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Event driven manufacturing systems as time domain control systems

B.J.P. Roset, H. Nijmeijer, J.A.W.M. van Eekelen, E. Lefeber and J.E. Rooda

***Abstract*— Manufacturing systems are event driven systems and are therefore often considered from an event domain perspective. Notions from control system theory are all characterized in a time domain setting. In this paper the coupling between both domains is investigated. Also the relevance of this interconnection if control is applied to manufacturing systems is shown.**

I. INTRODUCTION

Manufacturing systems are mostly modelled and characterized as discrete event systems. Their behaviors are driven only by occurrences of different types of events. See [1] for an overview of discrete event systems. One of the major difficulties of analyzing discrete event systems is the fact that those systems are hard to tackle in a general mathematical framework which allows for analytical analysis of the systems. Especially, if one has the ambition to synthesize controllers for those systems, lack of a mathematical framework that allows for analytical manipulations is a disadvantage. For some subclass of discrete event systems an algebra that allows for some analytical study has been developed, see [2]. Based on this algebra some controller syntheses techniques have appeared, see for example [3], [4] or [5]. However, in all those papers modelling and controller synthesis is performed in the so called event domain in which the notion of time seems to be lost. In [9] an attempt is made to include the notion of time. Because we want to enforce a certain behavior in time of the manufacturing systems, by using control, the control objectives that one specifies are usually also defined in the time domain. A question that emerges is, how should one relate or transform time based analysis from control systems theory into event based notions as they are used in for example [2], [3], [4] and [5]?

In this paper it is spelled out how a class of event driven manufacturing systems, modelled in an event domain setting, can be considered in a time domain context. The motivation of this is to relate and apply conventional control system theory notions as stability, robustness, controllability, observability, and time based formulations of control objectives to event driven manufacturing systems modelled in an event domain setting.

The paper is organized as follows. First the considered manufacturing systems are formally defined in an event domain setting. Next, the same manufacturing systems are formally defined in a time domain frame work. The relation between both settings is discussed in the next section. Then a manufacturing example in which the ideas are illustrated is given. Finally some conclusions are drawn and potential future work will be discussed.

All authors are with the Department of Mechanical Engineering, Technische Universiteit Eindhoven, The Netherlands, [b.j.p.roset,h.nijmeijer,j.a.w.m.v.eekelen, a.a.j.lefeber,j.e.rooda]@tue.nl

II. TIMED MANUFACTURING SYSTEMS IN THE EVENT DOMAIN

Discrete event systems can be specified by logic rules. Events occurring should then satisfy these logic specifications. In general one can consider discrete event systems that can evolve without time elapsing. In other words events are occurring in the system while no time is elapsing. A subclass of discrete event systems are timed discrete event systems. In those systems, as in discrete event systems in general, events are evolving as a result of other events that are taking place, however, the key issue in timed discrete event systems is that some time between events can elapse as well. Events occur over time. This paper focusses on manufacturing systems that can be categorized in this class of timed discrete event systems, and will be called timed manufacturing systems. We formally define timed manufacturing systems or timed discrete event systems as

$$\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B}), \tag{1}$$

with $\mathbb{K} = \mathbb{Z}$ an *event* counter, $\mathbb{W} = \mathbb{R}^n$ the signal space, and $\mathscr{B} \subseteq \mathbb{W}^{\mathbb{K}}$ the behavior. In words, the manufacturing system (1) is defined by $\mathbb{K}$ representing events of interest, $\mathbb{W}$ representing the space in which the event driven signals, containing *time* instances, take on their values and $\mathscr{B}$ a subset of event trajectories $\mathbb{W}$. The subclass of behaviors $\mathscr{B}$ that we are considering are the behaviors $\mathscr{B}$ that at least guarantee that all signals $\mathbf{w}(\cdot)$ belonging to $\mathscr{B}$ have the property $\mathbf{w}(k-1) \leq \mathbf{w}(k)$ or $\gamma\mathbf{w} \leq \mathbf{w}$, where γ is the event shift operator. This property is a property that is characterizing for the timed manufacturing systems that we are considering. The sets $\mathbb{K}$, $\mathbb{W}$ and the special property of $\mathscr{B}$ ($\mathbf{w}(k-1) \leq \mathbf{w}(k)$) define the setting of a general manufacturing system, while further restrictions on $\mathscr{B}$ formalizes the laws of a specific considered manufacturing system. An example of a behavior of a manufacturing system in this framework is

$$\mathscr{B} = \{\mathbf{w} : \mathbb{Z} \rightarrow \mathbb{R}^n \mid \gamma\mathbf{w} \leq \mathbf{w}, \text{"Physics of a specific manufacturing system are satisfied"}\}. \tag{2}$$

In the following example an example of a manufacturing system that fits in the considered framework is given.

***Example** 2.1:* Consider a manufacturing system, which consists of 2 processing units M_1 and M_2 with processing times d_1 and d_2 respectively. Raw products are coming from two sources, knowing product stream A and B. In Fig. 1 an iconic model of a simple manufacturing system is given.

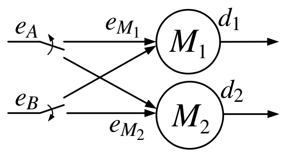

Fig. 1. Considered manufacturing system ($d_1 < d_2$).

In which e_A, e_B, e_{M_1} and e_{M_2} are events defined as:

- e_A :=Raw product from product stream A arrives.
- e_B :=Raw product of product stream B arrives.
- e_{M_1} :=Machine M_1 starts processing.
- e_{M_2} :=Machine M_2 starts processing.

