Recent Advances and Challenges in Process Identification

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Abstract

Process identification is undergoing tremendous developments as computational capabilities improve. Theories are rapidly catching up with the needs of practical applications but practical process identification experiences still reveal significant gaps between theory and practice. This review attempts to highlight the present gaps and challenges. With this objective, the review treats recent progress in process identification with data gathered in closed loop, and in the tailoring of an entire identification process to a given control objective.

Keywords

Process identification, Model uncertainty, Robust control, Parameter estimation, Closed-loop identification, Identification for control, Multivariable systems, Nonlinear systems, Time-varying systems

Introduction

Process identification is concerned with using observed process data to form a model of the process that can be used in various constructive ways for process improvement. It is unquestionably one of the most important steps of control system design—accounting for as much as 80-90% of the cost and time involved. It encompasses a diverse set of tasks that include plant testing, selection of a model structure, parameter estimation, and model validation. Prior to actual controller implementation, it is the only step that requires direct interaction with the process. Consequently, any erroneous decision here can jeopardize success of an entire control project. This fact demands that all decisions involved in identifying a process model be made carefully and systematically based on sound scientific principles and methods—a fact that perhaps explains the good synergy between researchers and practitioners of the field.

The paper of Åström and Eykhoff (1971) was one of the first to review the research in system identification. During the 70s and 80s, the theoretical foundations for system identification were laid by the pioneering work of Ljung (1987), Söderström and Stoica (1989) and their coworkers. The work of Ljung centered around a particular paradigm called *Prediction Error Minimization (PEM)*, which today is the norm of industrial practice. Process identification at the beginning of the 90s was reviewed by Andersen et al. (1991) in CPC-IV.

The decade has seen several major developments. One of them is the *subspace approach*, which was motivated to overcome some drawbacks of PEM for multivariable system identification (Van Overschee and De Moor, 1996; Verhaegen and Dewilde, 1992; Larimore, 1990). Significant advances have also come along in closed-loop identification and "control-oriented" (or "control-relevant") identification. In these areas the goal is to tailor the whole identification procedure to a given control objective. The past decade has also seen an unprecedented range and volume of applications of process identification in industries, mainly to provide models for predictive control. After a decade of such explosive developments, it is indeed apt to reflect upon the progress and the state of the field at this CPC meeting.

The gap between research and practice, though narrower than in most fields, is nevertheless significant and therefore is worth elaborating a bit:

- *Plant Test:* Industrial plant tests use simple signals like steps or PRBSs. In addition, it is almost always limited to perturbing one input at a time, mostly out of the concern for unpredictable effects on process behavior (Qin and Badgewell, 1997). Literature is replete with optimal test signal design methods including those that attempt to incorporate specific control requirements and process characteristics into design in an iterative manner (Rivera et al., 1993; Asprey and Macchietto, 2000; Pearson, 1998; Cooley and Lee, 2001). However, such tailored and iterative designs have rarely been attempted in practice, if ever. The single-input testing will inevitably emphasize accuracy of individual SISO dynamics but several studies have shown that accurate identification of SISO dynamics may be inadequate for multivariable control of certain types of plants (*e.g.*, ill-conditioned plants) (Andersen et al., 1989; Koung and MacGregor, 1994; Li and Lee, 1996).
- *Model Structure:* Popular structures are Finite Impulse Response (FIR) models and ARX models, both of which lead to linear regression problems. Other structures, like ARMAX models, OE models, and Box-Jenkins models, which require non-convex optimization, are less common but are used in some occasions. In almost all cases, Multiple-Input-Single-Output (MISO) structures are used, in which a separately parameterized model is fitted for each output (Andersen et al., 1991). This practice

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is clearly inefficient, both with respect to model order and accuracy, in view of the fact that most industrial process outputs exhibit significant levels of cross-correlation. The preference for (or insistence on) MISO structure is clearly linked to ease of parameter estimation as explained below.

- *Model Estimation:* PEM is by far the most dominant method for estimating model parameters, perhaps owing to its flexibility and sound theoretical basis as well as the ready availability of software tools. However, with multivariable structures, PEM requires special parameterizations and nonconvex optimization (Ljung, 1987; Van Overschee and De Moor, 1994), a fact that perhaps explains the industry's proclivity toward use of MISO structures. The subspace approach is designed to obviate these problems but requires relatively large data sets. In fact, the two approaches are best combined into one: The subspace approach can be used to provide a good initialization for PEM, which should alleviate the aforementioned problems. However, extensive use of such MIMO identification methods in the industry is not at all evident.
  Statistical methods like the Maximum Likelihood Estimation or Bayesian estimation have not found much use. This is probably because of the lack of probability information or the fact that these complex methods often reduce to the same least squares calculation as PEM under commonly made probability assumptions. Likewise, use of nonparametric methods such as the frequency-domain Empirical Transfer Function Estimation seems rare, probably due to the lack of sufficiently large data sets.
- *Use of Estimated Model:* The PEM approach with many structures (e.g., ARX or ARMAX) as well as the subspace approach yield a combined deterministic-stochastic system model. However, the noise part of the model is seldom used in control system design. This may be due to the belief that disturbances experienced during an identification experiment are not representative of those encountered during real operation. However, this practice bears some danger as the two model parts are identified to work together as a *single predictor*. Also, many process monitoring and soft-sensing schemes require precisely information on how variables are correlated to one another in time due to unknown inputs and noises. Hence, the noise part of a multivariable model, when fitted with appropriate data, can serve a very useful function. In addition, some control applications, such as those involving inferential estimation, should clearly benefit from availability of an accurate noise model (Amirthalingam and Lee, 1998; Amirthalingam et al., 2000).
- *Model Validation:* Standard tests like residual analysis and cross validation are ubiquitous in the industrial practice. However, model uncertainties are seldom captured in a form that can be used for robust controller design. There is an extensive literature on how information on process noises (either statistics or bounds) can be translated into model bounds (Rivera et al., 1993; Ninness and Goodwin, 1995; Böling and Mäkilä., 1995). However, these methods are seldom used as process noise information itself is unavailable or highly inaccurate. In addition, there are many other contributing uncertainties, *e.g.*, process nonlinearities, actuator errors, etc. Good theories with wrong assumptions are just as ineffective as bad theories.
- *Closed-Loop Testing and Iterative Improvement:* Many industrial processes already have several working loops that may not be removed. Hence, we suspect that closed-loop testing has already been practiced to some extent. However, it is not clear whether practitioners are always aware of potential problems that can arise with usage of feedback correlated data. It is also less common to use closed-loop testing as a way to generate data that are highly informative for feedback controller design (Gevers and Ljung, 1986; Gevers, 2000; Forssell and Ljung, 1999a). In addition, many published papers deal with iterative improvement of model and controller through closed-loop testing (Van den Hof and Scharma, 1995; Hjalmarsson and Birkel, 1998). The idea of continually improving the closed-loop performance by using data collected from a working loop is very attractive from a practical viewpoint. It connects well with the industry's growing concern over maintaining performance of advanced controllers. However, it is not clear that the industry at large has seriously considered this possibility.
- *Nonlinear Process Identification:* While systematic tools for formulating first engineering principle models have begun to appear, few systematic methods for *identifying* first engineering principle models are available (Asprey and Macchietto, 2000; Lee, 2000). On the other hand, some implementations of nonlinear model predictive control have been reported (Qin and Badgewell, 1998; Young et al., 2001). The most common industrial approach to deal with process nonlinearities is by use of multiple models. Switching rules among different models are *ad hoc* and seldom systematically designed. Some applications of artificial neural networks are reported but their effectiveness as *causal* models, *i.e.*, as optimization and control would use them, is questionable at best. Despite the vigorous research in this area during the past decade, the field still lacks a basic framework and a unifying theoretical foundation (Johansen and Foss, 1995).

These gaps in Table 1 serve to motivate our selection of topics covered in this review paper. We have

Issue	Practice	Theory
Model Structure	SISO/MISO ARX or FIR	MIMO State Space
Parameter estimation	PEM (Least Squares)	Subspace and PEM
Noise info usage	Rarely used	Disturbance estimation, inferential control
Model validation	Residual analysis	Model error bounds
Plant Test	One input at a time	Simultaneous testing of multiple inputs
Closed Loop test	Little understood	Vigorously researched
Nonlinear Process Identification	Seldom used	Systematic tools lacking

Table 1: Summary of current gaps between practice and theory.

chosen topics, for which theories, in our view, have advanced to a point that some of the aforementioned gaps can be closed to bring significant benefits to the practice. For that, we concentrate mainly on two topics, control-oriented identification and closed-loop identification, which in our view have seen the most significant research and progress related to process control during the past decade. We attempt to highlight the progress as well as the current limitations, and point out obstacles an engineer may face in adopting the new approaches in practice.

We have chosen to leave out the subspace approach in our review, other than developments relevant to closed-loop identification, as the topic has already been well publicized. Current literature on this topic includes a book (Van Overschee and De Moor, 1996) and several review papers, e.g., Viberg (1995) and Shi and MacGregor (2000). We have also chosen to leave out a review on nonlinear process identification, for which only limited progress has occurred but further developments are very much needed. Marquardt (2001)in this conference gives a comprehensive coverage of this topic.

The rest of the paper is organized as follows. In Section 2, we review the current state of knowledge in *closed-loop identification*. The asymptotic behavior of various identification approaches are discussed but our focus remains on practical implications of the theories. In Section 3, we discuss the problem of tailoring the entire identification process to a given specific control objective. We discuss why this consideration naturally leads to iterative identification and review both open-loop and closed-loop strategies. In Section 4, we conclude.

Our main objective for writing this paper is to fuel an honest and substantive debate among researchers and practitioners on the current state of identification theories as related to potential closure of the existing gaps. We hope that such a debate will clarify the strengths and limitations of the existing theories and methodologies for practitioners. We also hope that the factors previously passed over by the researchers but must be accounted for theories to be practicable will also be brought out in the open.

