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Closed-Loop PARSIMonious Subspace Identification: Theory and Application to MPC ⋆

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Abstract: We propose in this paper a novel subspace identification method, based on PARSIMonious parameterization (Qin et al., 2005), and we show that such algorithm guarantees consistent estimates of the Markov parameters with open-loop and closed-loop data. The method uses the predictor form and it effectively exploits in all steps the Toeplitz structure of the Markov parameters' matrices. After evaluation of $(A_K = A - KC, C)$ from the identified observability matrix, the method computes $(B_K = B - KD, D, K)$ and the initial condition by solving a single (well conditioned even for unstable systems) Least Squares problem. We use such method to obtain linear models for MPC design, and we show how the proposed method compares favorably with other existing subspace algorithms in two examples.

Keywords: Closed-loop identification, subspace algorithms, MPC, process control

1. INTRODUCTION

Model Predictive Control (MPC) represents an extraordinary success case in the history of automation (Qin and Badgwell, 2003). MPC algorithms use (linear) multi-variable models to forecast the process behavior over a future horizon, and solve on-line optimization problems to determine the optimal control strategy (Rawlings and Mayne, 2009). Such technologies have allowed a tremendous increase of profits, e.g. to refinery and petrochemical companies due to overall energy usage reduction, tighter quality control and throughput maximization.

The first step of an MPC application is "always" the determination, from data, of a dynamic model of the effect of the inputs (MVs, Manipulated Variables) on the outputs (CVs, Controlled Variables). In general, systems identification algorithms can be divided into two major categories, namely Prediction Error (PE) and Subspace IDentification (SID) methods. PE methods have been studied for long time and many fundamental theoretical aspects can be considered mature [see (Ljung, 1999) for an exhaustive discussion]. SID methods, are relatively young and have experienced large attention during the last 15 years [see e.g. (Verhaegen and Dewilde, 1992; Van Overschee and de Moor, 1994; Larimore, 1996; Gustafsson, 2002; Jansson, 2003; Huang et al., 2005; Chiuso and Picci, 2005; Qin et al., 2005; Katayama et al., 2005; Qin, 2006; Wang and Qin, 2006; Li et al., 2006; Katayama and Tanaka, 2007; Chiuso, 2007b; Micchi and Pannocchia, 2008) and references therein].

Model identification is usually the most critical and time-consuming phase of an MPC project (Zhu, 2001), at least, for three reasons. First of all, typically 20-50 MVs have to be perturbed, and the traditional (open-loop) patterns consisted of non-overlapped step signals (i.e. each MV is changed from a reference value with the remaining MVs fixed), whose duration allows (most) CVs to reach the new steady state. It is clear that this approach is overly time-consuming, resulting in several weeks of plant testing. Secondly, because the product quality is significantly affected during such periods, operators do not allow large steps to be introduced, thus resulting in weak signals from an identification point of view. Thirdly, a continuous human supervision during these tests is unavoidable.

In some cases, multi-variable PRBS or GBN signals may be used to mitigate such problems. However, closed-loop data collection has emerged as a nice potential solution to these issues (Jorgensen and Lee, 2001; Gevers, 2006), because the introduction of a controller can reduce duration of the identification tests significantly. Furthermore, product quality can be maintained (to some extent) within suitable limits, and the automatic controller reduces the necessity for close (continuous) test supervision. However, as it is well known from the literature (Ljung, 1999; Zhu, 2001), a fundamental problem arises from the correlation between the input variables (MVs) and the unknown measurement noise of the outputs (CVs). As a consequence of this correlation, traditional SID methods (e.g. N4SID, MOESP) may deliver biased estimates if data are collected in closed loop (Qin, 2006).

The objective of this paper is twofold: (i) propose a novel closed-loop SID algorithm with *PARSIMonious* parameterization (see later on), (ii) evaluate its performance against other existing SID methods when the identified models are used in MPC systems.

Notation. The superscript $'$ denotes the transpose operator, the superscript $^+$ denote the Moore-Penrose pseudo-inverse, I is the identity matrix and 0 is the zero matrix of appropriate dimensions. The symbol $\widehat{(\cdot)}$ denotes the estimate of $(\cdot)$, the symbol $\mathcal{E}(\cdot)$ denotes the expected value of a stochastic variable. Given a matrix $B \in \mathbb{R}^{n\times m}$, we denote with $\mathrm{vec}(B)$ the nm column vector obtained by stacking the columns of B on each other. Given four matrices (A, B, C, D) of suitable dimensions and an

