Analysis of a Class of Statistical Techniques for Estimating the Economic Benefit from Improved Process Control

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Abstract

This paper analyzes three statistical techniques commonly applied to estimate the potential benefit from improved process control. A brief overview of process control benefit estimation and a derivation of the statistical approaches used to estimate process control benefits is presented. The assumptions made in the derivation are outlined and the merits of each technique are discussed. The results from applying these techniques are then compared to the value actually obtained with improved process control using a semi-continuous distillation process.

Keywords

Process control improvement, Process control benefit estimation, Process control economic analysis

Introduction

Estimation of the economic benefit that can be obtained from implementing advanced process control and/or control system upgrades is essential for both justification and prioritization of these projects. Process control improvements result in both qualitative and quantitative benefits. The qualitative benefits can include more efficient evaluation of process and control system performance, improved access to real-time and historical process information, and better evaluation and management of abnormal conditions. Although improving the process operation, it can often be difficult to accurately and consistently assign a direct economic value to these benefits. The quantitative benefits can include improved energy efficiency, increased production rate, and decreased off-specification production. The direct economic benefits are more straightforward to determine for these cases. In this work, we consider three statistical techniques used for *a priori* estimation of the quantitative economic benefit from improved process control. These techniques have commonly been applied to justify the investment in process control projects for the petroleum and petrochemical industries (Tolfo, 1983), (Sivasubramanian and Penrod, 1990), (Martin et al., 1991), and (Latour, 1992).

These techniques are only applicable to controlled variables with an operating constraint or product specification limit and an economic incentive to operate as close as possible to this limit without excessive violation. Separation processes that require a minimum product purity are examples of a controlled variable with a product specification limit. A fired gas heater constrained by maximum tube skin or flue gas temperatures is an example of a controlled variable with an operating constraint.

Process Control Benefit Estimation

Quantitative estimation of the economic benefit from improved process control begins with determining the *base operation*. Process data, which includes the key economic controlled variables, are collected during a period of normal closed-loop operation. This collection period should be representative of the typical closed-loop operation of the process with the current control system. If the process is operated at a number of different conditions, a base operation is developed for each operating condition. The base operation mean value and variance for the controlled variables are determined from this data.

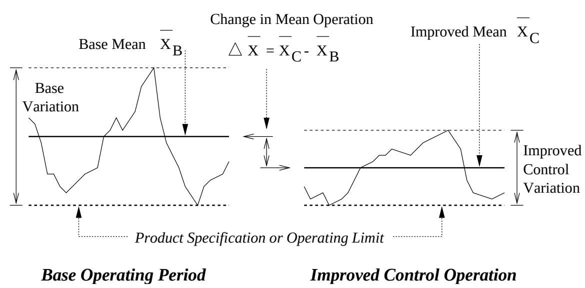

Figure 1: Improved process control operation.

Process control improvements are expected to reduce the variance of the controlled variables. Because of the reduction in the controlled variable variation, the mean operating value can be shifted closer to the product specification or operating constraint without increasing the frequency of violation. This operation is referred to as the *improved control operation* as shown in Figure 1.

The economic benefit is realized from operation at this new mean value. Quantification of the economic benefit is performed by using some form of a process model to determine the steady-state material and energy balance changes resulting from the improved control operation. Economic values are then used to estimate the monetary benefit. If there are a number of different operating conditions, this analysis is carried out for each and an average monetary benefit is determined by weighting the benefit realized from each operating mode by the fraction of time the process operates in that mode.

Predicting the change in the steady-state mean op-

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erating value obtainable from improved process control is performed in a variety of ways. Heuristic approaches include assuming the controller can consistently achieve the same performance as the best base operating data (Tolfo, 1983), assuming the controller can operate exactly at the specification or constraint (Martin et al., 1991), and assuming the process can be described by a low-order polynomial function (Stout and Cline, 1976). In this work, we discuss statistical techniques that determine the change in the mean operating value based on the reduction in the controlled variable variance.

Additional benefit may also be achieved from reducing the controlled variable variation. For many processes, such as polymerization, the product quality specifications are set by the desired end-use properties and cannot be changed due to a variance reduction. In these cases, the quantitative economic benefit comes from the reduction in variation alone. The statistical estimation techniques presented here are not appropriate for estimating this benefit and it will not be considered further.