It is given, that once a raw product arriving for the k-th time through one of the product streams is followed by a raw product arriving for the k-th time through the other product stream. Since, M_2 is slower than M_1 the following policy is applied. The first raw product arriving for the k-th time, either from A or B is processed on M_2 and the second raw product arriving for the k-th time, from A if B was first or from B if A was first, is processed on M_1. Further the machines start processing as soon as a raw product is available for the machines and the previous processes on the machines have been finished. The considered manufacturing system can be modelled in the framework of (1). This is done by defining the following variables $w_A(k)$, $w_B(k)$, $w_{M_1}(k)$ and $w_{M_2}(k)$ which represent time instances at which the events e_A, e_B, e_{M_1} and e_{M_2} occurred for the k-th time respectively. The behavior $\mathscr{B}$ that defines the considered manufacturing system is now defined as

$$\begin{aligned}\mathscr{B} = \Big\{ & \mathbf{w} = [w_A, w_B, w_{M_1}, w_{M_2}]^T : \mathbb{Z} \to \mathbb{R}^4 \mid \gamma\mathbf{w} \leq \mathbf{w}, \\ & \gamma w_A \leq w_B,\ \ w_{M_1} = \max(\gamma w_{M_1} + d_1, \max(w_B, w_A)) \\ & \gamma w_B \leq w_A,\ \ w_{M_2} = \max(\gamma w_{M_2} + d_2, \min(w_B, w_A)) \Big\}. \end{aligned} \quad (3)$$

The system in (2) can have certain properties. One nice property (2) can possess is for example event shift invariance.

***Definition** 2.1: A system $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ is said to be event shift invariant iff*

$$\mathbf{w}(\cdot) \in \mathscr{B} \Rightarrow \gamma^c \mathbf{w}(\cdot) \in \mathscr{B}, \quad \forall c \in \mathbb{Z}. \quad (4)$$

Note that $\gamma^c \mathbf{w}(\cdot)$ means that the signal $\mathbf{w}(\cdot)$ is shift c event counter steps. Another system property is L-completeness,

***Definition** 2.2: A system $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ is said to be L-complete if for some $L \in \mathbb{N}$*

$$\mathbf{w}(\cdot) \in \mathscr{B} \Leftrightarrow \gamma^c \mathbf{w}(\cdot)|_{[0,L]} \in \mathscr{B}|_{[0,L]}, \quad \forall c \in \mathbb{Z}. \quad (5)$$

We call Σ static if $L = 0$. A question that arises is: when can one write a manufacturing system, as for example given in Example 2.1, as a behavioral difference equation? What properties of the behavior $\mathscr{B}$ allow the system to be represented by a difference equation which is in the general case defined as

$$\begin{aligned}\mathscr{B} = \big\{ & \mathbf{w} : \mathbb{Z} \to \mathbb{R}^n \mid \gamma\mathbf{w} \leq \mathbf{w}, \mathbf{f}_1(\mathbf{w}, \gamma\mathbf{w}, ..., \gamma^{L-2}\mathbf{w}, \\ & \gamma^{L-1}\mathbf{w}) = \mathbf{f}_2(\mathbf{w}, \gamma\mathbf{w}, ..., \gamma^{L-2}\mathbf{w}, \gamma^{L-1}\mathbf{w}) \big\}, \end{aligned} \quad (6)$$

where L is called the lag of the system. In [7] a proposition is given that answers the question for discrete time dynamical systems. Since in the world of discrete time dynamical systems the time axis is of a discrete nature and in (1) the event axis $\mathbb{K}$ is also of a discrete nature, the proposition in [7] can trivially be be adapted to our context and then reads

***Proposition** 2.1: Consider $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ in* (1). *The following conditions are equivalent*

i) Σ is event shift invariant and L-complete;
ii) Σ can be described by a behavioral difference equation with lag L.

III. TIME BASED REPRESENTATION

The same event driven timed manufacturing system Σ in (1) can also be considered in time. In [7] a way to present the system dynamics as function of time is proposed. For the timed manufacturing systems that we are considering a formulation in this modelling framework would look like

$$\Sigma_{\mathscr{T}} = (\mathbb{T}, \mathbb{W}_{\mathscr{T}}, \mathscr{B}_{\mathscr{T}}), \quad (7)$$

with $\mathbb{T} = \mathbb{R}$ the time axis, $\mathbb{W}_{\mathscr{T}} = \mathbb{Z}^n$ the signal space, and $\mathscr{B}_{\mathscr{T}} \subseteq \mathbb{W}_{\mathscr{T}}^{\mathbb{T}}$ the behavior. In words, the manufacturing system (7) is defined by $\mathbb{T}$ representing time instants of interest, $\mathbb{W}_{\mathscr{T}}$ representing the space in which event counters take on their values, and $\mathscr{B}_{\mathscr{T}}$ a subset of $\mathbb{W}_{\mathscr{T}}$ to which all allowable time trajectories of the system belong to. The subclass of behaviors $\mathscr{B}_{\mathscr{T}}$ that we are considering are the behaviors $\mathscr{B}_{\mathscr{T}}$ that at least guarantee that all signals $\mathbf{w}_{\mathscr{T}}(\cdot)$ satisfying $\mathscr{B}_{\mathscr{T}}$ have the property $\mathbf{w}_{\mathscr{T}}(t-\tau) \leq \mathbf{w}_{\mathscr{T}}(t), \forall \tau > 0$ or $\sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}}, \forall \tau > 0$, where σ is a time shift operator. The sets $\mathbb{T}$, $\mathbb{W}_{\mathscr{T}}$ and the special property of $\mathscr{B}_{\mathscr{T}}$ ($\sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}}, \forall \tau > 0$) define the setting of a general manufacturing system, while further restrictions on $\mathscr{B}_{\mathscr{T}}$ formalizes the laws of a specific considered manufacturing system. An example of a behavior of a manufacturing system in this frame work is

$$\begin{aligned}\mathscr{B}_{\mathscr{T}} = \{ & \mathbf{w}_{\mathscr{T}} : \mathbb{R} \to \mathbb{Z}^n \mid \sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}}, \forall \tau > 0, \text{ "Physics} \\ & \text{of a specific manufacturing system are satisfied"} \}. \end{aligned} \quad (8)$$

