of determining values of $(k_p,\ k_i,\ k_d)$ for which the following conditions hold:

(1) $\rho(s,\ k_p,\ k_i,\ k_d)$ is Hurwitz;

(2) $\psi(s,\ k_p,\ k_i,\ k_d,\ \theta,\ \phi)$ is Hurwitz for all θ and $\phi \in [0,\ 2\pi)$;

(3) $|W_1(0)S(0)| + |W_2(0)C(0)S(0)| < 1$.

In view of the above conditions, the problem of synthesizing PID controllers for robust performance has been reduced to simultaneous complex polynomial stabilization. With a fixed θ and a fixed ϕ, both $\psi(s,\ k_p,\ k_i,\ k_d,\ \theta,\ \phi)$ and $\rho(s,\ k_p,\ k_i,\ k_d)$ are in the form of:

$$L(s) + (k_d s^2 + k_p s + k_i)M(s) \tag{6}$$

where $L(s)$ and $M(s)$ are some given complex polynomials. In [20]-[22], a linear-programming-based synthesis procedure was provided for determining all stabilizing $(k_p,\ k_i,\ k_d)$ values, if any, for which (6) is Hurwitz. In particular, it was shown that for a fixed k_p, the stabilizing $(k_i,\ k_d)$ values are the feasible solutions of a set of linear inequalities. Accordingly, by sweeping over k_p the linear-programming techniques were used to generate the entire parametric space of the stabilizing $(k_p,\ k_i,\ k_d)$ values for (6). The necessary ranges of stabilizing k_p can be prescribed by using the root locus method presented in [14], [21].

To determine the admissible $(k_p,\ k_i,\ k_d)$ values satisfying the robust performance criterion (4), we first sweep over θ and $\phi \in [0,\ 2\pi)$ and use the method presented in [14], [21] to determine the necessary ranges of k_p such that the admissible $(k_i,\ k_d)$ values of conditions (1) and (2) may exist. Then with a fixed k_p in the necessary ranges, using the results from [20]-[22], we are able to determine the entire admissible $(k_i,\ k_d)$ region such that condition (1) is satisfied. The resulting admissible set is denoted by $\mathcal{S}_1$. With the same k_p, sweeping over θ and $\phi \in [0,\ 2\pi)$, and using the results of [20]-[22] again, we can determine the entire admissible $(k_i,\ k_d)$ region such that condition (2) is satisfied. Let $\mathcal{S}_2$ denote the resulting admissible set. Condition (3) gives a set of linear inequalities in k_i and the resulting admissible set is denoted by $\mathcal{S}_3$. Then for a fixed k_p, the admissible $(k_i,\ k_d)$ region achieving robust performance is given by

$$\mathcal{S} = \mathcal{S}_1 \cap \mathcal{S}_2 \cap \mathcal{S}_3.$$

By sweeping over k_p in the necessary ranges and determining the corresponding $\mathcal{S}$ at each stage, we can obtain the entire set of the admissible $(k_p,\ k_i,\ k_d)$ gain values such that the robust performance criterion (4) is satisfied.

Now we will proceed to design PID controllers for the flexible-link system to meet the following step-response performance specifications:

- Settling time is approximately 6 seconds for the system output to achieve 95 % of its final value.
- Overshoot is less than 20 %.

Based on weight selection given in [4], the performance weighting function $W_1(s)$ is chosen as the approximation of the inverse sensitivity function of an acceptable closed-loop system. The resulting expression of $W_1(s)$ is given by

$$W_1(s) \ \ = \ \ \frac{(0.7217s + 1.4874)^2}{(s+1)^2}. \tag{7}$$

In this study, since $|W_1(0)S(0)| + |W_2(0)C(0)S(0)| < 1$ holds for any stabilizing PID controller, condition (3) will not impose any constraint on $(k_p,\ k_i,\ k_d)$ gain values. Now using the PID robust performance design procedure stated above, we first determine the necessary range of k_p values for the existence of admissible $(k_i,\ k_d)$ gain values. It is given by $k_p \in [0,\ 1.9387]$. With a fixed $k_p \in [0,\ 1.9387]$, for instance $k_p = 0.85$, and following the design procedure, we obtain the admissible set of $(k_i,\ k_d)$ gain values sketched in Fig. 5. By sweeping over $k_p \in [0,\ 1.9387]$, and determining the corresponding admissible region at each stage, we can obtain the entire admissible set of $(k_p,\ k_i,\ k_d)$ gain values such that the closed-loop system satisfies the robust performance criterion (4). This admissible set is sketched in Fig. 6.