Statistical Estimation Background

The probability that a normally distributed random variable X is less than a given value X_L is $P(X < X_L)$

$$P(X < X_L) = P(Z < Z_L) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_L} \exp\left(-\frac{Z^2}{2}\right) dZ$$

$$Z = \frac{X - \bar{X}}{S_X}, \qquad Z_L = \frac{X_L - \bar{X}}{S_X} \tag{1}$$

in which Z is the standard normal variable, $\bar{X}$ is the mean of X, and S_X is the standard deviation of X. The probability that a normally distributed random variable X is greater than a given value X_L is $P(X > X_L) = 1 - P(Z < Z_L) = P(Z < -Z_L)$ where the equalities follow from the symmetry of the normal distribution. The standard normal variable is used in these relationships to determine the probability since the integral is independent of the mean and variance of the process data.

Assuming that $P(Z < Z_L)$ also represents the fraction of samples below a given value X_L in a time series realized from a normally distributed stochastic process, $F_{\max}(Z_L) = P(Z < Z_L)$, in which $F_{\max}(Z_L)$ represents the fraction of samples in the base or improved control time series that is below the maximum limit. Under the same assumptions used in the maximum limit case, $F_{\min}(Z_L) = P(Z < -Z_L)$ where $F_{\min}(Z_L)$ represents the fraction of samples in the time series that is above the minimum limit. Note that Z_L is a negative number in the minimum limit case since the limit is less than the mean. Therefore, the fraction of samples in the time series below a maximum limit or above a minimum limit can be expressed as

$$F(Z_L) = P(Z < |Z_L|) \tag{2}$$

in which Z_L is the standard normal limit value determined from the actual process limit X_L using Equation 1.

The standard deviation of the base and improved control operation is required to determine the normalized limit values. The base operation variance, S_B^2, is determined from the base operating data. Assuming that the sensor noise is independent of the controlled variable, $S_B^2 = S_P^2 + S_M^2$, which is the sum of the contribution from the variance of the process, S_P^2, and the variance of the sensor, S_M^2. Under the same assumption, the improved control variance is the sum of the process variance after implementing the process control improvements and the sensor variance, $S_C^2 = S_I^2 + S_M^2$, in which the improved process variance is some function of the base process variance, $S_I^2 = \boldsymbol{f}(S_P^2)$. The measurement variance is typically neglected in this analysis since it is usually much smaller than the process variance. The exception is new and/or improved controlled variable sensors.

Estimating the reduction in the process variance obtainable from control system improvement is typically based on heuristics and prior experience. This reduction will depend on the process, the current control system, and the control system improvement under study. A typical assumption for advanced control implementation is a 50% reduction in the controlled variable variance (Sharpe and Latour, 1986) or standard deviation (Martin et al., 1991). Reductions in the standard deviation as large as 90% have been claimed (Tolfo, 1983). A lower bound on the achievable variance can also be estimated from the base operating data using controller performance assessment techniques (Qin, 1999) and a multiple of this value used. In this work, the improved control variance is determined from the experimental data and this value is used with each of the techniques.

Statistical Estimation Techniques

In this section, three published statistically-based techniques for determining an improved control mean operating value are presented. Each uses the base operation mean and variance and an improved control variance estimate. These techniques implicitly assume that the controlled variable time series for the base and improved control operation are realized from a strictly stationary, normally distributed, stochastic process. In addition, it is implicitly assumed that the controlled variable setpoints are not changed during the base operating period. If the setpoints change, there will be a contribution to the base variance due to the tracking control action. In this case, the setpoint deviation should be analyzed and these techniques implemented on a differential basis.

Method 1: Equal Operation at the Limit

The first method is referred to as *equal operation at the limit*. In this method, the improved control operation is required to respect the limit or specification the same fraction of the time as the base operation. Since the variation in the improved control operation is reduced, the mean operation can be moved closer to the limit. This technique is claimed to be the most common method to estimate the change in the mean operation for product quality controlled variables (Sharpe and Latour, 1986) and is applicable if an acceptable fraction of the base operation violates the limit or specification (Sharpe and Latour, 1986), (Martin et al., 1991).