Closed-Loop Identification

Introduction

During the past decade, interest in closed-loop identification by the community has risen noticeably. Closed-loop identification is motivated by the fact that many industrial processes have already in place one or more loops that cannot be removed for safety and/or economic reasons. Beyond this lies the attractive idea of being able to improve the closed-loop performance continually by making use of data being collected from a working loop. It has also been claimed that data collected from a closed-loop operation better reflect the actual situation in which the developed model will be used, and therefore could yield better overall results (Gevers and Ljung, 1986; Hjalmarsson et al., 1996; Gevers, 2000).

On the other hand, a closed-loop condition presents some additional complications for system identification. The fundamental problem is the correlation between the output error and the input through the feedback controller. Because of the correlation, many identification methods that are proven to work with open-loop data can fail. This is true for the prediction error approach as well as the subspace approach and nonparametric approaches like empirical transfer function estimation.

Awareness of the potential failings has engendered significant research efforts, which in turn have led to better understanding of the properties of the existing methods when used with closed-loop data as well as some remedies and special measures needed to circumvent the potential problems. We will try to give a concise and pragmatic summary of the recent developments, concentrating on the practical implications of the theoretical results. See Gustavsson et al. (1987); Van den Hof and Scharma (1995); Forssell and Ljung (1999a) for more comphensive surveys and formal disquisitions on the topic. We will focus on the prediction error approach (Ljung, 1987), which is the standard at the moment, but we will also point to some potential problems and remedies for the subspace approach at the end.

The closed-loop identification methods can be classified into three broad categories. In the *direct* approach, the feedback is largely ignored and the open-loop system is identified directly using measurements of the input and

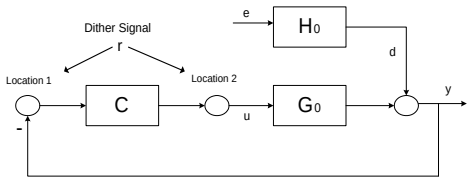

Figure 1: Typical setup for closed-loop identification.

output. In the *indirect* approach, the closed-loop transfer function between an external perturbation signal and the output (or input) is first identified and an open-loop system model is determined from it with knowledge of the controller. Finally, in the *joint input-output* approach, measurements of both the input and output are used to identify a joint system, which has the plant input and output as its outputs and the external perturbation signal as its input. From the joint system, an open-loop system model is extracted. We will examine each approach one at a time, concentrating on asymptotic properties and their implications.

The block diagram in Figure 1 displays a typical setup of closed-loop identification experiment. External perturbations may be added to the setpoint of the controller (at Location 1 in Figure 1) or directly to the controller output (at Location 2 in Figure 1). For the simplicity of exposition, we will assume from now on that external perturbations enter through Location 2. Note that, in the case that the controller is linear, this assumption can be made without loss of generality since perturbations introduced at Location 1 can always be rewritten as equivalent perturbations at Location 2.

Disturbances in the plant output are described collectively as a white noise signal (denoted by e) passed through some linear filter H_0. Without loss of generality, we will assume that H_0 is stably invertible and monic ($H_0(\infty) = I$) and the white noise signal e has the mean of zero and the covariance of P_e.

Assuming the plant G_0 and the controller C are both linear, we can write down the following closed-loop relationships:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \underbrace{(I + G_0C)^{-1}}_{S_0} G_0 & (I + G_0C)^{-1}H_0 \\ \underbrace{(1 + CG_0)^{-1}}_{S_0^r} & -C(I + G_0C)^{-1}H_0 \end{bmatrix} \begin{bmatrix} r(t) \\ e(t) \end{bmatrix} \tag{1}$$

In the above, S_0 and S_0^r represent the sensitivity function and reverse sensitivity function respectively.

Of course, both the assumption of linear plant and the particular way of describing the disturbance are great simplifications but are typical of developing and analyzing linear identification methods. In addition, the assumption of linearity of the controller, when made, may be untenable in many industrial situations where the controllers are equipped with various anti-windup and override features.

Direct Approach

In the direct approach, measurements of the plant input and output are used to build a model for the open-loop system directly, as in open-loop identification. The main advantage of the direct method is that the controller is not restricted to linear ones and its identity needs not be known. However, the correlation introduced by the feedback can cause problems and successful application demands some additional knowledge as we shall see.

The Method. The generic model structure used in linear identification is as follows:

$$y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \tag{2}$$

θ is the vector of unknown parameters of the model and e is a white noise sequence. We may restrict the search to $\theta \in \Theta_M$. In this case, $\mathcal{G}_M \triangleq \{G(q, \theta) \mid \theta \in \Theta_M\}$ and $\mathcal{H}_M \triangleq \{H(q, \theta) \mid \theta \in \Theta_M\}$ represent the allowable set for plant model G and noise model H, respectively. Let the available data be denoted by

$$D^N = [y(1), u(1), \cdots, \cdots, y(N), u(N)] \tag{3}$$

The prediction error minimization (PEM) chooses $\hat{\theta}_N$, the estimate of θ based on D^N, according to

$$\hat{\theta}_N = \arg \min_{\theta \in \Theta_M} \left[V_N(\theta, D^N)\right] \tag{4}$$

where

$$V_N(\theta, D^N) = \frac{1}{N} \sum_{t=1}^{N} \|\varepsilon(\theta, t)\|_W \tag{5}$$

$$\varepsilon(\theta, t) = H^{-1}(q, \theta) \left[y(t) - G(q, \theta)u(t)\right] \tag{6}$$

and $\|x\|_W \triangleq x^T W x$ denotes the weighted quadratic norm.

Notation: We will often use symbols $\hat{G}_N$ and $\hat{H}_N$ to denote $G(q, \hat{\theta}_N)$ and $H(q, \hat{\theta}_N)$. The same symbols may also be used to compactly represent $G(e^{j\omega}, \hat{\theta}_N)$ and $H(e^{j\omega}, \hat{\theta}_N)$ when the context makes their meanings clear. Similarly, we will use G_θ and H_θ to denote $G(q, \theta)$ and $H(q, \theta)$, or sometimes $G(e^{j\omega}, \theta)$ and $H(e^{j\omega}, \theta)$.

Convergence Behavior. If all the signals are quasi-stationary, $V_N(\theta, D^N) \to \bar{V}$ and $\hat{\theta}_N \to \bar{\theta}$ w.p. 1 as $N \to \infty$, where

$$\bar{V}(\theta) \triangleq \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E\{\|\varepsilon(\theta, t)\|_W\} \tag{7}$$

and

$$\bar{\theta} \triangleq \arg\min_{\theta\in\Theta_M} \bar{V}(\theta) \tag{8}$$

From Parseval's relation,

$$\bar{\theta} = \arg\min_{\theta\in\Theta_M} \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{tr}\left[W\Phi_\varepsilon\right] d\omega \tag{9}$$

where Φ_ε is the spectrum of the prediction error ε.

Since from (6),

$$\varepsilon(t) = H_\theta^{-1}\left[G_0u(t) + H_0e(t) - G_\theta u(t) - H_\theta e(t)\right] + e(t) \tag{10}$$

and $e(t)$ is independent of $(G_0 - G_\theta)u(t)$ and $(H_0 - H_\theta)e(t)$ because $(G_0 - G_\theta)$ and $(H_0 - H_\theta)$ both contain at least one delay,

$$\Phi_\varepsilon = H_\theta^{-1}\Delta_{GH}H_\theta^{-1*} + P_e \tag{11}$$

where

$$\Delta_{GH} = \begin{bmatrix}(G_0 - G_\theta) & (H_0 - H_\theta)\end{bmatrix}\begin{bmatrix}\Phi_u & \Phi_{ue}\\ \Phi_{eu} & P_e\end{bmatrix}\begin{bmatrix}(G_0^* - G_\theta^*)\\ (H_0^* - H_\theta^*)\end{bmatrix} \tag{12}$$

Hence,

$$\bar{\theta} = \arg\min_{\theta\in\Theta_M}\int_{-\pi}^{\pi} \mathrm{tr}\left\{\Delta_{GH}H_\theta^{-1*}WH_\theta^{-1}\right\} d\omega \tag{13}$$

The expression in (13) provides some good insights into the convergence behavior under the direct approach.

- Suppose $G_0 \in \mathcal{G}_M$ and $H_0 \in \mathcal{H}_M$ (i.e., the true plant and noise process both lie inside the parameterized model sets). Then, it is clear from the objective function that the minimum corresponds to $G_{\bar{\theta}} = G_0$ and $H_{\bar{\theta}} = H_0$ (implying consistent estimation) if

$$\Phi_x = \begin{bmatrix}\Phi_u & \Phi_{ue}\\ \Phi_{eu} & P_e\end{bmatrix} > 0 \qquad \forall\omega.$$

- In the case of a linear controller,

$$\begin{aligned}&\begin{bmatrix}\Phi_u & \Phi_{ue}\\ \Phi_{eu} & P_e\end{bmatrix}\\ &= \begin{bmatrix}\underbrace{S_0^r\Phi_r S_0^{r*}}_{\Phi_u^r} + \underbrace{CS_0H_0P_eH_0^*S_0^*C^*}_{\Phi_u^e} & \underbrace{-CS_0H_0P_e}_{\Phi_{ue}}\\ \underbrace{-P_eH_0^*S_0^*C^*}_{\Phi_{eu}} & P_e\end{bmatrix}\\ &= \begin{bmatrix}\Phi_u^r + \Phi_{ue}P_e^{-1}\Phi_{eu} & \Phi_{ue}\\ \Phi_{eu} & P_e\end{bmatrix}\\ &= \begin{bmatrix}I & \Phi_{ue}P_e^{-1}\\ 0 & I\end{bmatrix}\begin{bmatrix}\Phi_u^r & 0\\ 0 & P_e\end{bmatrix}\begin{bmatrix}I & 0\\ P_e^{-1}\Phi_{eu} & I\end{bmatrix}\end{aligned} \tag{14}$$

  Note that Φ_u is expressed as sum of two components, Φ_u^r representing the contribution from external dithering and Φ_u^e the contribution from noise feedback. From (14), it is clear that $\Phi_x > 0$ iff $\Phi_u^r > 0$.