***Example** 3.1:* We consider again the system in Example 2.1 of which the iconic representation is given in Fig. 1. The considered manufacturing system in this example can also be modelled in the framework of (7). To make this concrete we define the functions $w_{\mathscr{T}_A}(t)$, $w_{\mathscr{T}_B}(t)$, $w_{\mathscr{T}_{M_1}}(t)$ and $w_{\mathscr{T}_{M_2}}(t)$. The functions are "counter" functions that count how many times the events e_A, e_B, e_{M_1} and e_{M_2} have occurred in time respectively. The behavior $\mathscr{B}_{\mathscr{T}}$ that defines the considered manufacturing system is now defined as

$$\begin{aligned}\mathscr{B}_{\mathscr{T}} = \Big\{ & \mathbf{w}_{\mathscr{T}} = \left[w_{\mathscr{T}_A}, w_{\mathscr{T}_B}, w_{\mathscr{T}_{M_1}}, w_{\mathscr{T}_{M_2}} \right]^T : \mathbb{R} \to \mathbb{Z}^4 \quad | \\ & \sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}},\ \tau > 0,\ w_{\mathscr{T}_A} \geq w_{\mathscr{T}_B} + 1,\ w_{\mathscr{T}_B} \geq w_{\mathscr{T}_A} + 1, \\ & w_{\mathscr{T}_{M_1}} = \min(\sigma^{d_1} w_{\mathscr{T}_{M_1}} + 1, \min(w_{\mathscr{T}_B}, w_{\mathscr{T}_A})) \\ & w_{\mathscr{T}_{M_2}} = \min(\sigma^{d_2} w_{\mathscr{T}_{M_2}} + 1, \max(w_{\mathscr{T}_B}, w_{\mathscr{T}_A})) \Big\}. \end{aligned} \quad (9)$$

IV. INTERCONNECTION EVENT DOMAIN AND TIME BASED REPRESENTATION

In this section the coupling between the modelling frameworks considered in sections II and III is discussed. Assume one has a timed manufacturing system of which a description can be obtained in the form of (1) and (7). A natural question that arises is whether or not the signals obeying the laws of (1) can somehow be related to the signals obeying (7) and vice versa. If a relation exits that links the signals of both domains in a unique manner, then we call the manufacturing system considered in either (1) or (7) similar.

***Definition** 4.1: Let $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ and $\Sigma_{\mathscr{T}} = (\mathbb{T}, \mathbb{W}_{\mathscr{T}}, \mathscr{B}_{\mathscr{T}})$ be a description for a timed manufacturing system, then Σ and $\Sigma_{\mathscr{T}}$ are similar iff there exists a bijection $\pi : \mathbb{W}_{\mathscr{T}} \to \mathbb{W}$ such that $\mathbf{w}_{\mathscr{T}} \in \mathscr{B}_{\mathscr{T}} \Leftrightarrow \pi(\mathbf{w}_{\mathscr{T}}) \in \mathscr{B}$.*

For π to be a bijection the following properties must hold

1) Injective (one-to-one): For every $\mathbf{w}_{\mathscr{T}_1}$, $\mathbf{w}_{\mathscr{T}_2} \in \mathscr{B}_{\mathscr{T}}$, $\pi(\mathbf{w}_{\mathscr{T}_1}(t)) = \pi(\mathbf{w}_{\mathscr{T}_2}(t)) \Rightarrow \mathbf{w}_{\mathscr{T}_1}(t) = \mathbf{w}_{\mathscr{T}_2}(t)$
2) Surjective (onto): For every $\mathbf{w}(k) \in \mathscr{B}$, there exists $\mathbf{w}_{\mathscr{T}}(t) \in \mathscr{B}_{\mathscr{T}}$ such that $\pi(\mathbf{w}_{\mathscr{T}}(t)) = \mathbf{w}(k)$.

A necessary condition for the injective property, is that the system should at least be observed for a time span of Δ. The time span Δ is a measure for the *memory span* of the system and is defined in [7] as

***Definition** 4.2: Let $\Sigma_{\mathscr{T}} = (\mathbb{T}, \mathbb{W}_{\mathscr{T}}, \mathscr{B}_{\mathscr{T}})$ be a dynamical system, then $\Sigma_{\mathscr{T}}$ is said to have memory span Δ ($\Delta \in \mathbb{T}, \Delta > 0$) if $\mathbf{w}_{\mathscr{T}_1}$, $\mathbf{w}_{\mathscr{T}_2} \in \mathscr{B}_{\mathscr{T}}$, $\mathbf{w}_{\mathscr{T}_1}(t) = \mathbf{w}_{\mathscr{T}_2}(t)$ for $t_1 \le t < t_1 + \Delta \Rightarrow \mathbf{w}_{\mathscr{T}_1} \wedge \mathbf{w}_{\mathscr{T}_2} \in \mathscr{B}_{\mathscr{T}}$. Where $\wedge$ denotes concatenation (at time t_1), defined as*

$$(\mathbf{w}_{\mathscr{T}_1} \wedge \mathbf{w}_{\mathscr{T}_2})(t) = \begin{cases} \mathbf{w}_{\mathscr{T}_1}(t) & \text{for} \quad t < t_1 \\ \mathbf{w}_{\mathscr{T}_2}(t) & \text{for} \quad t \ge t_1. \end{cases} \tag{10}$$

A system with a memory span Δ must in general be observed for a time span Δ to be able to conclude whether or not two signals $\mathbf{w}_{\mathscr{T}_1} \in \mathscr{B}_{\mathscr{T}}$, $\mathbf{w}_{\mathscr{T}_2} \in \mathscr{B}_{\mathscr{T}}$ are equivalent ($\mathbf{w}_{\mathscr{T}_1} = \mathbf{w}_{\mathscr{T}_2}$). And thus it is necessary to observe a signal at least for a time span Δ to obtain the injectivity property for π.

***Theorem** 4.1: Let $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ and $\Sigma_{\mathscr{T}} = (\mathbb{T}, \mathbb{W}_{\mathscr{T}}, \mathscr{B}_{\mathscr{T}})$ be a description of a timed manufacturing system. Assume $\Sigma = (\mathbb{K}, \mathbb{W}, \mathscr{B})$ is an event shift invariant description which is L-complete (Definition 2.2). If the timed manufacturing system is observed for a time span Δ in time and the function $\mathbf{w}_{\mathscr{T}}(t)$ is right continuous [1], then $\pi : \mathbb{W}_{\mathscr{T}} \to \mathbb{W}$ defined as,*

$$\mathbf{w}_i(k) = \pi(\mathbf{w}_{\mathscr{T}_i}(t)) = \inf_{\mathbf{w}_{\mathscr{T}_i}(t) \ge k,\ t \in \mathbb{R}} t,\ i = \{1, ..., n\}, \forall k \in \mathbb{Z} \tag{11}$$

is a bijection such that $\mathbf{w}_{\mathscr{T}} \in \mathscr{B}_{\mathscr{T}} \Leftrightarrow \pi(\mathbf{w}_{\mathscr{T}}) \in \mathscr{B}$, if the signal $\mathbf{w}$ and $\mathbf{w}_{\mathscr{T}}$ correspond to the same physical events in the system.