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integer $n > 2$, we denote with $\mathcal{T}^n(A,B,C,D)$ the following lower block triangular Toeplitz matrix:

$$\begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{n-2}B & \cdots & CB & D \end{bmatrix} = \begin{bmatrix} \mathcal{T}^{n1}(\cdot) & 0 & \cdots & 0 \\ \mathcal{T}^{n2}(\cdot) & \mathcal{T}^{n1}(\cdot) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \mathcal{T}^{nn}(\cdot) & \cdots & \mathcal{T}^{n2}(\cdot) & \mathcal{T}^{n1}(\cdot) \end{bmatrix}$$

with $\mathcal{O}^n(A,C)$ the extended observability matrix:

$$\mathcal{O}^n(A,C) = \begin{bmatrix} C' & A'C' & \cdots & (A^n)'C' \end{bmatrix}'$$

with $\mathcal{C}^n(A,B)$ the reversed extended controllability matrix:

$$\mathcal{C}^n(A,B) = \begin{bmatrix} A^nB & A^{n-1}B & \cdots & B \end{bmatrix}$$

Given a sequence of variables $(y_0, y_1, \ldots, y_{L-1})$ of length L and two integers p, f, we define:

$$\mathbf{y}_{fi} = \begin{bmatrix} y_{p+i-1} & y_{p+i} & \cdots & y_{L-f+i-1} \end{bmatrix} \quad \text{for } i = 1, \ldots, f$$
$$\mathbf{y}_{pi} = \begin{bmatrix} y_{i-1} & y_i & \cdots & y_{L-f-p+i-1} \end{bmatrix} \quad \text{for } i = 1, \ldots, f$$
$$Y_f = \begin{bmatrix} \mathbf{y}'_{f1} & \mathbf{y}'_{f2} & \cdots & \mathbf{y}'_{ff} \end{bmatrix}' \quad Y_p = \begin{bmatrix} \mathbf{y}'_{p1} & \mathbf{y}'_{p2} & \cdots & \mathbf{y}'_{pf} \end{bmatrix}'$$

in which we notice that Y_f and Y_p are block Hankel matrices. If $F(q)$ is an $l \times n$ transfer function matrix and $\{u_k\}$ is a sequence of m dimensional vectors, we define $\varphi_k \in \mathbb{R}^{l \times nm}$ as follows:

$$\varphi_k = F(q) \otimes u'_k = \begin{bmatrix} F_{11}(q)u'_k & \cdots & F_{1n}(q)u'_k \\ \vdots & & \vdots \\ F_{l1}(q)u'_k & \cdots & F_{ln}(q)u'_k \end{bmatrix}$$

2. BASICS AND RELATED WORKS

2.1 Introduction

We consider linear time-invariant systems in *innovation* form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k \end{aligned} \tag{1}$$

in which $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the manipulated input, $y \in \mathbb{R}^l$ is the measured output, $e \in \mathbb{R}^l$ is the innovation and K is the Kalman filter gain matrix (to be identified along with the system matrices A, B, C, D). An alternative form of system (1) is the so-called *predictor* form, which can be written as:

$$\begin{aligned} x_{k+1} &= A_K x_k + B_K u_k + Ky_k \\ y_k &= Cx_k + Du_k + e_k \end{aligned} \tag{2}$$

in which $A_K = A - KC$, $B_K = B - KD$.

We consider the following standard (Qin, 2006) assumptions.

- A1. (A,B) is controllable, (A,C) is observable, and $A_K = A - KC$ is strictly Hurwitz (in discrete-time sense).
- A2. The innovation $\{e_k\}$ is a stationary, zero mean, white noise process with covariance:
$$\mathcal{E}(e_j e'_j) = R_e, \qquad \mathcal{E}(e_i e'_j) = 0 \quad \text{for } i \neq j \tag{3}$$
in which R_e is positive definite.
- A3. Data are collected for L sampling times $(0, 1, \ldots, L-1)$.
  - A3.1 In open loop, the condition $\mathcal{E}(u_i e'_j) = 0$ holds $\forall i, j$.
  - A3.2 In closed loop, if $D = 0$, the condition $\mathcal{E}(u_i e'_j) = 0$ holds for $i < j$, i.e. u_i can be in feedback with y_i. If $D \neq 0$, the condition $\mathcal{E}(u_i e'_j) = 0$ holds for $i \leq j$, i.e. u_i is in feedback with y_{i-1} (or earlier outputs).
- A4. The input $\{u_k\}$ is quasi-stationary and persistently exciting of order $f + p$ (Ljung, 1999), with f, p defined later.

Many different SID methods exist, and a number of them are briefly recalled in this section for comparison purposes and because of the relations with the proposed algorithm.