- If the controller is nonlinear or time-varying, Φ_x may be positive definite even without $\Phi_r > 0$.
- Suppose $G_0 \in \mathcal{G}_M$ but $H_0 \notin \mathcal{H}_M$ (implying the noise part is undermodelled). Then, from (13), we see that $\hat{G}_N$ does not converge to G_0 as $G_\theta = G_0$ does not achieve the minimum of (13) in this case.
- When the noise model is completely fixed a priori (as in an *Output Error* structure), $\hat{G}_N$ does not converge to G_0 in general. From expression (13), we see that $\hat{G}_N \to G_0$ only when the assumed noise model is perfect or $\Phi_{ue} = 0$, which corresponds to the open-loop case.

Error Analysis. The error behavior in the limit can be formalized as follows.

Bias

Bias refers to the expected error $E\{\theta_0 - \hat{\theta}_N\}$. In terms of our notations, it is $\theta_0 - \bar{\theta}$ (or $G_0 - G_{\bar{\theta}}$ in terms of frequency-domain transfer function). An expression for bias in the limit can be obtained by further manipulating equation (13) into the following expression (Forssell and Ljung, 1999b).

$$\begin{aligned}\bar{\theta} = \arg\min_{\theta\in\Theta_M}\int_{-\pi}^{\pi} \mathrm{tr}\{&[(G_0 + E_\theta - G_\theta)\Phi_u(G_0 + E_\theta - G_\theta)^*\\ &+ (H_0 - H_\theta)(P_e - \Phi_{eu}\Phi_u^{-1}\Phi_{ue})\\ &(H_0 - H_\theta)^*]\,H_\theta^{-1*}WH_\theta^{-1}\}\, d\omega\end{aligned} \tag{15}$$

where

$$E_\theta = (H_0 - H_\theta)\Phi_{eu}\Phi_u^{-1} \tag{16}$$

From (15), we can conclude the followings:

1. If the parameterization of the plant/noise model is flexible enough that $G_\theta \in \mathcal{G}_M$ and $H_\theta \in \mathcal{H}_M$,

   $$G_{\bar{\theta}} = G_0 \text{ and } H_{\bar{\theta}} = H_0$$

   provided that the minimum is unique, which is guaranteed by $\Phi_u^r > 0$. In this case, $\hat{G}_N$ is an unbiased estimate of G_0.
2. Suppose noise model is fixed a priori as H_M, which does not have any dependence on θ. (Alternatively, assume that separate sets of parameters are used for G and H as in Box-Jenkins model). Also suppose that $G_\theta \in \mathcal{G}_M$. Then, we can conclude from the above that

   $$(G_0 - G_{\bar{\theta}}) = (H - H_M)\Phi_{eu}\Phi_u^{-1} \tag{17}$$

   Hence, $(H - H_M)\Phi_{eu}\Phi_u^{-1}$ is the bias. The bias will be zero if one or both of the following conditions is satisfied.

   - The assumed noise model is perfect, i.e., $H_M = H_0$.

- $\Phi_{eu} = 0$, which implies open-loop testing.

We also note that the size of bias depends on the following two things:

- $(H - H_M)$, which is the error in the assumed noise model.
- $\Phi_{eu}\Phi_u^{-1}$, which is affected by, among many things, the power of the external perturbation signal relative to the noise signal. Note that

$$\Phi_{eu}\Phi_u^{-1} = (P_e\Phi_u^{-1}) \times (\Phi_u^e\Phi_u^{-1}) \tag{18}$$

$P_e\Phi_u^{-1}$ is the noise-to-signal ratio in an open-loop sense. What multiplies this is $\Phi_u^e\Phi_u^{-1}$, which can be interpreted as the relative contribution of the noise feedback to the total input power. The larger the noise feedback's contribution, the bigger the bias. Note that increasing the controller gain will decrease $P_e\Phi_u^{-1}$ but will also increase $\Phi_u^e\Phi_u^{-1}$, thus making its effect on the bias inconclusive.

3. In the case of undermodeling such that $G_\theta \notin \mathcal{G}_M$ and/or $H_\theta \notin \mathcal{H}_M$, θ will be chosen to make both $G_0 - G_\theta$ and $H_0 - H_\theta$ small. Bias in the frequency domain will be distributed according to the weightings given in (15).

Variance

The other part of error is variance, which refers to the error due to an insufficient number of data points relative to the number of parameters. This error is mathematically formalized as

$$\mathrm{Cov}(\hat{\theta}_N) \triangleq E\left\{\left(\hat{\theta}_N - \bar{\theta}\right)\left(\hat{\theta}_N - \bar{\theta}\right)^T\right\} \tag{19}$$

in the parameter domain and

$$\mathrm{Cov}\left(\mathrm{vec}\hat{G}_N\right) \triangleq E\left\{\left(\mathrm{vec}\hat{G}_N - \mathrm{vec}G_{\bar{\theta}}\right)\left(\mathrm{vec}\hat{G}_N - \mathrm{vec}G_{\bar{\theta}}\right)^T\right\} \tag{20}$$

in the frequency domain, where the notation $\mathrm{vec}(\cdot)$ refers to a vectorized form of a matrix obtained by stacking the columns of the matrix into a single column.

In Zhu. (1989), it is shown for open-loop identification that, as $n \to \infty$ and $N \to \infty$,

$$\mathrm{Cov}(\mathrm{vec}\hat{G}_N) \sim \frac{n}{N}(\Phi_u)^{-T} \otimes \Phi_d \tag{21}$$

where

$$\Phi_d = H_0 P_e H_0^* \tag{22}$$

and $\otimes$ denotes the Kronecker product. The above expression shows that the asymptotic variance distribution in the frequency domain is shaped by the signal-to-noise ratio.

For a closed-loop system, it follows directly (Forssell and Ljung, 1999b) that

$$\mathrm{Cov}(\mathrm{vec}\hat{G}_N) \sim \frac{n}{N}(\Phi_u^r)^{-T} \otimes \Phi_d \tag{23}$$

Recall that $\Phi_u = \Phi_u^r + \Phi_u^e$. Thus the above says that the excitation contributed by noise feedback does not contribute to variance reduction, at least in the asymptotic case of $n \to \infty$. One can make some sense of this result by considering the extreme case that input excitation is entirely due to noise feedback. In this case, measurements of output and input contain just the information about closed-loop transfer functions $(I + G_0C)^{-1}H_0$ and $C(I + G_0C)^{-1}H_0$. If the model order is allowed to approach infinity, even perfect knowledge of the closed-loop functions yields no information about G_0 and H_0 independently. This is because, for an arbitrary choice of H_0, one can always choose G_0 to match any given closed-loop functions and vice versa (barring an invertibility problem). In such a case, variance would be infinite at all frequencies, which is consistent with expression given in (23).

An exceptional case is when noise model is perfectly known a priori. In this case, one can show that asymptotic variance distribution follows the open-loop case. Hence, noise feedback contributes just as much as external dithering to variance reduction. This is consistent with the foregoing argument as the perfect knowledge of H_0 would enable direct translation of information about the closed-loop functions into that about the open-loop function.

In situations where model order can be constrained to a finite number, the noise feedback would make some contribution to the information content for the estimation of open-loop functions G and H.

Notation: The Kronecker product between $A \in \Re^{n\times m}$ and $B \in \Re^{p\times r}$ is defined as

$$A \otimes B = \begin{bmatrix} a_{1,1}B & \cdots & \cdots & a_{1,m}B \\ a_{1,2}B & \ddots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ a_{n,1}B & \cdots & \cdots & a_{n,m}B \end{bmatrix} \tag{24}$$

Practical Implications. The following are the important points to take away from the foregoing analysis.

- The main advantage of the direct approach is that the controller is completely taken out of the picture in the estimation step. Not only is it unnecessary to know the controller, but it is also not required for the controller to be linear and time-invariant. It is an important advantage considering that most industrial controllers are not adequately represented by a linear, time-invariant operator.

- The most serious problem for the direct approach is that noise model (structure) needs to be perfect in order for it to yield consistent estimates. Fixing noise model completely a priori, as is often done in practice, results in a biased-esimate of G_0 in general. For example, the popular least squares identification of finite impulse response parameters would give a biased model. This is the most important difference from the open-loop identification case.

- The theoretical result indicates that consistent estimates can be obtained by leaving noise model sufficiently flexible (so that $H_\theta \in \mathcal{H}_M$). Though this would indeed help reduce bias, it may not get rid of the bias problem completely in practice as most disturbances in industrial processes are non-stationary and are not accurately captured by a white noise through a linear filter. In addition, increasing the order of model would mean increased variance, thus demanding heavier external dithering.

- In case that noise part is undermodelled, the size of the resulting bias in G depends on the signal-to-noise ratio and the relative contribution of external dithering to the total input power (compared to noise feedback). Hence, in principle, *bias can be made negligible by overwhelming noise feedback with heavy external dithering.* On the other hand, boosting the signal-to-noise ratio by increasing the controller gain has no ameliorating effect on bias as it increases the relative contribution of noise feedback to the input power as well the signal-to-noise ratio .

- External dithering with a persistently exciting signal is necessary for consistent estimation. It is generally not sufficient to have an input that is persistently exciting. The requirement for external dithering may be removed by using a nonlinear controller or a time-varying controller.

- Asymptotic variance of a frequency transfer function estimate is shaped by input excitation *achieved through external dithering.* The excitation through noise feedback contributes little to variance reduction when model order is high. This means relying solely on noise feedback for input excitation can lead a very bad result (an estimate of infinite variance). Exception is the case where the noise model is accurately known a priori. However, such a case would be rare in practice, and any error in the a priori fixed noise model could translate into a bias in the estimate for G_0.

- External dithering is motivated from both the viewpoint of bias and of variance. If the noise structure is left very flexible, variance would be a significant problem without external dithering, no matter how much input excitation is there through noise feedback. On the other hand, if the noise structure is very restricted (for example, fixed completely), external dithering would be needed to reduce bias.