Proof: First the "specific physics of a manufacturing system" for both event and time domain, see (1) and (7) respectively, is ignored. The following behaviors then follow

$$\mathscr{B}^* = \{\mathbf{w} : \mathbb{Z} \to \mathbb{R}^n \quad | \quad \gamma \mathbf{w} \le \mathbf{w}\} \quad \text{and} \tag{12}$$

$$\mathscr{B}^*_{\mathscr{T}} = \{\mathbf{w}_{\mathscr{T}} : \mathbb{R} \to \mathbb{Z}^n \quad | \quad \sigma^{\tau} \mathbf{w}_{\mathscr{T}} \le \mathbf{w}_{\mathscr{T}}, \forall \tau > 0\}. \tag{13}$$

Substituting (11) in the property ($\gamma \mathbf{w} \le \mathbf{w}$) defined in the behavior given in (12) leads to the following inequality

$$\left(\inf_{\mathbf{w}_{\mathscr{T}_i}(t) \ge (k-1),\ t \in \mathbb{R}} t \right) \le \left(\inf_{\mathbf{w}_{\mathscr{T}_i}(t) \ge k,\ t \in \mathbb{R}} t \right), \quad \forall k \in \mathbb{Z}. \tag{14}$$

[1] $\lim_{t \to a^+} \mathbf{w}_{\mathscr{T}}(t) = \mathbf{w}_{\mathscr{T}}(a), \quad \forall a \in \mathbb{T}$

Note that the inequality in (14) can only be satisfied for all k if the non-decreasing property ($\sigma^{\tau} \mathbf{w}_{\mathscr{T}} \le \mathbf{w}_{\mathscr{T}}$, $\forall \tau > 0$) in (13) is satisfied as well.

Let us now take the "specific physics of a manufacturing system", that has been ignored, into account. The systems behavior $\mathscr{B}$ under consideration is event shift invariant and L-complete. According to Proposition 2.1 this means that the systems behavior can be formulated as a difference equation with lag L in the domain presented in (1). The behavioral difference equation in its general form has already been given in (6). Note that (11) says that

$$\begin{aligned} \mathbf{w} = \pi(\mathbf{w}_{\mathscr{T}}),\ \ \gamma \mathbf{w} = \pi(\mathbf{w}_{\mathscr{T}} + \mathbf{1}), ..., \gamma^{L-2} \mathbf{w} = \\ \pi(\mathbf{w}_{\mathscr{T}} + (L-2)\mathbf{1}), \gamma^{L-1} \mathbf{w} = \pi(\mathbf{w}_{\mathscr{T}} + (L-1)\mathbf{1}). \end{aligned} \tag{15}$$

Using (6) and the relations given in (15) and using the derived equivalence relation for $\gamma \mathbf{w} \le \mathbf{w}$, found earlier, the time domain behavior must then consequentially be of the form

$$\begin{aligned} \mathscr{B}_{\mathscr{T}} = \{ & \mathbf{w}_{\mathscr{T}} : \mathbb{R} \to \mathbb{Z}^n \ \ | \ \ \sigma^{\tau} \mathbf{w}_{\mathscr{T}} \le \mathbf{w}_{\mathscr{T}}, \forall \tau > 0, \\ & \mathbf{f}_{\mathscr{T},1}(\mathbf{w}_{\mathscr{T}}, \mathbf{w}_{\mathscr{T}} + \mathbf{1}, ..., \mathbf{w}_{\mathscr{T}} + (L-2)\mathbf{1}, \mathbf{w}_{\mathscr{T}} + (L-1)\mathbf{1}) = \\ & \mathbf{f}_{\mathscr{T},2}(\mathbf{w}_{\mathscr{T}}, \mathbf{w}_{\mathscr{T}} + \mathbf{1}, ..., \mathbf{w}_{\mathscr{T}} + (L-2)\mathbf{1}, \mathbf{w}_{\mathscr{T}} + (L-1)\mathbf{1}) \}. \end{aligned} \tag{16}$$

The question however, is whether or not the arguments of $\mathbf{f}_{\mathscr{T},1}$ and $\mathbf{f}_{\mathscr{T},2}$ as they appear in (16) actually belong to $\mathscr{B}_{\mathscr{T}}$. The answer is yes, because it was assumed that the signals $\mathbf{w}$ and $\mathbf{w}_{\mathscr{T}}$ correspond to the same physical events in the system, hence

$$\gamma^c \mathbf{w} \in \mathscr{B} \Leftrightarrow \mathbf{w}_{\mathscr{T}} + c\mathbf{1} \in \mathscr{B}_{\mathscr{T}}, \quad \forall c \in \mathbb{Z}. \tag{17}$$

This means that the arguments of $\mathbf{f}_{\mathscr{T},1}$ and $\mathbf{f}_{\mathscr{T},2}$ as they appear in (16) belong to $\mathscr{B}_{\mathscr{T}}$. ■

***Example** 4.1:* Consider again the timed manufacturing system from Example 2.1 presented in Fig. 1. The timed manufacturing system can be described in the domain of (1) with the behavior defined as

$$\begin{aligned} \mathscr{B} = \Big\{ & \mathbf{w} = [w_A, w_B, w_{M_1}, w_{M_2}]^T : \mathbb{Z} \to \mathbb{R}^4 \ \ | \ \ \gamma \mathbf{w} \le \mathbf{w}, \\ & \gamma w_A \le w_B,\ \ \gamma w_B \le w_A,\ \ d_1 \in [5,9],\ \ d_2 \in [4,5], \\ & w_{M_1} = \max(\gamma w_{M_1} + d_1, \max(w_B, w_A)) \\ & w_{M_2} = \max(\gamma w_{M_2} + d_2, \min(w_B, w_A)) \Big\}. \end{aligned} \tag{18}$$