Given an integer f, referred to as the *future horizon*, the starting point is to define the block Hankel matrices $U_f \in \mathbb{R}^{mf \times N}$, $E_f \in \mathbb{R}^{lf \times N}$, $Y_f \in \mathbb{R}^{lf \times N}$, with $N = L - f - p + 1$, and write:

$$Y_f = \Gamma^f \mathbf{x}_f + H^f U_f + G^f E_f \tag{4}$$

in which $\Gamma^f = \mathcal{O}^f(A,C) \in \mathbb{R}^{lf \times n}$, $H^f = \mathcal{T}^f(A,B,C,D) \in \mathbb{R}^{lf \times mf}$ and $G^f = \mathcal{T}^f(A,K,C,I) \in \mathbb{R}^{lf \times lf}$, and $\mathbf{x}_f = \mathbf{x}_{f1} \in \mathbb{R}^{n \times N}$. Next, we define the matrices $(\mathbf{x}_p, Y_p, U_p)$ as $(\mathbf{x}_f, Y_f, U_f)$ but with elements shifted p times backward, in which p is referred to as the *past horizon*. Then, we express $\mathbf{x}_f$ as follows: $\mathbf{x}_f = A_K^p \mathbf{x}_p + L_z Z_p$ in which $L_z = [\mathcal{C}^p(A_K, K), \ \mathcal{C}^p(A_K, B_K)]$ and $Z_p = [Y'_p, \ U'_p]'$. Since A_K is strictly Hurwitz, we assume that p is sufficiently large that $A_K^p \mathbf{x}_p \cong 0$, and we rewrite (4) as:

$$Y_f = \Gamma^f L_z Z_p + H^f U_f + G^f E_f \tag{5}$$

We observe that for finite past horizon p, in general, this approximation introduces a bias in the model estimate.

If data are collected in open loop, E_f is uncorrelated with U_f, i.e. $\frac{1}{N} E_f U'_f \to 0$ as $N \to \infty$. Thus, (5) can be solved by Least Squares (LS) to obtain estimates of $(\Gamma^f L_z, H^f)$, and after a model reduction step that involves a truncated Singular Value Decomposition (SVD), the system matrices can be estimated. Each of these three steps is extensively reviewed in (Qin, 2006) for several open-loop SID algorithms.

For closed-loop data instead, a direct estimation of $(\Gamma^f L_z, H^f)$ from (5) is typically biased due to the correlation between U_f and E_f. Different approaches can be considered to overcome this problem. The CSIMPCA (Closed-loop Subspace Identification Method via Principal Component Analysis) algorithm (Wang and Qin, 2006) and the CSOPIM (Closed-loop Subspace Orthogonal Projection Identification Method) algorithm (Huang et al., 2005) assume to multiply (on the right) both sides of (4) by a matrix W' such that: $\lim_{N\to\infty} \frac{1}{N} E_f W' = 0$. Other methods (Jansson, 2003; Larimore, 2004) use a high-order ARX model to pre-estimate the Markov parameters, or a state sequence (Ljung and McKelvey, 1996), followed by an SVD to estimate low order system matrices. The Whitening Filter approach (Chiuso and Picci, 2005) constructs bases of two state sequences (shifted each other by one sampling time) by oblique projections, and then it estimates the system matrices by LS. The Innovation Estimation method (Qin and Ljung, 2003), instead, pre-estimates the innovation sequence and then uses the basic relation (5) to compute $(\Gamma^f L_z, H^f)$. The asymptotic properties of several closed-loop SID methods are analyzed in (Chiuso and Picci, 2005; Chiuso, 2007b). In particular, Chiuso (2007b) shows how different algorithms (Jansson, 2003; Larimore, 2004; Chiuso and Picci, 2005) can be seen within a unifying framework of SID methods based on ARX modeling. Moreover, Chiuso (2007a) proposes a variant of the Whitening Filter method called PBSID_OPT (Optimized Predictor Based SID), in which the Toeplitz structure of appropriate matrices used during the oblique projections is exploited, although differently from the PARSIM methods discussed next. Furthermore, Chiuso (2007b) proves that PBSID_OPT outperforms several SID algorithms in closed-loop conditions (Jansson, 2003; Larimore, 2004; Chiuso and Picci, 2005). For such reasons, PBSID_OPT will serve as a challenging SID algorithm in Section 4.

2.2 The PARSIMonious subspace identification methods

Traditional SID methods (e.g. MOESP, N4SID, etc.) do not exploit the fact that the matrix H^f is block lower triangular, i.e.