- Choice of location for dithering (between Location 1 and Location 2) is not important as a perturbation made at one location can always be translated into an equivalent perturbation at the other location. On the other hand, the perturbation signal (*e.g.*, its spectrum) should be designed accordingly. To see this, let us compare the expressions for the resulting input spectrum (Φ_u^r), which appears in the asymptotic variance expressions of (23), under the two dithering strategies. For dithering at Location 1, we obtain

$$\Phi_u^r = CS_0\Phi_r S_0^* C^* \tag{25}$$

  Dithering at Location 2 gives

$$\Phi_u^r = S_0^r\Phi_r S_0^{r*} \tag{26}$$

  Loops with integral controllers yield S_0 (or S_0^r) that starts from 0 at $\omega = 0$ and increases to 1 at high frequencies. Hence, a white noise perturbation signal at Location 2 would give Φ_u^r that is zero at $\omega = 0$ and very low in the frequency region well below the controller's bandwidth. In view of (23), this would make the low frequency part of the model very poor. Estimation of the low frequency dynamics is exacerbated by the typical shape of noise spectrum Φ_v, which is high in the low frequency region for most process control problems. On the other hand, white noise dithering at Location 1 does not suffer from this problem to a same degree since $CS_0 \approx G_0^{-1}$ in the low frequency region and $\approx I$ in the high frequency region. However, white noise dithering signal is generally not optimal, regardless of the location, and its spectrum could be designed systematically based on estimated noise spectrum and desired variance distribution.

- In many practical situations, disturbances process sees during an identification experiment are non-stationary and better represented by a model that contains integrators such as the one shown below:

$$y = G(q)u + H(q)\frac{1}{1-q^{-1}}e$$
$$\Rightarrow \quad \Delta y = G(q)\Delta u + H(q)e \tag{27}$$

  From the right-hand-side of the arrow in the above, we see that differencing the input and output data prior to applying the PEM makes this case equivalent to the standard case. All the foregoing discussions regarding the signal spectra hold with respect to the differenced signals. For example, to distribute the variance fairly evenly across the frequency, one should use a dithering strategy where

an *integrated white noise* signal (or an integrated PRBS) is added to the controller's setpoint. This would make Δr a white noise signal in our analysis. On the other hand, differencing of noisy data can amplify the noise effect and make the identification more difficult.

Indirect Approach

In the indirect approach, measurements of plant output (or input) is used along with record of an external dither signal to build a model for the closed-loop system first. Then, based on knowledge of the controller, an estimate for open-loop function is extracted from the estimate of the closed-loop function. Note that

$$y(t) = \underbrace{(I + G_0C)^{-1}G_0}_{T_0^{yr}} r(t) + \underbrace{(I + G_0C)^{-1}H_0}_{T_0^{ye}} e(t) \quad (28)$$

Hence, in the first step, T_0^{yr} is estimated using data record for r and y, and in the second step, G_0 is extracted from the estimate.

The main advantage of the indirect approach is that the first step is in essence the same problem as open-loop identification because one does not have to be concerned with any feedback-induced correlation between the system input (r) and the noise (e). This removes the requirement of perfect noise model (structure) for consistent estimation. On the other hand, the second step requires a mathematical representation of the controller, which generally has to be assumed linear in order to extract the open-loop functions from the closed-loop function. In addition, the resulting model can be of very high order, depending on the parameterization used.

The Method. The generic model structure used here is

$$y(t) = T^{yr}(q, \theta)r(t) + T^{ye}(q, \theta)e(t) \quad (29)$$

The same PEM can be applied to the above model structure with data record of

$$D^N = [y(1), r(1), \cdots, \cdots, y(N), r(N)]$$

to obtain estimates $\hat{T}_N^{yr}$ and $\hat{T}_N^{ye}$. Since $T_0^{yr} = (I + G_0C)^{-1}G_0$, we can obtain an estimate for open-loop function G_0 as follows:

$$\hat{G}_N = \hat{T}_N^{yr}(I - \hat{T}_N^{yr}C)^{-1} \quad (30)$$

The above calculation can result in $\hat{G}_N$ of very high order. One may obviate this problem by using the following parameterization of the closed-loop function:

$$T^{yr}(q, \theta) = (I + G(q, \theta)C)^{-1}G(q, \theta) \quad (31)$$

However, adoption of such a parameterization would make parameter estimation (via PEM) more complex.

Another concern may be that resulting $\hat{G}_N$ may not even give a stable closed loop under the controller C. To ensure that the model is stabilized by the controller, one can first parameterize the set of all linear plants that are stabilized by C using the dual Youla parameterization and then perform the search over the set (de Callafon and Van den Hof, 1996).

By applying the expression of (13) to the indirect identification, convergence behavior can be easily analyzed. It is not discussed here since the problem is essentially the same as the open-loop case. Error analysis can also be carried out in a similar manner as before. Asymptotic variance of $\hat{G}_N$ under the indirect method is the same as in the direct method and follows (23).

Key Points.

- To obtain the closed-loop function, one can use any proven open-loop identification method without modification.
- Consistent estimation is possible even without a perfect noise model (structure), which is the main attractive point for the indrect approach.
- The most serious problem for the indirect approach is the assumption of linear controller, which may be untenable in many industrial situations. Most industrial controllers are equipped with special anti-windup/override features and their behavior may not be represented accurately by a single linear function. One needs to be careful that these special features do not become active during data collection. An error in the controller representation will translate into an error in extracting the open-loop function from the closed-loop function.
- Another potential disadvantage is high model order that often results from the two step calculation. This problem may be obviated by employing a particular parameterization involving a parameterized form of the open-loop function (such as the one in (31)). On the other hand, with such a specialized structure, the prediction error minimization becomes more demanding computationally.
- The asymptotic variance distribution in the frequency domain follows the same expression as in the direct identification case. Hence, all the previous remarks regarding shaping of variance distribution apply here as well.

Joint Input-Output Approach

The idea behind the joint input-output approach is to use measurements of both the output and input to remove the requirement of a known controller. Since

$$y(t) = \underbrace{G_0(I + CG_0)^{-1}}_{T_0^{yr}} r(t) + \underbrace{(I + G_0C)^{-1}H_0}_{T_0^{ye}} e(t) \quad (32)$$

and

$$u(t) = \underbrace{(I + CG_0)^{-1}}_{T_0^{ur}} r(t) + \underbrace{-C(I + G_0C)^{-1}H_0}_{T_0^{ue}} e(t) \quad (33)$$

we have

$$G_0 = T_0^{yr} \left(T_0^{ur}\right)^{-1} \quad (34)$$

The above relation can be used to calculate the open-loop function from the two closed-loop functions without knowledge of the controller.

In effect, in the joint input-output approach, the controller is "identified." Even though the controller is not required to be known, it is implicitly assumed to be linear and time-invariant, an assumption that may not always be justified in practical situations.

The Method. In the first step, the following model form is used:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} T^{yr}(q,\theta) \\ T^{ur}(q,\theta) \end{bmatrix} r(t) + \begin{bmatrix} T_M^{ye}(q) \\ T_M^{ue}(q) \end{bmatrix} e(t) \quad (35)$$

In the above, we assumed that the noise part of the model is fixed a priori but it too can be parameterized for estimation. Note that consistent estimates can be obtained with a fixed noise model in this case since r and e are uncorrelated.

To the above, PEM can be applied with the data record of

$$D^N = [y(1), u(1), r(1), \cdots\cdots\cdots, y(N), u(N), r(N)]$$

to obtain estimates $\hat{T}_N^{yr}$ and $\hat{T}_N^{ur}$. Then open-loop plant estimate $\hat{G}^N$ is obtained by

$$\hat{G}_N = \hat{T}_N^{yr} \left(\hat{T}_N^{ur}\right)^{-1} \quad (36)$$

One drawback is that $\hat{G}_N$ so obtained may be of very high order. This can be circumvented by using a model form that recognizes the underlying structure, such as the one below:

$$T^{yr}(q,\theta) = G(q,\theta_1)T^{ur}(q,\theta_2) \quad (37)$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

By adopting the above structure, $G(q,\theta_1)$ is identified directly, rather than through the inversion in (36).

Two-Stage Method and Projection Method. Among the variations of the above method are the so called *two-stage method* by Van den Hof and Schrama. (1993) and *projection method* by Forssell and Ljung (2000a). Note that

$$u(t) = \underbrace{(I + CG_0)^{-1}r(t)}_{u^r(t)} + \underbrace{-C(I + G_0C)^{-1}H_0e(t)}_{u^e(t)} \quad (38)$$

Hence, u_r represents the portion of the input generated by signal r and u^e is the portion due to noise feedback. Now,

$$y(t) = G_0u^r(t) + G_0u^e(t) + H_0e(t) \quad (39)$$

Since u^r is uncorrelated with the rest of the right-hand-side, a consistent estimate of G_0 can be obtained with data for y and u^r. This consideration leads to the following method:

1. Start with the parameterized structure

$$u(t) = T^{ur}(q,\theta)r(t) + T_M^{ue}(q)e(t) \quad (40)$$

   Apply PEM to the above model with data record D^N and obtain $\hat{T}_N^{ur}$.

2. Obtain data for u^r through

$$\hat{u}_N^r(t) = \hat{T}_N^{ur}r(t), t = 1, \cdots, N \quad (41)$$

3. Apply PEM to the model structure

$$y(t) = G(q,\theta)u^r(t) + T_M^{ye}(q)e(t) \quad (42)$$

   using the data record of

$$D^N = [\hat{u}_N^r(1), y(1), \cdots, \cdots, \hat{u}_N^r(N), y(N)]$$

   to obtain $\hat{G}_N$.

In the projection method, T^{ur} is allowed to be an *acausal* operator by parameterizing it as a two-sided FIR filter:

$$T^{ur}(q,\theta) = \sum_{i=-n_1}^{n_2} \theta_i q^{-i} \quad (43)$$

This is to ensure that the resulting $\hat{u}_N^r$ is uncorrelated ("orthogonal" in the least squares language) asymptotically with $u - \hat{u}_N^r$, which becomes a part of the residual in the second PEM. Recall that, in the least squares estimation, the residual has to be orthogonal to the regressor in order to obtain a consistent estimate (Ljung, 1987). If the controller is linear, all the preceding argument holds and the orthogonality of the residual can be assured (asymptotically) with a sufficiently rich yet causal T_θ^{ur}. However, with a nonlinear or time-varying controller, $\hat{u}_N^r$ obtained with any casual $\hat{T}_N^{ur}$ and $u - \hat{u}_N^r$ may be correlated, thereby destroying the consistency of estimation in the final step. By employing an acausual FIR filter with sufficiently large n_1 and n_2 for $T^{ur}(q,\theta)$, one gets $\hat{u}_N^r$ that is uncorrelated (orthogonal) with $u - \hat{u}_N^r$, even when the controller is nonlinear or time-varying.