Note that the processing times of the machines are not constant this time. Every time the machines M_1 and M_2 processes a job, their processing times d_1 and d_2 change within some bounded sets $d_1 \in [5,9]$ and $d_2 \in [4,5]$ respectively. The considered system is event shift invariant, because for any signal $\mathbf{w}(\cdot) \in \mathscr{B}$ that is shifted some arbitrary event steps $\gamma^c \mathbf{w}$ $\forall c \in \mathbb{Z}$, there holds

$$\begin{aligned} \Big\{ & \gamma^c \mathbf{w} : \mathbb{Z} \to \mathbb{R}^4 \mid \gamma^{(c+1)} \mathbf{w} \le \gamma^c \mathbf{w},\ \ \gamma^c w_A \le w_B, \\ & \gamma^c w_B \le w_A,\ \ d_1 \in [5,9],\ \ d_2 \in [4,5], \\ & \gamma^c w_{M_1} = \max(\gamma^{(c+1)} w_{M_1} + \gamma^c d_1, \max(\gamma^c w_B, \gamma^c w_A)) \\ & \gamma^c w_{M_2} = \max(\gamma^{(c+1)} w_{M_2} + \gamma^c d_2, \min(\gamma^c w_B, \gamma^c w_A)) \Big\} = \mathscr{B}. \end{aligned} \tag{19}$$

Hence, Definition 2.1 is true for the considered timed manufacturing system. Note that the lag L of the timed manufacturing system is 1. All signals belonging to $\mathscr{B}$, which are arbitrarily event shifted ($\gamma^c \mathbf{w}\ \forall c \in \mathbb{Z}$) and observed over an interval $[0,1]$ of the event axis $\mathbb{K}$, also belong to the behavior $\mathscr{B}$ specified only on an interval $[0,1]$ of the event axis $\mathbb{K}$. The system is thus L-complete (Definition 2.2). The manufacturing system is event shift invariant and L-complete, and hence the proposed bijection π proposed in Theorem 4.1 can be applied to (18). This results in a behavior of the type considered in (8), which for this example reads

$$\begin{aligned} \mathscr{B}_{\mathscr{T}} = \Big\{ & \mathbf{w}_{\mathscr{T}} = \left[w_{\mathscr{T}_A}, w_{\mathscr{T}_B}, w_{\mathscr{T}_{M_1}}, w_{\mathscr{T}_{M_2}} \right]^T : \mathbb{R} \to \mathbb{Z}^4 \quad | \\ & \sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}}, \forall \tau > 0,\ w_{\mathscr{T}_B} \geq w_{\mathscr{T}_A} + 1,\ w_{\mathscr{T}_A} \geq w_{\mathscr{T}_B} + 1 \\ & w_{\mathscr{T}_{M_1}} = \mathbf{f}_{\mathscr{T},2_1}(w_{\mathscr{T}_{M_1}} + 1, w_{\mathscr{T}_B}, w_{\mathscr{T}_A}, d_1),\ d_1 \in [5,9] \\ & w_{\mathscr{T}_{M_2}} = \mathbf{f}_{\mathscr{T},2_2}(w_{\mathscr{T}_{M_2}} + 1, w_{\mathscr{T}_B}, w_{\mathscr{T}_A}, d_2),\ d_2 \in [4,5] \Big\}. \end{aligned} \tag{20}$$

An analytical expression for $\mathbf{f}_{\mathscr{T},2_1}$ and $\mathbf{f}_{\mathscr{T},2_2}$ cannot trivially be derived. However, if one assumes d_1 and d_2 are constant processing times (20) can be written as

$$\begin{aligned} \mathscr{B}_{\mathscr{T}} = \Big\{ & \mathbf{w}_{\mathscr{T}} = \left[w_{\mathscr{T}_A}, w_{\mathscr{T}_B}, w_{\mathscr{T}_{M_1}}, w_{\mathscr{T}_{M_2}} \right]^T : \mathbb{R} \to \mathbb{Z}^4 \quad | \\ & \sigma^\tau \mathbf{w}_{\mathscr{T}} \leq \mathbf{w}_{\mathscr{T}}, \forall \tau > 0,\ w_{\mathscr{T}_B} \geq w_{\mathscr{T}_A} + 1,\ w_{\mathscr{T}_A} \geq w_{\mathscr{T}_B} + 1 \\ & w_{\mathscr{T}_{M_1}} = \min(\sigma^{d_1}(w_{\mathscr{T}_{M_1}} + 1), \min(w_{\mathscr{T}_B}, w_{\mathscr{T}_A})) \\ & w_{\mathscr{T}_{M_2}} = \min(\sigma^{d_2}(w_{\mathscr{T}_{M_2}} + 1), \max(w_{\mathscr{T}_B}, w_{\mathscr{T}_A})) \Big\}. \end{aligned} \tag{21}$$

And hence, this boils down to the derived time domain behavior in Example 3.1.

From the above example, one can learn that although the analytical structure of a timed manufacturing system can be derived straightforwardly in the event domain, the analytical structure of the same manufacturing system in the time domain appears to be very hard or impossible to obtain. In a control environment one usually wants to manipulate the behavior of the system in time, therefore it would be beneficial to use models in the time domain as well. But we have just shown (Example 4.1) that it is difficult to obtain a useful analytical model for the systems under consideration in the time domain. However, if the relation (π) between both domains is known, it is not necessary to know the mathematical structure of the system in time domain a priori to conclude about the behavior of the system in the time domain. The notion of the time and event domain interconnection could be used to investigate what issues as stability and for example robustness, as we know from time domain, mean in an event-driven timed manufacturing systems environment. Also with issues as controllability and observability properties of timed manufacturing systems, the knowledge of the linkage between event and time domain might be valuable. Of course not all of these issues can be discussed here. As to give an illustration in which the notion of the coupling between both domains can be used, a practical timed manufacturing example is spelled out in the next section.