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they do not remove non-causal terms. Clearly, no consistency issues arise from an asymptotic point of view, at least if data are collected in open loop, but exploitation of the matrix structure can improve both calculation efficiency and convergence towards to true solution for finite amount of data (Qin, 2006). Here, we focus on the so-called Parsimonious Subspace Identification Method with Innovation Estimation (PARSIM-E) (Qin and Ljung, 2003). Other algorithms of the same family are the Parallel and Sequential Parsimonious Subspace Identification Methods, denoted by PARSIM-P and PARSIM-S (Qin et al., 2005), which however may be biased in closed loop.

In PARSIM-E, the lower block triangular Toeplitz structure of H^f is taken into account by writing the basic relation (5) as f separate relations:

$$\mathbf{y}_{f1} = \Gamma^{f1} L_z Z_p + H^{f1}\mathbf{u}_{f1} + \mathbf{e}_{f1} \tag{6a}$$

$$\mathbf{y}_{fi} = \Gamma^{fi} L_z Z_p + \underbrace{\begin{bmatrix} H^{fi} & \cdots & H^{f1} \end{bmatrix}}_{H^{f\to i}} \underbrace{\begin{bmatrix} \mathbf{u}_{f1} \\ \vdots \\ \mathbf{u}_{fi} \end{bmatrix}}_{U_{f\to i}} + \underbrace{\begin{bmatrix} G^{fi} & \cdots & G^{f2} \end{bmatrix}}_{G^{f\to i-1}} \underbrace{\begin{bmatrix} \mathbf{e}_{f1} \\ \vdots \\ \mathbf{e}_{f(i-1)} \end{bmatrix}}_{E_{f\to i-1}} + \mathbf{e}_{fi} \qquad i = 2, \dots, f \tag{6b}$$

The key idea is to estimate the innovation terms $\mathbf{e}_{fj}$, $j = 1, \dots, i-1$, from the residuals of (6) and to build the matrix $\widehat{E_{f\to i-1}}$ accordingly, so that the effect of all past innovations is taken into account during the LS regressions. More in details, the following steps are performed:

$$[\widehat{(\Gamma^{f1} L_z)},\ H^{f1}] = \mathbf{y}_{f1} \begin{bmatrix} Z_p \\ \mathbf{u}_{f1} \end{bmatrix}^+ \tag{7a}$$

$$\widehat{\mathbf{e}_{f1}} = \mathbf{y}_{f1} - [\widehat{(\Gamma^{f1} L_z)},\ H^{f1}] \begin{bmatrix} Z_p \\ \mathbf{u}_{f1} \end{bmatrix} \tag{7b}$$

and for $i = 2, \dots, f$:

$$[(\Gamma^{fi} L_z),\ \widehat{H^{f\to i}}, G^{f\to i-1}\ ] = \mathbf{y}_{fi} \begin{bmatrix} Z_p \\ U_{f\to i} \\ \widehat{E_{f\to i-1}} \end{bmatrix}^+ \tag{7c}$$

$$\widehat{\mathbf{e}_{fi}} = \mathbf{y}_{fi} - [(\Gamma^{fi} L_z),\ \widehat{H^{f\to i}}, G^{f\to i-1}\ ] \begin{bmatrix} Z_p \\ U_{f\to i} \\ \widehat{E_{f\to i-1}} \end{bmatrix} \tag{7d}$$

Subsequently the matrix $\widehat{\Gamma^f L_z}$ is formed by stacking the f terms $\widehat{\Gamma^{fi} L_z}$, $i = 1, \dots, f$, and a truncated SVD is performed to compute $\widehat{\Gamma^f}$ from which estimates of the systems matrices (A, C) are readily computed by LS using the shift-invariance property of Γ^f. The computation of K, described in (Lin et al., 2004), uses the estimates of $G^{f\to i-1}$ and of the extended observability matrix. Finally, given the estimates of (A, C, K), the remaining matrices (B, D) and the initial state are computed using the state-space model in predictor form. Such step is performed by pre-whitening the equation error, and it is a modified version of the method discussed in (Ljung, 1999, Sec. 10.6).

Remark 1. PARSIM-E provides consistent estimates of the system matrices under the closed-loop conditions discussed in Assumption A3.2, as long as p is sufficiently large that $A_K^p \to 0$.

Remark 2. Although all non-causal terms are structurally discarded, in PARSIM-E several Markov parameters included in the terms $(H^{f\to i}, G^{f\to i-1})$ are re-estimated at each step i.

3. PROPOSED METHOD

We now propose a novel PARSIMonious SID algorithm, based on the predictor form (2), which we will name PARSIM-K.