The convergence behavior is essentially the same as for the indirect approach and does not require an elaborate discussion here. Error behavior too is similar to that for the indirect approach. The variance also follows the same expression of (23). The same comments apply to the two-step method and the projection method.

Closed-Loop method	Perfect Noise Model	External Dithering	Linear Controller	Known Controller
Direct	Yes	Yes*	No	No
Indirect	No	Yes	Yes	Yes
Joint I/O	No	Yes	Yes	No
Two-Step	No	Yes	Yes	No
Projection	No	Yes	No	No

*Unless the noise model is perfectly known *a priori*

Table 2: Summary of the requirements of different closed-loop identification methods.

Key Points. Some key points for the joint I/O method and its off-springs are:

- As with the indirect approach, the main advantage of the joint input-output approach over the direct approach is that consistent estimates can be obtained even with an imperfect noise model.
- The main advantage of the joint input-output approach over the indirect approach is that explicit knowledge of the controller is not required. However, it does implicitly assume that the controller is linear.
- The two step method is essentially same as the standard joint input-output method and does not appear to offer any new advantage.
- The projection method, on the other hand, further improves it by removing the requirement of linear controller for consistent estimation. It also retains the aforementioned advantage over the direct approach. The relaxation of the requirement for linear controller behavior is practically significant in view of the fact that most industrial controllers show some degree of nonlinear behavior due to various fixes and add-ons.
- Measurement errors for the input do not destroy the consistency if they are uncorrelated with the dither signal.

The practical requirements of the different closed-loop identification approaches are summarized in table 2.

Subspace Identification with Closed-Loop Data

So far, our discussion has centered around PEM. An alternative to PEM is subspace identification, which has drawn much attention in recent years. The main attraction for the subspace approach is that it yields a *multivariable system model* without the need for a special parameterization, which requires significant prior knowledge and nonconvex optimization. Both represent significant hurdles for using PEM for multivariable system identification, which explains why it is hardly used for this purpose. Most subspace methods in the literature can fail, however, when used with closed-loop data. By "fail", we mean that the guarantee of an asymptotically unbiased estimate, a nice proven property for most subspace methods, is lost. A practical implication is that, with closed-loop data, the method may yield a poor model regardless of number of data points used. Here, we will briefly examine what fundamental problem closed-loop data pose for the subspace method and examine some available modifications to the standard approach in order to circumvent the problem.

Main Idea of Subspace Identification. We first review the subspace method. Though many versions exist in the literature, the essential ideas are same and the practical outcomes from applying different algorithms should not differ much (Van Overschee and De Moor, 1995). We will discuss one of the most popular methods, called N4SID, which was introduced by Van Overschee and De Moor (1995).

The underlying plant is assumed to be

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + w(t) \\ y(t) &= Cx(t) + Du(t) + \nu(t) \end{aligned} \tag{44}$$

where $w(t)$ and $\nu(t)$ are white noise processes. The state-space description is very general and subsumes most of the input-output structures studied in the system identification literature.

The following is an alternative representation of the above when (44) is viewed as an input-output system description:

$$\begin{aligned} x_\infty(t+1) &= Ax_\infty(t) + Bu(t) + K_\infty\varepsilon_\infty(t) \\ y(t) &= Cx_\infty(t) + Du(t) + \varepsilon_\infty(t) \end{aligned} \tag{45}$$

(45) can be interpreted as the steady-state Kalman filter for (44) and hence ε_∞ is the innovation sequence, which is white. The two are equivalent in an input-output sense and (45) is referred to as the innovation form for (44).

The N4SID method attempts to identify the following *non-steady-state Kalman filter* equation (within some state coordinate transformation) to obtain the parameters for the system equation in (45):

$$\begin{aligned} x_{n+1}(t+1) &= Ax_n(t) + Bu(t) + K_n\varepsilon_n(t) \\ y(t) &= Cx_n(t) + Du(t) + \varepsilon_n(t) \end{aligned} \tag{46}$$

From the notation, you may deduce that the above represents a non-steady-state Kalman filter started at $t-n$ with zero initial estimate and initial covariance set equal to the system's open-loop covariance. Successful identification of the above equation yields system matrices (A, B, C, D) for (45). In addition, with a large n, it gives a good approximation for stochastic part of the system in (45) since $K_n \to K_\infty$ and $\mathrm{Cov}\{\varepsilon_n\} \to \mathrm{Cov}\{\varepsilon_\infty\}$ as $n \to \infty$. Here n is assumed to be higher than the intrinsic system order.

For the simplicity of discussion, we will assume from hereafter that $u(t)$ is an independent (temporally uncorrelated) sequence.

The key to subspace identification is the following multi-step prediction equation:

$$\begin{aligned}\begin{bmatrix} y(t) \\ y(t+1) \\ \vdots \\ y(t+n-1) \end{bmatrix} &= \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} x_n(t) \\ &+ \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ CA^{n-2}B & \cdots & CB & D \end{bmatrix} \begin{bmatrix} u(t) \\ u(t+1) \\ \vdots \\ u(t+n-1) \end{bmatrix} \\ &+ \begin{bmatrix} \varepsilon_n(t|t-1) \\ \varepsilon_n(t+1|t-1) \\ \vdots \\ \varepsilon_n(t+n-1|t-1) \end{bmatrix} \end{aligned} \quad (47)$$

$\varepsilon_n(t+j|t-1)$ represents the $j+1$-step-ahead prediction error (based on the Kalman estimate $x_n(t)$). The above will be denoted by

$$\mathcal{Y}_n^{0+}(t) = \Gamma_n^o x_n(t) + H_n^f U_n^{0+}(t) + \mathcal{E}_n^{0+}(t) \quad (48)$$

A key point is that $\mathcal{E}_n^{0+}(t)$ is orthogonal to both $x_n(t)$ and $U_n^{0+}(t)$. Now, since the Kalman filter is linear with respect to the measurement and the input, we can express

$$x_n(t) = M_1 \mathcal{Y}_n^-(t) + M_2 \mathcal{U}_n^-(t) \quad (49)$$

where

$$\mathcal{Y}_n^-(t) = \begin{bmatrix} y(t-n) \\ y(t-n+1) \\ \vdots \\ y(t-1) \end{bmatrix}; \qquad \mathcal{U}_n^-(t) = \begin{bmatrix} u(t-n) \\ u(t-n+1) \\ \vdots \\ u(t-1) \end{bmatrix} \quad (50)$$

Substituting (49) into (48) gives

$$\mathcal{Y}_n^{0+}(t) = \underbrace{\Gamma_n^o \begin{bmatrix} M_1 & M_2 \end{bmatrix}}_{H_n^p} \begin{bmatrix} \mathcal{Y}_n^-(t) \\ \mathcal{U}_n^-(t) \end{bmatrix} + H_n^f U_n^{0+}(t) + \mathcal{E}_n^{0+}(t) \quad (51)$$

By arranging the data for y and u appropriately based on the above equation, one can obtain estimates for H_n^p and H_n^f through linear least squares. The least squares estimation yields consistent estimates of H_n^p and H_n^f as residual $\mathcal{E}_n^{0+}(t)$ is orthogonal to regressors $\mathcal{Y}_n^-(t), \mathcal{U}_n^-(t)$ and $U_n^{0+}(t)$.

Using estimate $\hat{H}_n^p$, one can determine the system order (by examining its rank) and obtain estimates for Γ_n^o,M_1 and M_2 (within a coordinate transformation). Finally, data for the Kalman state x_n can be constructed from the estimates as

$$\hat{x}_n(t) = \begin{bmatrix} \hat{M}_1 & \hat{M}_2 \end{bmatrix} \begin{bmatrix} \mathcal{Y}_n^-(t) \\ \mathcal{U}_n^-(t) \end{bmatrix}, \qquad t = n, \cdots, N-n \quad (52)$$

The whole calculation can be done by performing some matrix projections with appropriately arranged data matrices, which can be implemented in a computationally efficient and robust way (Van Overschee and De Moor, 1996).

By following a similar procedure, one can also obtain data for $x_{n+1}(t+1), t = 1, \cdots, N-n$.

After the data for $x_n(t)$ and $x_{n+1}(t+1)$ are obtained, the system matrices are estimated based on the following equation:

$$\begin{bmatrix} x_{n+1}(t+1) \\ y(t) \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} x_n(t) + \begin{bmatrix} B \\ D \end{bmatrix} u(t) + \begin{bmatrix} K_n \\ I \end{bmatrix} \varepsilon_n(t) \quad (53)$$

Again, since $\varepsilon_n(t)$ is orthgonal to $x_n(t)$ and $u(t)$, linear least squares gives consistent estimates of A, B, C, and D. One can also obtain consistent estimates of K_n and $P_n(\triangleq \mathrm{Cov}\{\varepsilon_n(t)\})$ using the residuals from the least squares.

In overall, one obtains consistent estimates of the Kalman filter matrices in (46) since the first least squares yields a consistent estimate of H_n^p and therefore of $x_n(t)$ (within a coordinate transformation), and the second least squares yields a consistent estimate of (A, B, C, D, K_n, P_n) provided consistent estimates of $x_n(t)$ and $x_{n+1}(t+1)$ are used.

With the obtained estimates denoted hereafter by $\left(\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{K}, \hat{P}\right)$, one can form the state-space model

$$\begin{aligned} x(t+1) &= \hat{A}x(t) + \hat{B}u(t) + \hat{K}e(t) \\ y(t) &= \hat{C}x(t) + \hat{D}u(t) + e(t) \end{aligned} \quad (54)$$

where e is assumed to be a white noise process of covariance $\hat{P}$. The deterministic part of the model is unbiased and the stochastic part is slightly biased due to mismatch between K_n and K_∞ as well as that between P_n and P_∞. *Note:* If the input u is not white, the assumed initialization of the non-steady-state Kalman filter should be altered slightly in order to make $\mathcal{E}_n^{0+}$ uncorrelated with $x_n(t)$ and $U^{0+}(t)$. In this case, the proper initial estimate to assume for the Kalman Filter is not zero, but a function of $\mathcal{U}_n^-(t)$ and $\mathcal{Y}_n^-(t)$. This creates a slight inconsistency between the assumed initialization of the underlying Kalman filter for $x_n(t)$ and that for $x_{n+1}(t+1)$,

and complicates the procedure somewhat. The details of the modifications needed to save the nice asymptotic property are worked out in (Van Overschee and De Moor, 1994, 1996). This is mostly a theoretical concern, however. Since the effect of initialization on the Kalman estimate becomes negligible as $n \to \infty$, the basic algorithm works well with a large n, even when the input is not white.