V. ILLUSTRATIVE MANUFACTURING EXAMPLE

Consider a timed manufacturing system as depicted in Fig. 2. The considered system is a manufacturing line, which

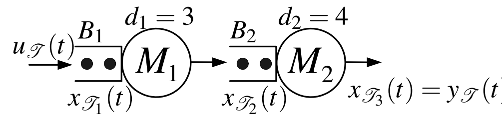

Fig. 2. An example of a manufacturing line.

consists of 2 processing units M_1 and M_2 with processing times d_1 and d_2 respectively. Raw products enter the line from one source at the beginning of the line. Before products can enter either machine M_1 or M_2, they have to pass through buffers B_1 and B_2 respectively. Both buffers have a capacity for at most two products. A machine will start processing a product, if there is a product present in the buffer in front of it and if the previous product has left the machine. The variables that can be used to model the system are given in Fig. 2. Here $u_{\mathscr{T}}(t)$, $x_{\mathscr{T}_1}(t)$, $x_{\mathscr{T}_2}(t)$ and $x_{\mathscr{T}_3}(t) = y_{\mathscr{T}}(t)$ represent the number of times, a raw product is released, a product has entered buffer B_1, a product has entered buffer B_2 and a product has left the manufacturing line at time t respectively. Note that the signals are of the type that belong to the modelling framework presented in (7), $\mathbf{w}_{\mathscr{T}}(t) = \left[u_{\mathscr{T}}(t)\ \ x_{\mathscr{T}_1}(t)\ \ x_{\mathscr{T}_2}(t)\ \ x_{\mathscr{T}_3}(t) \right]$ which belong to a specific behavior $\mathscr{B}_{\mathscr{T}}$. In the system the function $u_{\mathscr{T}}(t)$ is a function which can be manipulated within certain bounds specified in $\mathscr{B}_{\mathscr{T}}$ and is called the input of the system. It can be shown, for the considered manufacturing system, that the behavior in the event domain $\mathscr{B}$ is defined as

$$\begin{aligned} \mathscr{B} = \big\{ & \mathbf{w} = [u, x_1, x_2, x_3] : \mathbb{Z} \to \mathbb{R}^4 \mid \gamma \mathbf{w} \leq \mathbf{w}, \\ & x_1 = \max(\gamma^3 x_2, u),\ \ x_2 = \max(x_1 + d_1, \gamma x_2 \\ & + d_1, \gamma^3 x_3),\ x_3 = \max(x_2 + d_2, \gamma x_3 + d_2) \big\}. \end{aligned} \tag{22}$$

A. Formulation of an MPC scheme

Given is a manufacturing system observed in the time domain. Take as example the system defined in Fig. 2. Assume that all variables of the manufacturing system, which are present in $\mathbf{w}_{\mathscr{T}}$, have been observed in the past until the current time $t = t_c$ for at least Δ time (Definition 4.2). Based on these observations, together with the input function $u_{\mathscr{T}}(t)$ being defined for $t \in [t_c, t_c + T_p]$, a signal $\mathbf{w}_{\mathscr{T}}(t) \in \mathscr{B}_{\mathscr{T}}$ is uniquely defined for $t \in [t_c, t_c + T_p]$. In other words, a prediction of the future behavior of the system is obtained for $t \in [t_c, t_c + T_p]$. Based on the given arguments, an MPC strategy can be formulated for the example. In an MPC strategy an input signal ($\bar{u}_{\mathscr{T}}$) of the system over a time horizon of $t \in [t_c, t_c + T_p]$ is obtained by minimizing a cost criterion $J_{\mathscr{T}}$ subject to all the system's allowable signals $\bar{\mathbf{w}}_{\mathscr{T}}(t)|_{[t_c, t_c + T_p]}$ that belong to $\mathscr{B}_{\mathscr{T}}$. The obtained optimal input function $\bar{u}^*_{\mathscr{T}}(t)|_{[t_c, t_c + T_p]}$ is then applied to the system during a certain time interval $t \in [t_c, t_c + \delta]$ (open-loop). For some fixed $\delta > 0$ the problem can be formally written as

$$\min_{\bar{u}_{\mathscr{T}}(\cdot)} J_{\mathscr{T}}(\bar{\mathbf{w}}_{\mathscr{T}}(t)) = \int_{t_c}^{t_c + T_p} L_{\mathscr{T}}(\bar{\mathbf{w}}_{\mathscr{T}}(\tau)) d\tau \tag{23}$$

$$\text{subject to} \qquad \begin{aligned} & \bar{\mathbf{w}}_{\mathscr{T}}(t) \in \mathscr{B}_{\mathscr{T}} && t \in [t_c, t_c + T_p], \\ & \bar{\mathbf{w}}_{\mathscr{T}}(t) = \mathbf{w}_{\mathscr{T}}(t) && t \leq t_c, \\ & u_{\mathscr{T}}(t) = \bar{u}_{\mathscr{T}}(t) && t \in [t_c, t_c + \delta]. \end{aligned}$$

Note that a distinction is made between the true system signals and the internally obtained signals in the controller by writing a bar on the internally obtained signals. Due to disturbances and model uncertainties the internally obtained system trajectories will differ from the real systems trajectories. To introduce a feedback mechanism we solve the optimization problem given in (23) repeatedly after each δ time intervals. A typical cost function $J_{\mathscr{T}}$ that one would like to minimize in a timed manufacturing environment is for example.

$$J_{\mathscr{T}} = \int_{t_c}^{t_c+T_p} \Big(|\bar{y}_{\mathscr{T}}(\tau) - y_{r_{\mathscr{T}}}(\tau)| + \lambda |\bar{u}_{\mathscr{T}}(\tau) - u_{r_{\mathscr{T}}}(\tau)| \Big) d\tau. \tag{24}$$

Here $\bar{y}_{\mathscr{T}}(t)$ (internally obtained output) represents a signal from $\bar{\mathbf{w}}_{\mathscr{T}}(t)$ that one wants to control, and $\bar{u}_{\mathscr{T}}(t)$ (internally obtained input) is a signal from $\bar{\mathbf{w}}_{\mathscr{T}}(t)$ that the controller can still define within certain bounds that are specified in $\mathscr{B}_{\mathscr{T}}$. The signal $\bar{y}_{\mathscr{T}}(t)$ is enforced to track a certain prescribed reference signal $y_{r_{\mathscr{T}}}(t)$ by minimizing the cost function in (24). Also the input $\bar{u}_{\mathscr{T}}(t)$ is enforced to track a certain prescribed reference $u_{r_{\mathscr{T}}}(t)$ by minimizing the cost function. The reference signals $y_{r_{\mathscr{T}}}(t)$ and $u_{r_{\mathscr{T}}}(t)$ are predetermined signals satisfying the behavior $\mathscr{B}_{\mathscr{T}}$ of the system. In a manufacturing system environment the signals $y_{r_{\mathscr{T}}}(t)$ and $u_{r_{\mathscr{T}}}(t)$ can be thought of as the "schedule" for the system. Further λ is a weighting parameter, which can be used to specify the ratio of importance of tracking $\bar{y}_{\mathscr{T}}(t)$ or $\bar{u}_{\mathscr{T}}(t)$.