V. Experimental Results and Controller Tuning

The controller designed in the previous section is implemented on the DSP system as shown in Fig. 1. Again, implementation is done by discretizing the PID controller (3) with the Tustin transformation and a sampling frequency of 200 Hz. To assess the performance of the closed-loop system, a 30° set-point reference is considered. With the controller gain values $k_p = 0.85$, $k_i = 0.06$, and $k_d = 0.025$, the tip position response and control voltage are shown in Fig. 7 and 8, respectively. From Fig. 7, we know that the time-response specifications are satisfied. However, the steady state error reduces toward zero quite slowly. From Fig. 8, we also observe that actuator saturation occurs due to the large transient control voltage. To improve the time response of the system and to prevent actuator saturation, one can re-define the time-response specifications, re-select

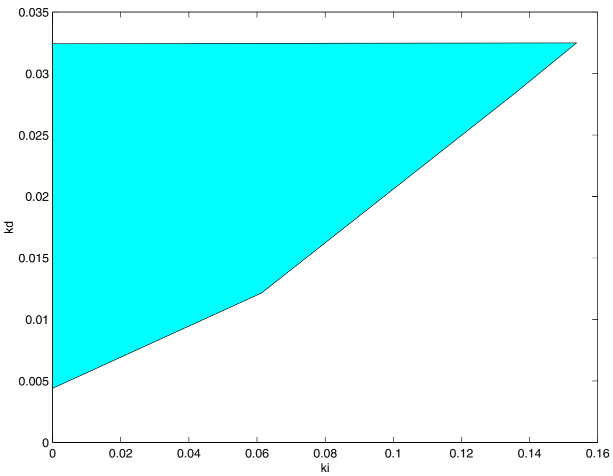

Fig. 5. Admissible set of $(k_i,\ k_d)$ values for $k_p = 0.85$.

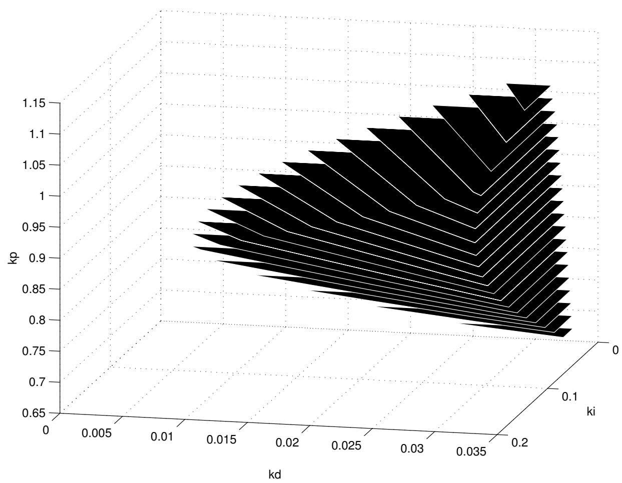

Fig. 6. Admissible set of $(k_p,\ k_i,\ k_d)$ values.

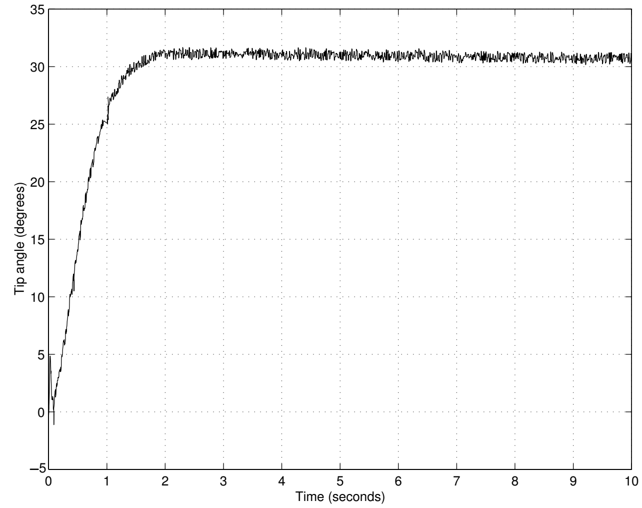

Fig. 7. Tip angular position response for controller with $k_p = 0.85$, $k_i = 0.06$, and $k_d = 0.025$.

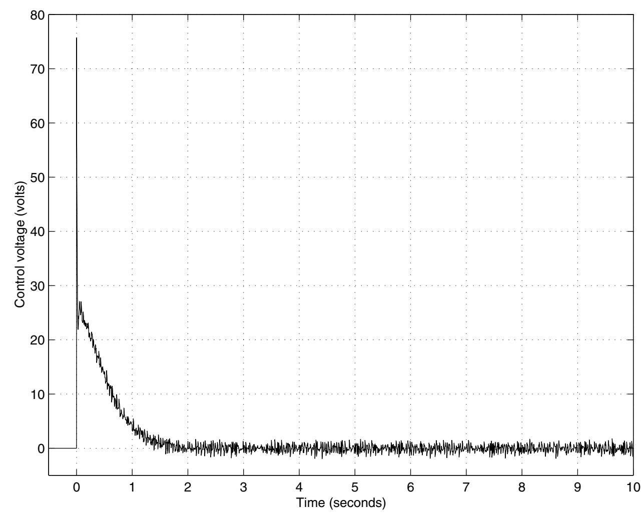

Fig. 8. Control voltage for controller with $k_p = 0.85$, $k_i = 0.06$, and $k_d = 0.025$.