As in other SID methods, we express $\mathbf{x}_f = A_K^p \mathbf{x}_p + L_z Z_p$, but we use the predictor form evolution (2) to obtain:

$$\begin{aligned} Y_f &= \Gamma_K^f \mathbf{x}_f + H_K^f U_f + G_K^f Y_f + E_f \\ &= \Gamma_K^f (A_K^p \mathbf{x}_p + L_z Z_p) + H_K^f U_f + G_K^f Y_f + E_f \end{aligned} \tag{8}$$

in which $\Gamma_K^f = \mathcal{O}^f(A_K, C)$, $H_K^f = \mathcal{T}^f(A_K, B_K, C, D)$, $G_K^f = \mathcal{T}^f(A_K, K, C, 0)$. We now exploit the lower block triangular structure of H_K^f and G_K^f to rewrite (8) as follows (notice that G_K^f is a strictly lower block Toeplitz matrix):

$$\mathbf{y}_{f1} = \Gamma_K^{f1}(A_K^p \mathbf{x}_p + L_z Z_p) + H_K^{f1}\mathbf{u}_{f1} + \mathbf{e}_{f1} \tag{9a}$$

$$\mathbf{y}_{fi} = \Gamma_K^{fi}(A_K^p \mathbf{x}_p + L_z Z_p) + H_K^{fi}\mathbf{u}_{f1} + G_K^{fi}\mathbf{y}_{f1} + \tilde{\mathbf{y}}_{fi} + \mathbf{e}_{fi} \quad \text{for } i = 2, \dots, f \tag{9b}$$

in which $\tilde{\mathbf{y}}_{f2} = H_K^{f1}\mathbf{u}_{f2}$ and $\tilde{\mathbf{y}}_{fi} = \sum_{j=1}^{i-1} H_K^{fj}\mathbf{u}_{f(i-j+1)} + \sum_{j=1}^{i-1} G_K^{fj}\mathbf{y}_{f(i-j+1)}$ for $i = 3, \dots, f$. Next, given that A_K is a strictly Hurwitz matrix, we assume that p is such that $A_K^p \mathbf{x}_p \approxeq 0$ and we estimate $(\Gamma^{fi} L_z)$, H_K^{fi}, G_K^{fi} sequentially as follows:

$$[\widehat{(\Gamma_K^{f1} L_z)},\ \widehat{H_K^{f1}}] = \mathbf{y}_{f1} \begin{bmatrix} Z_p \\ \mathbf{u}_{f1} \end{bmatrix}^+ \tag{10a}$$

$$[\widehat{(\Gamma_K^{fi} L_z)},\ \widehat{H_K^{fi}},\ \widehat{G_K^{fi}}] = (\mathbf{y}_{fi} - \tilde{\mathbf{y}}_{fi}) \begin{bmatrix} Z_p \\ \mathbf{u}_{f1} \\ \mathbf{y}_{f1} \end{bmatrix}^+ \quad i = 2, \dots, f \tag{10b}$$

Remark 3. At each computation step i, the terms $\tilde{\mathbf{y}}_{fi}$ for $i = 2, \dots, f$ are known because they depend on previously estimated parameters $(\widehat{H_K^{fj}}, \widehat{G_K^{fj}})$ with $j < i$. We also notice that in each step $i = 2, \dots, f$ the pseudo-inverse matrix is the same and hence it needs to be computed only at step $i = 2$.

Remark 4. If $D = 0$, (10a) simplifies into $\widehat{(\Gamma_K^{f1} L_z)} = \mathbf{y}_{f1} Z_p^+$ and $\widehat{H_K^{f1}} = 0$. Also notice that if $D \neq 0$, from Assumption A3.2 we have that $\mathbf{u}_{f1}$ is independent of $\mathbf{e}_{f1}$ even if data are collected in closed loop because u_k is in feedback with y_{k-1}.

Next, we stack the terms $\widehat{(\Gamma_K^{fi} L_z)}$ for $i = 1, \dots, f$ to obtain $\widehat{(\Gamma_K^f L_z)}$, and we perform a weighted SVD:

$$W_1 \widehat{(\Gamma_K^f L_z)} W_2 = \mathcal{U}_n \mathcal{S}_n \mathcal{V}_n' + \mathcal{R}_n \tag{11}$$

in which $(\mathcal{U}_n, \mathcal{S}_n, \mathcal{V}_n)$ are the SVD terms associated with the n largest singular values and $\mathcal{R}_n$ represents the error associated to neglecting the remaining $(fl - n)$ SVD terms. Next, we compute $\Gamma_K^f = W_1^{-1}\mathcal{U}_n \mathcal{S}_n^{1/2}$, and subsequently we estimate (A_K, C) by LS using the shift invariance property of Γ_K^f.