B. Solving the MPC problem

Solving the MPC problem, as formulated in (23), is a difficult task. However, using the assumptions and knowledge from Theorem 4.1, one can transform the MPC problem, as formulated in (23), to the event domain. This is done according to the following three steps. First the signal information, needed for re-initialization of the model in the controller, is translated to similar information needed to re-initialize a model in the event domain. The cost criterion $J_{\mathscr{T}}$ in (23) is then translated to the event domain (J). Next, the computed input sequence in the event domain, obtained by minimizing the converted cost criterion subject to a model in the event domain, is translated back to the time domain. The three steps will be explained in more detail in the following three subsections.

1) Signal conversion for re-initialization: As mentioned before, the model, used in the MPC controller, has to be re-initialized with new information by using measured signals of the system. In the previous section V-A it was explained that the needed information for re-initialization in time domain is given by the signal $\mathbf{w}_{\mathscr{T}}(t)|_{[t_c-\Delta,t_c]}$. If an event domain model in the form of (22) is used, one needs in general the $L-1$ latest event lags $\mathbf{w}(k)|_{[k_c-(L-1),k_c]}$ (see (6)) occurred just before the current time t_c for the re-initialization of the model. Together with the input signal in the event domain represented by $u(k)|_{[k_c+1,k_{p_u}]}$, which is a sequence of time instances of event occurrences known for the event counter interval $k \in [k_c+1, k_{p_u}]$, a signal $\mathbf{w}$ from $\mathscr{B}$ is uniquely defined over an interval of event counters $k \in [k_c+1, k_{p_u}]$. The signal $\mathbf{w}(k)|_{[k_c-(L-1),k_c]}$ can be obtained using the proposed bijection π defined in Theorem 4.1, hence

$$\mathbf{w}(k)|_{[k_c-(L-1),k_c]} = \pi(\mathbf{w}_{\mathscr{T}}(t), t \le t_c) \tag{25}$$

2) Conversion of cost criteria: A typical cost criterion that one would like to minimize in a timed manufacturing environment is for example the one given in (24). Due to the characteristic type of signals, the system allows for, the integral in (24), defining the cost functional $J_{\mathscr{T}}$, can be written as a finite sum in the event domain. It is assumed that Theorem 4.1 holds, thus one can consider the signals in (24) also in the event domain by interchanging the time and event axis (follows from (11)). In Fig. 3 a graphical representation of this for a single input output case is given. The vertical axis represents the time axis $\mathbb{T}$ from (7) and

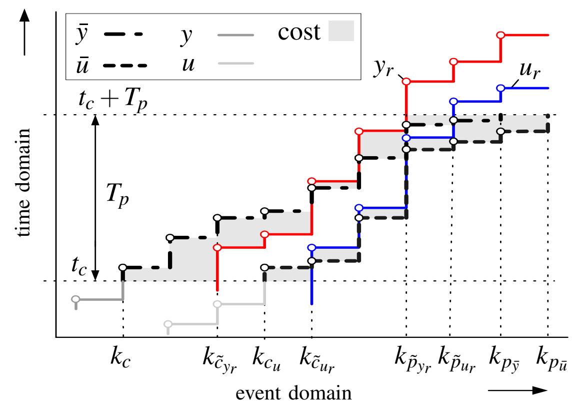

Fig. 3. Graphical representation of the costs.

the horizontal axis represents the event axis $\mathbb{K}$ from (1). The counters k_c, $k_{\tilde{c}_{y_r}}$, k_{c_u}, $k_{\tilde{c}_{u_r}}$, $k_{\tilde{p}_{y_r}}$, $k_{\tilde{p}_{u_r}}$, k_{p_y} and k_{p_u} represents the number of times a particular event, of which the counter evolution in time is described by the signals $\bar{y}_{\mathscr{T}}(t)$, $y_{\mathscr{T}}(t)$, $y_{r_{\mathscr{T}}}(t)$, $u_{r_{\mathscr{T}}}(t)$, $\bar{u}_{\mathscr{T}}(t)$ and $u_{\mathscr{T}}(t)$ respectively, occurred for the first time just before or on the time instance t_c and t_c+T_p respectively (see Fig. 3 for details). The grey coloured areas in Fig. 3 represent the costs that one wants to minimize. In case of time domain optimization, one will try to minimize the grey area by searching for $\bar{u}_{\mathscr{T}}(t)|_{[t_c,t_c+T_p]}$ (*theoretically an infinite number of design variables*), such that the grey area will become as small as possible. An optimization in the time domain will thus lead to a search through the feasible solution space which is along the *discrete* horizontal axis in Fig. 3. In case of an event domain optimization, one will try to minimize the grey area by searching for $\bar{u}(k)|_{[k_c+1,k_{p_u}]}$ (*a finite number of design variables*), such that the grey area in Fig. 3 will become as small as possible. An optimization in the event domain will thus lead to a search through the feasible solution space which is along the *continuous* vertical axis in Fig. 3. Note that the cost function $J_{\mathscr{T}}$ defined in the time domain as (24) can thus be rewritten in the event domain as

$$\int_{t_c}^{t_c+T_p} \left(|\bar{y}_{\mathscr{T}}(\tau) - y_{r_{\mathscr{T}}}(\tau)| + \lambda |\bar{u}_{\mathscr{T}}(\tau) - u_{r_{\mathscr{T}}}(\tau)| \right) d\tau \triangleq$$