the weighting functions $W_1(s)$ and $W_2(s)$, and repeat the design procedure from the beginning. The design may have to iterate between weight selection and the evaluation of the closed-loop time response many times before the time-response specifications are satisfied. Instead of carrying out such a design cycle, due to the structural simplicity of the PID controller, we can manually fine-tune the controller for possibly better performance. With the exact knowledge of the entire set of the admissible controller gain space provided by the H_∞ PID control synthesis method, controller tuning can be done much more efficiently and reliably. For instance, in order to prevent actuator saturation caused by the transition of the step input, based on Fig. 5 we can reduce the high frequency gain k_d of the PID controller to be $k_d = 0.008$. With $k_p = 0.85$ and $k_d = 0.008$, we fine-tune k_i within the admissible region given in Fig. 5. Without much effort, as $k_i = 0.006$, we obtain a better tip position response in the steady state and an acceptable control voltage in the transient state. The resulting tip position response and control voltage are shown in Fig. 9 and 10, respectively.

VI. Concluding Remarks

In this paper, the experimental results on PID control of a flexible-link manipulator were presented. To achieve high performance requirements, the PID control design problem was first cast into the H_∞ framework and an estimation of the finite-dimensional model was obtained by performing system identification. The discrepancy between the identified model and the actual system was treated as additive uncertainty and taken into account in controller synthesis. Then a recently proposed H_∞ PID control synthesis method was used to design PID controllers to guarantee robust performance in spite of uncertainty. In this PID control synthesis method, the modern control theory was incorporated into the design of the decades-old classical controllers. Significantly, unlike the standard H_∞ control synthesis techniques, this PID control synthesis method provided not just a single

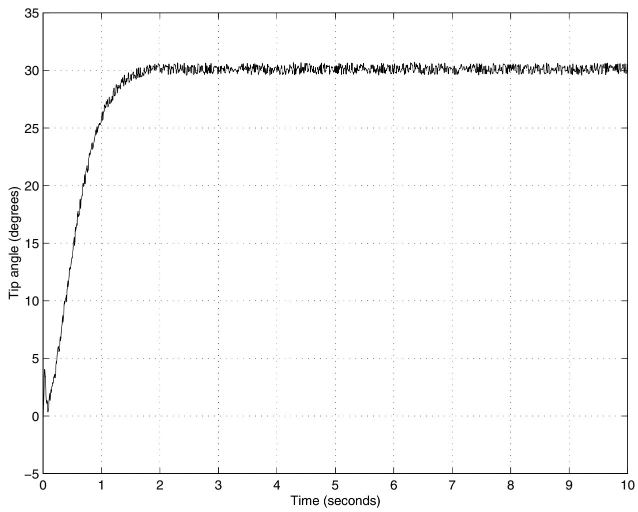

Fig. 9. Tip angular position response for controller with $k_p = 0.85$, $k_i = 0.006$, and $k_d = 0.008$.

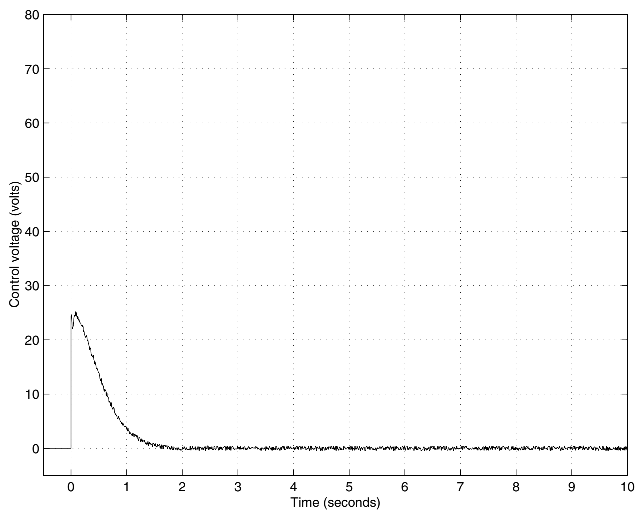

Fig. 10. Control voltage for controller with $k_p = 0.85$, $k_i = 0.006$, and $k_d = 0.008$.

solution, but the entire set of admissible PID gain values. In practice, the controller parameters always require fine tuning to optimize the actual performance. Despite the structural simplicity of the PID controller, PID tuning can be fairly time-consuming if done in an *ad hoc* manner. With the knowledge of the entire set of admissible PID gain values, PID tuning can be carried out much more efficiently and reliably as shown in the experimental investigations. Through the experimental studies, it was shown that the designed PID control met the performance requirements.

VII. ACKNOWLEDGMENTS

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