Problem with Closed-Loop Data. With closed-loop data, consistency for the first least squares estimation breaks down. This is because $\mathcal{E}_n^{0+}(k)$ is no longer uncorrelated with $\mathcal{U}^{0+}$. For example, $u(t)$ is a function of $y(t)$ and therefore of $\varepsilon_n(t|t-1)$, which is correlated with $\varepsilon_n(t+1|t-1)$. Note that $u(t)$ appears in the regressor for the two-step-ahead prediction model of $y(t+1)$. Hence, the regressor becomes correlated with its residual $\varepsilon_n(t+1|t-1)$. Since $\hat{H}_n^p$ is biased, the whole argument for convergence breaks down.

The Modifications. Since in the indirect approach, system input is an external perturbation signal which has no correlation with system noise, the standard subspace method can be used to obtain a closed-loop system model. Given a state-space representation of the controller, a state-space model for the open-loop system can easily be extracted from it.

Van Overschee and De Moor (1997) discuss a way to modify the projection algorithm they use in their N4SID algorithm to account for closed-loop nature of data. Though this method does not belong to the indirect approach, the method requires knowledge of the controller as is the case for the indirect approach.

The joint input-output approach can also be integrated seamlessly with the subspace method. The subspace method can be used to identify a joint system and then an open-loop system model can be conveniently extracted from it, as shown in (Verhagen, 1993).

In addition, the two-step method and the projection method discussed earlier in the context of PEM should be applicable here. Since the steps for applying these methods would be essentially the same as before, just with PEM replaced by a subspace method, they are not discussed here.

Finally, Ljung and McKelvey (1996) proposed an alternative way to construct state data. Instead of picking the state basis from the projected data matrix, which poses a consistency problem in the case of closed-loop data, they suggested to identify a high-order ARX model and use it to calculate the n-step predictions. Note that identifying a high-order ARX model amounts to identifying just the first of the n-step predictor in (51) with a very large n. The reason for identifying only the first of the n-step predictor is that consistent estimates of the one-step-ahead predictor can be obtained even with closed-loop data, since the regressor is uncorrelated with the residual. The identified ARX model is used successively to construct the n-step predictions of y, which are obtained with all the future inputs set equal to zero (*i.e.*, $u(t) = \cdots, u(t+n-1) = 0$). Denote the resulting n-step predictions by

$$\hat{\mathcal{Y}}_n^{0+}(t) = \begin{bmatrix} \hat{y}^T(t|t-1) & \hat{y}^T(t+1|t-1) \\ & \cdots \quad \hat{y}^T(t+n-1|t-1) \end{bmatrix}^T. \quad (55)$$

From here on, $\hat{\mathcal{Y}}_n^{0+}(t)$ is treated an object equivalent to $\hat{H}_n^p \begin{bmatrix} \mathcal{Y}_n^-(t) \\ \mathcal{U}_n^-(t) \end{bmatrix}$ in the standard method. Hence, the state is created as $\hat{x}_n(t) = L\hat{\mathcal{Y}}_n^{0+}(t)$, where L contains the basis picked by examining the data matrix for $\hat{\mathcal{Y}}_n^{0+}(t)$.

Control-Oriented Process Identification

The term *control-oriented process identification* refers to a tailoring of an entire identification process to requirements of intended control. It points directly to the much needed integration between model development and controller design. The term perhaps was born in the midst of a debate on the apparent disparity between system identification methodologies and robust controller design methodologies. It was pointed out that, since most robust controller designs require both a nominal model and error bounds, more than just a nominal model should be passed from the identification step to the controller design step.

Over the course of time, the term has come to mean much more than simply providing a model description suited to robust controller design. Since achievable performance of a model-based controller, regardless of design strategy one employs, depends on the model quality, it is beneficial to shape the whole identification process based on the ultimate goal of achieving the best possible closed-loop performance. For example, the data generation step has a direct bearing on the size and distribution of model error, which in turn influence the closed-loop performance achievable with a particular controller design strategy. Hence, rather than using some universal testing procedure that ignores the characteristics of underlying plant dynamics and control objective, one should *tailor-design* the procedure to befit the plant characteristics.

Integration of identification and control design has been presented by many authors going back to Ziegler and Nichols (1942) and Åström and Hägglund (1984) who aimed at model free tuning of simple controllers. The desire for development of identification for robust control was indicated by Andersen et al. (1991)at CPC-IV. Theoretical interest in identification for control design has grown significantly during the past decade and the intensive research has yielded some useful insights and methodologies.

One fundamental understanding that emanated from

the research is the necessity of iteration. For example, control-oriented design of a data-gathering experiment demands that one relates the design parameters to model error and then ultimately to the closed-loop performance. Under well-defined assumptions on the prior model set and data set (or prior probability distributions on the model parameters and measurement errors), it is possible to relate design parameters (*e.g.*, test signals) to model error. The more complicated half of the puzzle is how model error degrades closed-loop performance. This degradation depends on the nominal model as well as the controller design method employed (*i.e.*, how the nominal model and possibly model error information gets translated into a controller). Even though the controller design method may be known a priori, the nominal model is not. This leads to the necessity of approximating it through iteration (Cooley and Lee, 2001).

Even though the complex relationship between model error and closed-loop performance is the challenging aspect of the problem from a theoretical viewpoint, the issue of how design parameters influence model quality is by no means simple and in fact may be a more serious barrier to practical use of most available approaches. This is because the question inherently requires prior information on plant and measurement error (as deterministic bounds or probability distributions), which are not readily available or easily expressed in the required mathematical terms.

The problem of control-oriented identification can also be posed in the context of closed-loop identification, and in fact most research in this area has followed this route. The main result is that, to minimize the closed-loop error that will ultimately result from a model-based controller, the identification of the model should use closed-loop data produced with the very controller (Gevers, 2000). However, the controller is not known prior to an actual experiment and therefore iteration (between controller design and closed-loop identification) is performed with the hope that the controller converges through the iteration.

The following section presents key issues and results regarding identification of models with the aim of achieving the best possible closed-loop performance. First, we present some preliminaries related to tradeoffs between two types of identification errors. Understanding of how model error arises from different sources plays an important role during the development of identification for control. Thereafter control designs and model approximations are explained before development of identification for control is described and a methodology for joint optimization of modelling and control is presented.

Preliminaries

Identification. In identification for control, it is preferred to identify low order models, which subsequently may be used for robust controller design. Given that such low order models cannot represent the true plant over the entire relevant frequency range they will have a systematic error, *a bias error*, in addition to the inevitable noise-induced *variance error*. The relative contributions of the two types of error may be expressed in the frequency domain, by defining a model error $\tilde{T}$, where the model $T(q) = [P(q) \quad H(q)]$ contains a process and a noise transfer function $y(t+1) = P(q)u(t) + H(q)e(t)$. Thus a performance objective $\bar{J}$ may be formulated as

$$\tilde{T}(e^{i\omega}) = \hat{T}(e^{i\omega}) - T_0(e^{i\omega}) \tag{56}$$

$$\bar{J}(\tilde{T}) = \int_{-\pi}^{\pi} E\left\{\tilde{T}(e^{i\omega})C(\omega)\tilde{T}^T(e^{-i\omega})\right\} d\omega \tag{57}$$

where the Hermitian 2x2 block matrix weighting function $C(\omega)$ describes the relative importance of a good fit over the frequency range of interest as well as the relative importance of the fit of P and H respectively. Note that the expectation is due to the randomness of $\tilde{T}$. Using a first order approximation of the model error gives:

$$\begin{aligned}\tilde{T}_N(e^{i\omega},\Theta) &= \\ &= \hat{T}_N(e^{i\omega},\Theta) - T_0(e^{i\omega}) \\ &\simeq T(e^{i\omega},\Theta^*) - T_0(e^{i\omega}) + (\hat{\Theta}_N - \Theta^*)^T T'(e^{i\omega},\Theta^*) \\ &= B(e^{i\omega},\Theta) + (\hat{\Theta}_N - \Theta^*)^T T'(e^{i\omega},\Theta^*)\end{aligned} \tag{58}$$

where

$$T'(e^{i\omega},\Theta) = \frac{dT(i\omega,\Theta)}{d\Theta} \tag{59}$$

$$B(e^{i\omega}) = T(e^{i\omega}) - T_0(e^{i\omega}) \tag{60}$$

and Θ^* is the parameter estimate as $N \to \infty$.

Substituting this into (57) gives the following approximate expression Ljung (1999):

$$\bar{J} \simeq J_B + J_P \tag{61}$$

with

$$J_B = \int_{-\pi}^{\pi} B(e^{i\omega})C(\omega)B^T(e^{-i\omega})d\omega \tag{62}$$

$$J_P = \frac{1}{N}\int_{-\pi}^{\pi} tr[P(\omega)C(\omega)]d\omega \tag{63}$$

where

$$P(\omega) = T'^T(e^{-i\omega})[N\mathrm{Cov}(\hat{\Theta_N})]T'(e^{i\omega}) \tag{64}$$

The bias contribution J_B is mainly affected by the model set as well as by signal power spectrum. The variance contribution J_P on the other hand decreases with increasing amount of data and signal power, whereas it increases with the number of parameters estimated. Minimizing the objective function in (61) clearly involves

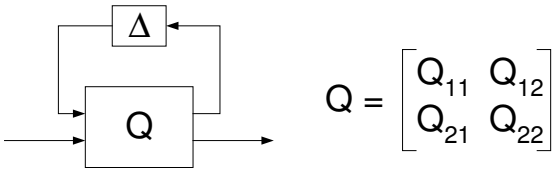

Figure 2: Representation of model perturbation by upper Linear Fractional Transformation (LFT) $F_u(Q, \Delta)$.