Remark 5. Similarly to the PARSIM algorithms (Qin et al., 2005), we use $W_1 = I$ and $W_2 = (Z_p \Pi_{U_f}^{\perp} Z_p')^{1/2}$ as weighting matrices. If the outputs have fairly different variance, scaling to same (or similar) variance in all outputs is advised. As discussed in (Qin et al., 2005, Sec.3.3) the choice of W_2 is equivalent to the CVA scaling.

Given the estimated matrices $(\widehat{A_K}, \widehat{C})$, we could obtain $(\widehat{B_K}, \widehat{D})$ from $\widehat{H_K^{fi}}$, and $\widehat{K}$ from $\widehat{G_K^{fi}}$ by LS operations. However, we here describe a different method that allows us to compute $(\widehat{B_K}, \widehat{D}, \widehat{K})$ and the initial state $\widehat{x_0}$ from a single LS problem.

Let q denote the forward shift operator and consider the following *known* stable (multi-variable) transfer function $F(q) = \widehat{C}(qI - \widehat{A_K})^{-1}$. We can write, for $k = 0, \dots, L-1$:

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$$\begin{aligned} y_k &= (F(q)B_K + D)u_k + F(q)Ky_k + CA_K^k x_0 + e_k \\ &= \varphi_k^1 \operatorname{vec}(B_K) + \varphi_k^2 \operatorname{vec}(D) + \varphi_k^3 \operatorname{vec}(K) + \varphi_k^4 x_0 + e_k \end{aligned} \tag{12}$$

in which $\varphi_k^1 = F(q) \otimes u_k'$, $\varphi_k^2 = I \otimes u_k'$, $\varphi_k^3 = F(q) \otimes y_k'$ and $\varphi_k^4 = CA_K^k$. We can now compute $(\widehat{\operatorname{vec}(B_K)}, \widehat{\operatorname{vec}(D)}, \widehat{\operatorname{vec}(K)}, \widehat{x_0})$ by LS solution of the *Ll* equations (12), i.e.

$$\begin{bmatrix} \widehat{\operatorname{vec}(B_K)} \\ \widehat{\operatorname{vec}(D)} \\ \widehat{\operatorname{vec}(K)} \\ \widehat{x_0} \end{bmatrix}' = \begin{bmatrix} y_0 \\ \vdots \\ y_{L-1} \end{bmatrix}' \begin{bmatrix} \varphi_0^1 & \varphi_0^2 & \varphi_0^3 & \varphi_0^4 \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{L-1}^1 & \varphi_{L-1}^2 & \varphi_{L-1}^3 & \varphi_{L-1}^4 \end{bmatrix}^+ \tag{13}$$

and after rebuilding $(\widehat{B_K}, \widehat{D}, \widehat{K})$, we finally compute:

$$\widehat{A} = \widehat{A_K} + \widehat{K}\widehat{C}, \quad \widehat{B} = \widehat{B_K} + \widehat{K}\widehat{D} \tag{14}$$

3.1 Properties

We have the following basic result for the proposed algorithm.

Theorem 6. Suppose that assumptions A1–A4 hold. Denote with $\left((\Gamma_K^f L_z), H_K^f, G_K^f\right)$ the true matrices in (8) and with $\left(\widehat{(\Gamma_K^f L_z)}, \widehat{H_K^f}, \widehat{G_K^f}\right)$ the estimates obtained with (10). Then, for each $\epsilon > 0$, there exist finite p and L^* such that:

$$\|\widehat{(\Gamma_K^f L_z)} - (\Gamma_K^f L_z)\|_\infty \le \epsilon \qquad \text{for all } L \ge L^* \tag{15a}$$

$$\|\widehat{H_K^f} - H_K^f\|_\infty \le \epsilon \qquad \text{for all } L \ge L^* \tag{15b}$$

$$\|\widehat{G_K^f} - G_K^f\|_\infty \le \epsilon \qquad \text{for all } L \ge L^* \tag{15c}$$

Proof. See (Pannocchia and Calosi, 2010).

Remark 7. Theorem 6 says that PARSIM-K gives *consistent* estimates of $[(\Gamma_K^f L_z),\ H_K^f,\ G_K^f]$ in the sense that the estimate error can be made arbitrarily small for finite p and L. This fact allows *consistent* estimation of the extended observability matrix upon a similarity transformation and, hence, of (A_K, C).