The fraction of time that the limit is respected for the base and improved control operation is determined using Equation 2. Assuming that these fractions are equal results in $F(Z_L^B) = F(Z_L^C)$ in which the superscript B refers to the base operation and the superscript C refers to the improved control operation. It then follows that

$$Z_L^B = Z_L^C \quad \Rightarrow \quad \frac{X_L - \bar{X}_B}{S_B} = \frac{X_L - \bar{X}_C}{S_C} \tag{3}$$

The change in mean operation is determined from Equation 3.

$$\Delta\bar{X} = \bar{X}_C - \bar{X}_B = \left(1 - \frac{S_C}{S_B}\right)(X_L - \bar{X}_B) \tag{4}$$

Method 2: Final Fractional Violation

The second method, referred to as *final fractional violation*, is recommended when the base operation does not violate the limit or specification (Sharpe and Latour, 1986). In this case, the improved control operation is allowed to violate the limit a specified fraction of the time f. The fraction of the time the improved control operation respects the limit is then $1 - f$ resulting in $F(Z_L^C) = 1 - f \;\Rightarrow\; Z_L^C = \alpha$ in which α is determined from f using the standard normal distribution $P(Z < |\alpha|) = 1 - f$. The change in the mean operation is determined from $Z_L^C = (X_L - \bar{X}_C)/S_C = \alpha$.

$$\Delta\bar{X} = \bar{X}_C - \bar{X}_B = X_L - \bar{X}_B - \alpha(S_C) \tag{5}$$

Note that *equal operation at the limit* is a special case of this method when the fractional violation and the distributions for the base and improved control operation are assumed to be the same. Substituting $(X_L - \bar{X}_B)/S_B$ for α in Equation 5 produces the expression in Equation 4.

Method 3: Equal Fractional Violation

The third method, referred to as *equal fractional violation*, is recommended when a significant fraction of the base operation data violates the specified limit (Sharpe and Latour, 1986), (Martin et al., 1991). In this method, the limit is replaced with one that results in a more reasonable fraction of violation by the base operation. The improved control operation is then allowed the same fractional violation of this new limit. The percent violation suggested to determine the new limit for this method is 5% (Sharpe and Latour, 1986), (Sivasubramanian and Penrod, 1990), (Martin et al., 1991), although no justification for the selection of this value is given.

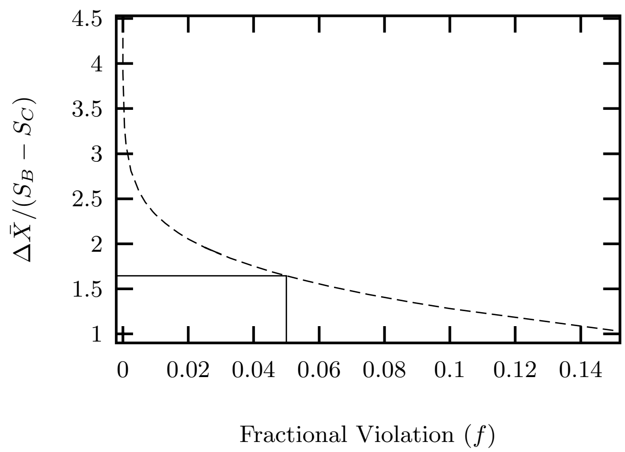

Figure 2: Equal fractional violation sensitivity to f.

The new limit, X_L^f, is that value violated by some fraction f of both the base and improved control operation. Therefore, the fraction of the time the base and improved control operation respect this limit is $1 - f$ resulting in $F(Z_f^B) = F(Z_f^C) = 1 - f$. It then follows that

$$Z_f^B = Z_f^C = \alpha \quad \Rightarrow \quad \frac{X_L^f - \bar{X}_B}{S_B} = \frac{X_L^f - \bar{X}_C}{S_C} = \alpha \tag{6}$$

The change in the mean operation is determined by eliminating the unknown limit X_L^f from Equation 6.

$$\Delta\bar{X} = \bar{X}_C - \bar{X}_B = \alpha(S_B - S_C) \tag{7}$$

The value of the new limit or specification, X_L^f, for this method is $\bar{X}_B + \alpha(S_B)$ which depends on the choice of f. The suggested value for f is 0.05 resulting in $|\alpha| = 1.645$. Note that the value of α is the ratio of the change in the mean to the change in the standard deviation, $\Delta\bar{X}/(S_B - S_C)$. As shown in Figure 2, this ratio is quite sensitive to f as the allowable violation is reduced.