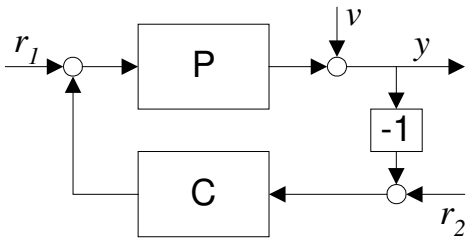

Figure 3: Feedback connection $\mathcal{T}(P_0, C)$ around a plant P_0.

making a tradeoff between bias and variance, i.e. between the model set and the number of parameters used. A proper decision on the tradeoff is key to success of any process identification scheme. The tradeoff is a factor through the design of an entire process identification process, including selection of model set and its parameterization, experimental design (e.g., test signals' power spectra), etc. The development of identification methods for control design has also exploited this tradeoff in various ways as described in the sequel.

Representation of Model Approximations. Knowledge of the nominal plant to be controlled is generally incomplete for robust controller design. Model error can be represented by a set of models. Such a set can be built up from a nominal model $\bar{P}$ along with a model perturbation Δ (Doyle et al., 1992) as illustrated in Figure 2. This Linear Fractional Transformation (LFT) model framework can represent additive as well as multiplicative uncertainty, each of which requires a different Q. In addition Q will contain the necessary information to characterize a set of models, $\mathcal{P}$. In Hansen and Fanklin (1988), a fractional model representation is described by a quotient of two stable factors parameterized via the dual Youla-Kucera parameterization. This approach is able to deal with estimation of a model set that includes both stable and unstable plants, even when operating under feedback controlled conditions. This approach is further explored by introducing a separate coefficient matrix Q (Zhou et al., 1996), where the entries depend on the nominal model $\bar{P}$ and the way in which the allowable perturbation Δ affects the nominal model. The set of models $\mathcal{P}$ may be characterized as

$$\mathcal{P} = \{P | P = F_u(Q, \Delta)\}, \text{ with } ||\Delta||_\infty < 1 \qquad (65)$$

where the upper LFT $F_u(Q, \Delta) \equiv Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12}$ assuming the inverse exists. With this representation the nominal model is $\bar{P} = Q_{22}$.

Control Design. The performance of a feedback connection $\mathcal{T}(P_0, C)$, where

$$\begin{pmatrix} y \\ u \end{pmatrix} = \mathcal{T}(P_0, C) \begin{pmatrix} r_2 \\ r_1 \end{pmatrix} \qquad (66)$$

$$\mathcal{T}(P_0, C) = \begin{pmatrix} P_0 \\ C \end{pmatrix} (I + CP_0)^{-1} \begin{pmatrix} C & I \end{pmatrix} \qquad (67)$$

which is constructed from a plant P_0 and a controller C as shown in Figure 3, can be characterized by a norm value of $J(P_0, C)$, which is a closed-loop-relevant operator. Minimization of this norm may be done through controller design directly, provided that measurements from the plant can provide information on the control objective function $J(P_0, C)$. If this norm can be assessed directly on the plant, then the controller may be tuned optimally iteratively. This approach may be termed *model-free controller tuning*. This idea has been used by several researchers. Hjalmarsson. et al. (1994) developed an iterative procedure where two to three batch experiments are performed at each iteration. This procedure has been further developed to the multivariable case by Hjalmarsson and Birkel (1998), and reviewed by Gevers (1998). The direct iterative tuning is restricted to those control objective functions that can be assessed directly from observations. However, this methodology originated from studying the interplay between identification and control. This interplay is further discussed in the following sections, where the key aspect is to relate the purpose of identification, i.e., achieving control performance, to the identification procedure.

Identification of Model for Control

The nominal model $\hat{P}$ within a model set plays an important role. Since the nominal model is the sole basis for many controller designs, its quality is very important. In this section, we review the work on identifying a nominal model that leads to a good control performance when a controller design based on nominal performance is used. The importance of the nominal model has been recognized by many researchers, (e.g., Rivera and Gaikwad, 1992; Scharma and Bosgra, 1993; Zang et al., 1995; Lee et al., 1995).

Problem Formulation. A key idea in *identification for control* is to tune the bias and variance error for control design. Such a tuning may be achieved

by bounding the actual control performance $||J(P_0, C)||$ through utilization of the nominal control performance $||J(\hat{P}, C)||$ and the performance degradation due to approximate modelling $||J(P_0, C) - J(\hat{P}, C)||$. The bounding is achieved through exploitation of the triangular inequality:

$$||J(\hat{P}, C)|| - ||J(P_0, C) - J(\hat{P}, C)|| \leq ||J(P_0, C)|| \leq ||J(\hat{P}, C)|| + ||J(P_0, C) - J(\hat{P}, C)|| \quad (68)$$

Thus a tight upper bound may be achieved by minimizing the performance degradation $||J(P_0, C) - J(\hat{P}, C)||$. For a given controller C, this minimization constitutes a "control relevant identification problem" (Gevers, 1993; Van den Hof and Scharma, 1995), where a nominal model $\hat{P}$ is found by minimizing the difference between the performance of the feedback connections $\mathcal{T}(P_0, C)$ and $\mathcal{T}(\hat{P}, C)$. From the triangular inequality it is clear that both the nominal model $\hat{P}$ and the controller C may be used to minimize the performance cost $||J(P_0, C)||$.

Alternating between minimizing the performance degradation $||J(P_0, C) - J(\hat{P}, C)||$ as an identification problem and minimizing the nominal performance $||J(\hat{P}, C)||$ as a control design problem provides an iterative scheme for subsequent identification and control design. By such a scheme it is hoped that $||J(P_0, C)||$ decreases. The development of these schemes is briefly reviewed below as they have led to several interesting identification schemes.

Exact Modeling. Some of the first attempts towards investigating the interaction between identification and control in case of exact modelling, i.e. without bias error, were presented by Åström and Wittenmark (1971) and Gevers and Ljung (1986). The latter authors mentioned that, in the case of exact modelling, to minimize norm-based performance degradation, a prediction error based estimation method can be devised that uses closed-loop experiments and appropriate data filters. However the data filters contain knowledge about the controller to be designed, and therefore an iterative procedure of identification and control design can be used to gain knowledge of the data filters. This was confirmed by Hjalmarsson et al. (1996) and Forssell and Ljung (2000b). The latter authors investigated particular experimental design issues involved in minimizing the performance degradation due to variance errors in the identified model with constraints on a linear combination of input and output variance. For the case where penalty is only upon the misfit in P, they show that the optimal controller is given by the solution to a standard LQ problem. Also the optimal reference signal is determined.

The idea of minimizing the parameter covariance matrix is also pursued by Cooley and Lee (2001) in an open-loop identification's context. The control oriented design here takes the form of a weighted trace optimal (L-optimal) design, where the weighting matrices depend on desired loop shapes as well as the estimate from the last iteration. The optimal design was originally formulated as a nonconvex optimization for the sampled data values of test inputs directly, but was later reformulated by Samyudia and Lee (2000) as a Linear Matrix Inequality problem cast in terms of the covariance of the inputs. The procedure was shown to perform much superior to the conventional PRBS tests on several ill-conditioned multivariable processes, which served as the main motivation for their development.

Although most of the above results are useful in pointing out the desire for iterative experiments, they suffer from the requirement of exact modelling of the plant P_0. This requirement inevitably leads to the requirement of estimating a high order nominal model (e.g., FIR model used in Cooley and Lee (2001)).

Approximate Modeling. Analysis of the more realistic situation, where $\hat{P}$ is considered to be an approximation of P_0 was presented by Wahlberg and Ljung (1986). These authors showed that a norm based expression can be used to characterize the bias of a model $\hat{P}$, and that this bias could be tuned provided a suitable model structure was used. Liu and Skelton (1990) proposed closed loop experiments to provide the proper weighting filters in an explicitly tunable bias expression from Wahlberg and Ljung (1986). However, *the controller to be used for closed loop experiments is still unknown.* The possibility of using closed loop experiments was proposed by Liu and Skelton (1990) and developed by Zang et al. (1995), Hakvoort et al. (1994) and Tay et al. (1997).

Gevers (2000) in his recent review discusses an approach based on the idea that minimization of the performance degradation caused by model error can be formulated as a prediction error minimization (weighted by the sensitivity function) with closed-loop data. However, for the equivalence to hold, the closed-loop data must be generated by the very controller, which is to be determined later and therefore unknown before the experiment. This naturally brings up an iterative scheme where the controller design with nominal model and the prediction error minimization with closed-loop data are alternated. Even though this approach has been applied with promising results on several examples, the convergence has been shown to fail.

The idea of providing proper weighting filters in a tunable bias expression during identification was also used by Rivera et al. (1993) and Rivera and Gaikwad (1992), where it is assumed that prefiltering of *open-loop* data from the plant can replace the benefits from closed-loop experiments. The variance error was not considered.

Although the model bias during the approximate identification is tuned towards the intended model applica-

tion in the above works, the model plant mismatch is not taken into account during the control design. In order to be able to account for both bias and variance errors in control design and also to judge the quality of the model against the performance requirement, estimation of a model set rather than just a nominal model is needed.

Identification and Robust Control Design

Estimating Model Sets. A set of models can be used to represent the incomplete knowledge of the plant P_0. The incompleteness is due to the limited availability of possible disturbed observations of the plant behaviour. Such a set of models can be considered to consist of all models that are validated by the data (Ljung, 1999). However, Smith et al. (1997) pointed out that it is never possible to validate models solely on the basis of finite number of experiments. Thus a set of models will consist of models that cannot be invalidated by the data from the plant P_0. The available data along with the prior assumptions give rise to a set of feasible models $\mathcal{F}$ (Hakvoort et al., 1994). If the prior assumptions are correct, then $P_0 \in \mathcal{F}$. However, the set $\mathcal{F}$ can be unstructured, and therefore estimation of a structured set $\mathcal{P}$ should be done such that $\mathcal{P}$ outer-bounds $\mathcal{F}$. Performance and robustness are conflicting requirements (Doyle et al., 1992), and in case of conservative control design, this conflict causes the performance of a designed controller to deteriorate. Thus $\mathcal{P}$ should be estimated in such a way that the performance degradation of a controller designed based upon $\mathcal{P}$ is as small as possible.