Remark 8. We notice that $(\widehat{\operatorname{vec}(B_K)}, \widehat{\operatorname{vec}(D)}, \widehat{\operatorname{vec}(K)}, \widehat{x_0})$ obtained from (13) are consistent if the true values of (A_K, C) are used in the computation of $(\varphi_k^1, \varphi_k^2, \varphi_k^3, \varphi_k^4)$. We also observe that (13) is unbiased for finite p because x_0 is estimated.

From a numerical point of view some comments and comparisons between PARSIM-E and PARSIM-K are appropriate. First of all, we notice that the first regression step, (7a) for PARSIM-E and (10a) for PARSIM-K, is identical. In fact, it computes $\Gamma^{f1} L_z = CL_z$ (or $\Gamma_K^{f1} L_z = CL_z$ in PARSIM-K) and $H^{f1} = D$ (or $H_K^{f1} = D$ in PARSIM-K). Subsequent steps instead differ, and it can be noticed that, unlike PARSIM-E, the proposed PARSIM-K does not recompute Markov parameters that were already computed in previous steps. Furthermore, in PARSIM-E the matrix that is pseudo-inverted in (7d) changes at each step i and increases in row size with i. On the other hand, the matrix that is pseudo-inverted in (10b) does not change at each step i, so it can be computed only once. Furthermore, such matrix has the same row size of that in (7d) for $i = 2$, but the one used in PARSIM-E increases in row size at subsequent steps. It is also important to notice that in the proposed PARSIM-K, the matrix K is computed along with the (B_K, D, x_0), whereas in PARSIM-S (Qin et al., 2005, Sec.4) and in PARSIM-E (Lin et al., 2004) a separate step for the evaluation of K is necessary. Finally, it must be remarked that since the proposed PARSIM-K uses the predictor form in all steps, i.e. the regressions (10) and the final regression (13), numerical stability for unstable systems is guaranteed (Chiuso and Picci, 2005; Qin, 2006).

4. CASE STUDIES

4.1 Introduction and MPC algorithm

We present in this section the simulation results obtained for the identification and control of multi-input multi-output (MIMO) processes. The performance of the proposed PARSIM-K algorithm is compared against the performance achieved by other subspace methods, namely PARSIM-S (Qin et al., 2005), PARSIM-E (Qin and Ljung, 2003), PBSID_OPT (Chiuso, 2007b) and N4SID with CVA weighting (as provided by the Identification Toolbox in Matlab R2007b).

For the first example we give a measure of the identified model quality as follows. Let $G(z) = C(zI - A)^{-1}B + D$ be the *true* process transfer matrix and let $\hat{G}^j(z) = \hat{C}(zI - \hat{A})^{-1}\hat{B} + \hat{D}$ be *identified* model transfer function matrix in the j−th run of M Monte-Carlo simulations. Then, we define the following (scalar) function $\epsilon(\omega)$ which measures the Relative Model Error (RME) at each normalized frequency: $\epsilon(\omega) = \frac{1}{M}\sum_{j=1}^{M} \sigma_{\max}\left(G(e^{i\omega}) - \hat{G}^j(e^{i\omega})\right)/\sigma_{\max}(G(e^{i\omega}))$, in which $\sigma_{\max}\{\cdot\}$ represents the largest singular value, i.e. the matrix 2-norm. We also consider the following *average* RME: $\bar{\epsilon} = \frac{1}{\pi}\int_0^\pi \epsilon(\omega)d\omega$. For the second example, instead, we compare the performance of offset-free MPCs based on models identified from closed-loop data using the five mentioned SID methods. In both examples f is chosen slightly larger than the model order, whereas the p is chosen by trial and error.

Let (A, B, C) be the model matrices identified with any of the considered SID methods (notice that in all SID methods we force $D = 0$), and consider the *augmented* model (Pannocchia and Rawlings, 2003):

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + B_d d_k + w_k^x \\ d_{k+1} &= d_k + w_k^d \\ y_k &= Cx_k + C_d d_k + v_k \end{aligned} \tag{16}$$

in which d is the fictitious (integrating) disturbance, included for offset-free purposes, and (w^x, w^d, v) are Gaussian random variables, with specified covariance (Q_x, Q_d, R_v) from which a Kalman filter gain is computed.