Discussion of the Statistical Estimation Methods

We begin our discussion by suggesting that the *equal fractional violation* method is not an appropriate estimation technique. This method is recommended when a significant fraction of the base operation violates the specified process limit. If this limit is violated too often during the base operation, it is either not the true process limit or the base operation control system is functioning poorly. If it is not the true process limit, a more realistic limit should be determined based on process engineering, operation, and economics. It should not come from this *ad hoc* statistical procedure. If the specified limit is the true process limit, the improved control operation should be determined from the fractional violation of this limit. There is no economic justification for the use of a different limit to estimate the benefit in this case.

The first method, *equal operation at the limit*, implicitly assumes that the base and improved control operation have the same distribution. Applying the same distribution to both the base and improved control operation may be a very poor assumption in many cases. The base operation fractional violation is also obtained from the base operating data mean and variance. The actual base operation fractional violation is not used. If this computed fractional violation deviates significantly from the actual fractional violation, this method can produce erroneous results. Finally, if equal violation of the limit is not acceptable, this method is not appropriate. These issues can limit the applicability of this technique.

The second method, *final fractional violation*, only requires an assumed distribution for the improved control operation. For advanced control applications, a normal distribution is often a reasonable assumption. The base operation fractional violation can be determined directly from the base operating data. If equal operation at the limit is desired, α in Equation 5 can be determined based on this value. If the base operation very seldom or never violates the limit, a value of α from a larger fractional violation can be used. If the limit is violated too often, the specified limit can be verified and either a more realistic value determined or a value of α based on more reasonable fractional violation selected.

Experimental Investigation

We present the results of an experimental investigation to compare the predicted mean operation from the three estimation techniques to the actual improved control operation of a semi-continuous distillation process.

Process Description

The process is a twenty tray ethanol/water distillation column used to produce concentrated ethanol from a dilute feed. The overhead ethanol product is recycled back to the process. The operating objective of the column is to maximize the recovery of ethanol subject to a minimum 74 wt% purity limit for the distillate product.

The improved control system for the column is shown in Figure 3. The column differential pressure is controlled by manipulating the reboiler steam flow rate. The differential pressure target is set slightly below the value in which jet flooding occurs in order to maximize separation. The concentration of ethanol in the distillate product is measured by an on-line density meter and used to reset the distillate to feed ratio target. Reflux is determined by the overhead liquid level. Since the total overhead liquid capacity in the system is very small, the composition responds quickly to changes in the distillate flow. The principal disturbances to the column are steam quality and reflux flow rate. The feed rate and

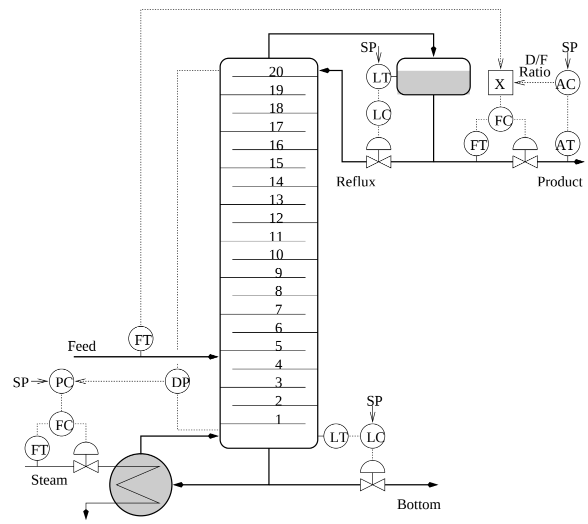

Figure 3: Distillation column control scheme.