$$\sum_{k=k_c+1}^{k_{\bar{c}y_r}} \max(\bar{y}(k)-t_c,0) + \sum_{k=k_{\bar{c}y_r}+1}^{k_{\bar{p}y_r}} \max(\bar{y}(k)-y_r(k),$$
$$y_r(k)-\bar{y}(k)) + \sum_{k=k_{\bar{p}y_r}+1}^{k_{py}} \max(t_c+T_p-\bar{y}(k),0) +$$
$$\lambda \sum_{k=k_c+1}^{k_{\bar{c}u_r}} \max(\bar{u}(k)-t_c,0) + \lambda \sum_{k=k_{\bar{c}u_r}+1}^{k_{\bar{p}u_r}} \max(\bar{u}(k)-u_r(k)$$
$$,u_r(k)-\bar{u}(k)) + \lambda \sum_{k=k_{\bar{p}u_r}+1}^{k_{pu}} \max(t_c+T_p-\bar{u}(k),0) = J(\bar{\mathbf{w}}(k)).$$

The optimization problem that we have to solve, can now be formulated as

$$\min_{\bar{u}(k_c+1),\ldots,\bar{u}(k_{p_u})} J(\bar{\mathbf{w}}(k)) \tag{26}$$

subject to $\quad \bar{\mathbf{w}}(k) \in \mathscr{B} \quad k \in [k_c+1, k_{p_u}],$
$\qquad\qquad\;\; \bar{\mathbf{w}}(k) = \mathbf{w}(k) \quad k \in [k_c-(L-1), k_c].$

For the example given in Fig. 2, of which the behavior $\mathscr{B}$ in the event domain is defined in (22), one can write the equations defining the specific physics of the manufacturing system in a recursive form $\mathbf{x}(k) = \mathbf{f}(\mathbf{x}(k-1), u(k))$. The term $\bar{\mathbf{w}}(k) \in \mathscr{B}$, $k \in [k_c+1, k_{p_u}]$ in (26) can then, for the example, be substituted by

$$\begin{array}{ll} \bar{\mathbf{x}}(k) = \mathbf{f}(\bar{\mathbf{x}}(k-1), \bar{u}(k)) & k \in [k_c+1, k_{p_u}], \\ \bar{u}(k) = u(k) & k \in [k_c+1, k_{c_u}], \\ \bar{u}(k+1) \geq \bar{u}(k) \geq t_c & k \in [k_{c_u}+1, k_{p_u}-1]. \end{array} \tag{27}$$

If the function $\mathbf{f}$ in (27) is of a max-min-plus-scaling type, one can use an efficient (convex) solution method proposed in [6] to solve (26). In our example the function $\mathbf{f}$ in (27) is of a max-plus linear type, which is a subclass of the max-min-plus-scaling functions. For max-plus linear systems a solution method for the optimization problem in (26) is proposed in [5].

3) Back to the time domain: The obtained signal $\bar{u}(k)|_{[k_c+1, k_{p_u}]}$ obtained by solving (26) can be converted to the time domain by using the dual relation of the bijection π proposed in (11) which is denoted by π^{-1}

$$u_{\mathscr{T}}(t) = \pi^{-1}(\bar{u}(k)) = \sup_{\bar{u}(k)\leq t,\; k\in\mathbb{Z}} k, \qquad t = [t_c, t_c+\delta]. \tag{28}$$

The control strategy is applied to the manufacturing line given in Fig. 2. A simulation result for $\delta = 1$, $T_p = 50$ time units and $\lambda = 1$ is shown in Fig. 4. After a machine failure the manufacturing line runs behind schedule. At time $t = 57$, for some initial configuration of the manufacturing system, a recovery to the schedule $y_{r_{\mathscr{T}}}(t)$ and $u_{r_{\mathscr{T}}}(t)$, in an optimal sense, has been obtained due to the proposed control strategy.

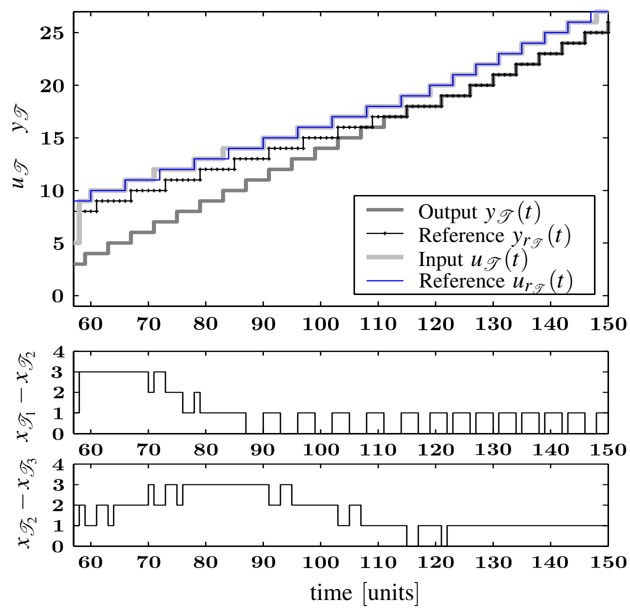

Fig. 4. Above: The response of the signals $y_{\mathscr{T}}(t)$ and $u_{\mathscr{T}}(t)$ compared to $y_{r_{\mathscr{T}}}(t)$ and $u_{r_{\mathscr{T}}}(t)$ respectively. Below: Amount of products present in B_1, M_1 $(x_{\mathscr{T}_1}(t) - x_{\mathscr{T}_2}(t))$ and B_2, M_2 $(x_{\mathscr{T}_2}(t) - x_{\mathscr{T}_3}(t))$ as function of time t.

VI. CONCLUSIONS

The relation between event domain modelling of a class of event driven timed manufacturing systems and the time domain has been obtained. The practical relevance of it has been shown on a typical manufacturing control problem which is untractable in the time domain, but is relatively easily solved in the event domain. Future investigations will be about how the notions of stability, robustness, observability and controllability that we have in the time domain can be used in the event domain by using the knowledge of the interconnection between both domains.

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