Hence, to enable incorporation of robustness into the design of a model based controller, a set of models must be estimated. Mainly two different approaches for estimating error bounds have appeared, which differ by the nature of underlying assumptions on the error bounds. When the prior assumptions are stochastic, so called 'soft' error bounds result. Examples here of are Goodwin et al. (1992), Bayard (1992), Rivera et al. (1993), Ninness and Goodwin (1995), and Cooley and Lee (1998). When deterministic assumptions on the data or the plant are used, non-probabilistic error bounds result. Consequently, these are called 'hard' bounds. Examples here of are Wahlberg and Ljung (1992) and Böling and Mäkilä. (1995). Combinations of both types of model error bounding may give the combined advantages (de Vries and Van den Hof, 1995; Hakvoort and Van den Hof, 1994). Some of the error bounding approaches take the intended control application into account by estimating control relevant, possibly low order, nominal models with an additive or multiplicative bound on the modelling error, e.g. Bayard (1992).

Although the approaches referenced in the two previous paragraphs treat bias and variance aspects separately, the estimation of a set of models $\mathcal{P}$ should include both bias and variance aspects. Thus the identification and construction of the Q coefficient matrix in Figure 2, is crucial for the design of a well-performing robust controller (de Callafon, 1998). Furthermore the set must be suitable for robust control design, in order to enable performance improvement of the controlled plant. Thus the structure and the estimation of the set $\mathcal{P}$ should take the performance cost $||J(P_0, C)||$ into account.

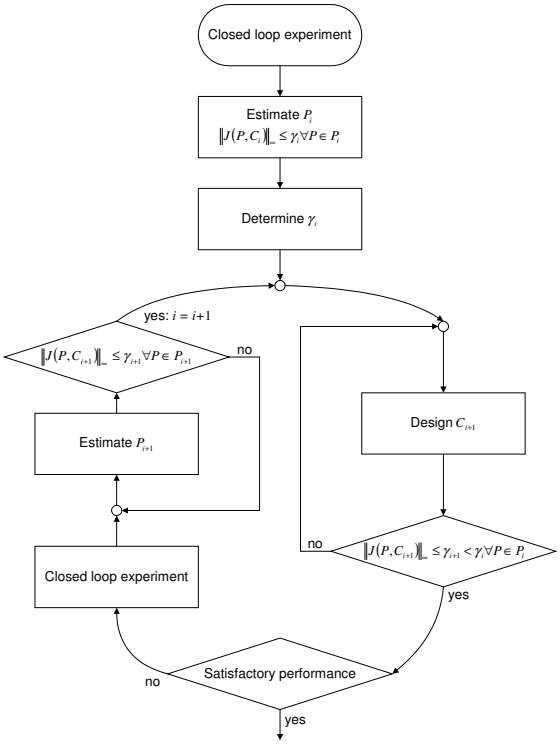

Figure 4: Iterative identification cycle for robust control.

Iterative Identification and Robust Control Design Procedure. The ultimate quest for joint optimization of modelling and control is a very complicated task, which still needs considerable research effort. A suboptimal procedure, depicted in Figure 4, has been proposed by de Callafon and Van den Hof (1997) and de Callafon (1998). One step in the procedure is that given a controller C_i at the i^{th} iteration, which together with the plant P_0 forms a stable feedback connection that satisfies the performance specification $||J(P_0, C_i)||_\infty \le \gamma_i$, then design a controller C_{i+1} that satisfies:

$$||J(P_0, C_{i+1})||_\infty \le \gamma_{i+1} \le \gamma_i \tag{69}$$

Another key step of the model-based procedure where the knowledge of the plant P_0 is represented by a set of models $\mathcal{P}_i$ is that evaluation of $||J(P_0, C_i)||_\infty$ can be achieved by evaluating $||J(P, C_i)||_\infty$ for all $P \in \mathcal{P}_i$. This enables specification of a control objective function that

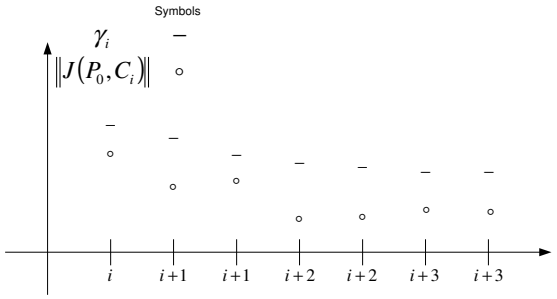

Figure 5: Progress during iterative identification cycle for robust control.

cannot be measured directly from data. Thereby three procedural steps can be outlined:

1. **Initial identification:** Use experimental data from $\mathcal{T}(P_0, C_i)$ and prior information on data or plant, to estimate a set of models $\mathcal{P}_i$ such that γ_i is minimized while $P_0 \in \mathcal{P}_i$, where
$$||J(P, C_i)||_\infty \leq \gamma_i \qquad \forall P \in \mathcal{P}_i \tag{70}$$
2. **Control Design:** Design C_{i+1} such that
$$||J(P, C_{i+1})||_\infty \leq \gamma_{i+1} \leq \gamma_i \qquad \forall P \in \mathcal{P}_i \tag{71}$$
3. **Re-Identification:** Use new experimental data from $\mathcal{T}(P_0, C_{i+1})$ and prior information to estimate a set of models $\mathcal{P}_{i+1}$ such that $P_0 \in \mathcal{P}_{i+1}$, subject to the condition
$$||J(P, C_{i+1})||_\infty \leq \gamma_{i+1} \qquad \forall P \in \mathcal{P}_{i+1} \tag{72}$$

Note that in the first step, a performance assessment is carried out to evaluate $||J(P_0, C_i)||_\infty$ for initialization purposes. Subsequently, the second step contributes a controller and the third step is a modelling validation step to enforce Equation 69. Subsequent to the initialization in step 1, repeated execution of steps 2 and 3 will provide a design procedure where the upper bound γ_i on a predetermined performance cost $||J(P_0, C_i)||_\infty$ can be progressively reduced as illustrated in Figure 5.

Implementation of this model based iterative procedure requires a procedure for estimating a set of models P tuned towards robust control design application. Such estimation of a modelling set may be carried out in a two step procedure described in de Callafon (1998). The set of models depend upon the nominal coprime factorization $(\hat{N}, \hat{D})$ and the weighting functions $(\hat{V}, \hat{W})$ that bound the model uncertainty. For the estimation, the closed loop performance based optimization
$$\min_{N,D,V,W} \sup_{P \in \mathcal{P}} ||J(P, C)||_\infty \tag{73}$$
is considered to ensure a model uncertainty set which is suitable for robust control design. The two estimation steps involve, (1)estimation of a nominal factorization such that the closed loop criterion is being minimized, subjected to internal stability of $\mathcal{T}(\hat{P}, C)$, and (2)estimation of model uncertainty, which consists of characterization of a frequency dependent upper bound on the uncertainty such that the closed-loop criterion is minimized using (V, W) subject to $P_0 \in \mathcal{P}$ (Hakvoort and Van den Hof, 1997).

Discussion. A number of important identification-related issues emerge from the above procedure:

- *Identification can be performed by open-loop experiments but more naturally by closed-loop experiments:* In order to account for the link between identification and control, identification should be performed with data that are representative of the eventual closed-loop condition. Although such data can be generated by carefully designing open-loop test signal, they are more naturally obtained by closing a loop with a sequence of controllers that gets progressively closer to the eventual controller.
- *Need for an iterative scheme:* Accounting for the intended application requires an iterative scheme of identification and feedback controller design. As the iteration continues, the feedback controller's robust performance continues to improve and this in turn generates information on the dynamics of plant that is more and more relevant for the ultimately intended robust control.
- *Model error information:* While performing iterations, information is developed on the complexity of nominal model, shape of allowable model uncertainty, and attainable robust performance. The characterization of modelling errors enables us to avoid performance degradation during the iterations. This also opens the possibility to formulate invalidation criteria for refusing models and controllers during iterations.
- *Unstable plants:* Both stable and unstable plants can be handled using the algebraic framework of stable factorizations, where a possible unstable dynamics is split into two stable factors. This split opens the possibility of an open-loop equivalent identification of the factors. The algebraic framework allows possible model errors to be described in a dual-Youla parameterization. Thus the effect of model perturbations can be studied under feedback controlled conditions. By considering a nominal stable factorization perturbed by an unknown but bounded stable operator, the set of models describe all models that are stabilized by a given feedback controller.

The above iterative procedure builds upon a large body of literature. The proposed procedure may be too complex for most applications, where the desire simply is to retune the feedback controller to improve the plant

performance. Simplifications of the procedure should be considered. Such simplifications could in fact become the standard practice of tomorrow for improving closed loop performance when requested by the operator on a routine basis as the need arises.

Summary

Closed-loop identification has progressed significantly over the past decade. Sound albeit limited theories have emerged to point out the limitations and tradeoffs for various closed-loop identification approaches. Final analysis reveals that there is no single methodology to be preferred on a universal basis and understanding of the advantages and disadvantages for the various available options is indispensable for making a right choice for a given situation. Potential benefits of closed-loop identification are huge and multi-faceted, ranging from the increased safety of the process and reduced harmful disruption to ongoing operation to the engineer's ability to collect more informative data in shorter time.

The area of identification for control holds significant promise to enable efficient, "plant-friendly" identification of multi-variable plants, including ill-conditioned plants, for which the traditional open-loop SISO identification is known to fail. Shaping of an identification process according to a specific control objective is naturally iterative in that various choices involved, e.g., shape of test signals, complexity of nominal model, shape of allowable model uncertainty and attainable performance, are highly dependent on the underlying plant. The shaping can be done in an open-loop experimental context where the data collected from previous experiments are used to improve the design of a new experiment. However, the shaping is more naturally done in a closed-loop experimental context, where data are made more and more representative of the eventual closed loop application by iteratively improving the controller. Despite the promise, more methodological developments are needed to address the practical issues, e.g., the requirement for ensuring the integrity of on-going operation, before routine application can be foreseen. Potential benefits of such methodological developments are significant in that process identification may be substantially facilitated when plants can be operated in their standard closed-loop configuration.

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