Operation	*Mean*	*Std. Dev.*	*% Violation*
Base	79.82 wt%	2.495	4.2
Improved	74.54 wt%	0.263	4.3

Table 1: Base and improved control operation.

composition were constant for this study. Base operation composition control was accomplished by manually adjusting the distillate to feed ratio target. The base operation differential pressure controller was poorly tuned.

Process Data

Overhead composition data was sampled every two minutes for both the base and improved control operation. Base operating data was collected for 240 minutes which is approximately the normal operating cycle for the column. Improved control data was collected for 90 minutes due to adjustments made to the control system. However, we believe that this data is representative of an operating cycle with improved control. Table 1 presents the mean, standard deviation, and percent violation of the distillate product purity limit for both the base and improved control operating cycles. This study is based on the comparison between these two operating cycles.

The normalized base operating data distribution presented in Figure 4 is a bimodal distribution. Assuming a normal distribution is not appropriate in this case. The normalized improved control data distribution is presented in Figure 5. It more closely resembles a normal distribution although the tail below the mean is skewed.

Experimental Results

The improved control mean distillate composition predicted by each benefit estimation method is presented in Table 2. The value in parentheses for methods 2 and 3 is

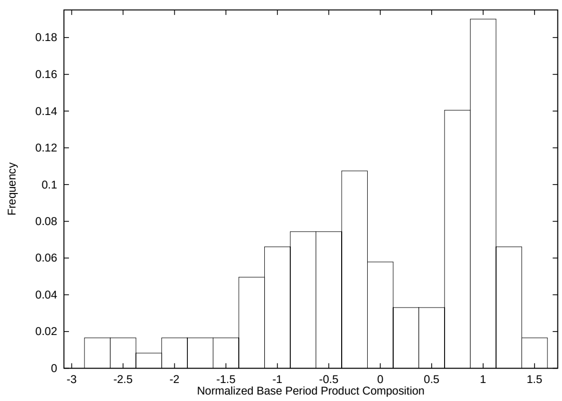

Figure 4: Distribution for base operation.

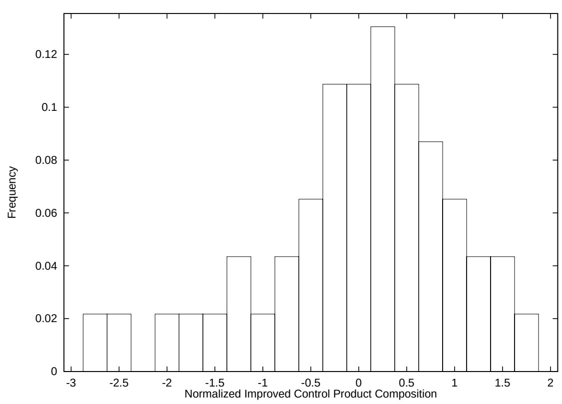

Figure 5: Distribution for improved control operation.

Benefit Estimation Method			
1	2 (5%)	2 (4.5%)	3 (5%)
74.62	74.43	74.44	76.15

Table 2: Estimated improved control mean comparison.

the percent violation specified for the method. The experimental base and improved control variance was used to determine these predictions. The actual improved control mean distillate composition was 74.54 wt%.

The first two methods, *equal operation at the limit* and *final fractional violation*, predict improved control mean distillate compositions quite close to the actual value. Since the first method assumes the same distribution for each operation, this result appears to be due to the interaction between the actual distributions in this case and is not believed to be a general result. The second method slightly under predicts the mean. Since the improved control distribution contains a larger fractional area that violates the limit than a normal distribution, this result is expected. The prediction from the third method, *equal fractional violation*, deviates significantly. The new limit assumed by this method is 75.72 wt% which helps explain the extent of the over prediction.

Conclusions

Three statistically-based techniques for *a priori* estimation of the mean operating controlled variable value after the application of process control improvements were presented. The *equal fractional violation* method is not recommended since the constraint limit or specification changes depending on the choice of the fractional violation. The *equal operation at the limit* method has limited applicability due to the restrictive assumptions made for the base operating data and equal fractional violation of the base and improved control operation. The *final fractional violation* method is the most general requiring an assumed distribution only for the improved control operation. The predictive capability of this method depends on how well the assumed distribution describes the improved control operation and how well the achievable improved control variance can be estimated.

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