Let $(\hat{x}_k, \hat{d}_k)$ be the updated estimates of (x, d) at time k and let $\bar{y}_k$ be the current output setpoint. We solve the target calculation problem to compute the equilibrium state and input targets:

$$\begin{aligned} \min_{x_k^s, u_k^s} \ & (Cx_k^s + C_d\hat{d}_k - \bar{y})'\bar{Q}(Cx_k^s + C_d\hat{d}_k - \bar{y}_k) + u_k^s\bar{R}u_k^s \\ \text{s.t. } & x_k^s = Ax_k^s + Bu_k^s + B_d\hat{d}_k, \quad u_{\min} \le u_k^s \le u_{\max} \end{aligned} \tag{17}$$

in which $(u_{\min}, u_{\max})$ represent the input bounds, and $\bar{Q}$, $\bar{R}$ are positive definite matrices. Next, we define the deviation variables: $\tilde{x}_k = x_k - x_k^s$, $\tilde{u}_k = u_k - u_k^s$ and consider the infinite-horizon optimal control problem:

$$\min_{\tilde{u}_k, \tilde{u}_{k+1}, \ldots} \sum_{j=k}^{\infty} \tilde{x}_j'C'QC\tilde{x}_j + (\tilde{u}_j - \tilde{u}_{j-1})'S(\tilde{u}_j - \tilde{u}_{j-1}) \quad \text{s.t.} \tag{18a}$$

$$\tilde{x}_k, \tilde{u}_{k-1} \text{ given}, \qquad \tilde{x}_{j+1} = A\tilde{x}_j + B\tilde{u}_j \tag{18b}$$

$$u_{\min} - u^s \le \tilde{u}_j \le u_{\max} - u^s \tag{18c}$$

in which Q, S are positive definite matrices. With a suitable state augmentation it is possible to re-write (18) in a conventional constrained LQR form (Rao and Rawlings, 1999). Given the optimal input sequence $(\tilde{u}_k^*, \tilde{u}_{k+1}^*, \ldots)$, obtained via Quadratic Programming of a suitable finite-horizon parameterization of (18), the injected input is given by $u_k = \tilde{u}_k^* + u_k^s$.

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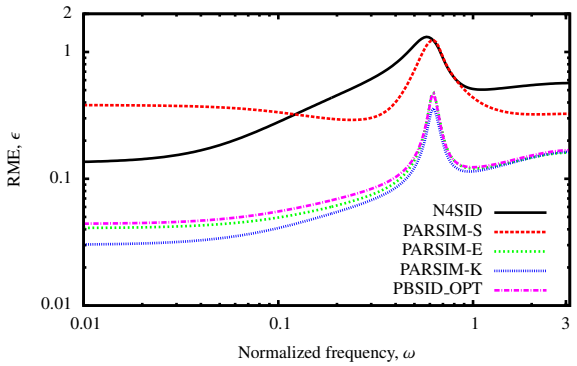

Fig. 1. Example 1. RME function $\epsilon(\omega)$ for N4SID, PARSIM-S, PARSIM-E, PARSIM-K and PBSID_OPT (closed-loop data, 2000 data-points, results are averaged over 50 Monte-Carlo simulations).

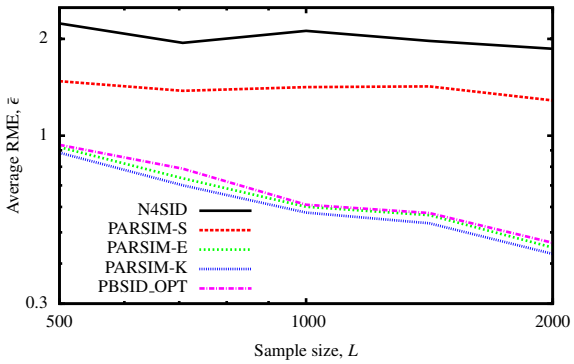

Fig. 2. Example 1. Average RME $\bar{\epsilon}$ for N4SID, PARSIM-S, PARSIM-E, PARSIM-K and PBSID_OPT as a function of the sample size (closed-loop data, results are averaged over 50 Monte-Carlo simulations).

4.2 Unstable MIMO example

As first example, we consider the unstable MIMO system:

$$x_{k+1} = \begin{bmatrix} 1.21 & -0.705 & 0.804 \\ 0 & 0.845 & -0.413 \\ 0 & 0.771 & 0.723 \end{bmatrix} x_k + \begin{bmatrix} 1.05 & 0 \\ 0 & 0.516 \\ 0.101 & 0 \end{bmatrix} u_k + \begin{bmatrix} 0.945 & -0.483 \\ 0 & 0.410 \\ 0 & 0.0375 \end{bmatrix} e_k$$
$$y_k = \begin{bmatrix} 0.256 & 0 & 0 \\ 0 & 1.25 & -1.14 \end{bmatrix} x_k + e_k$$

Data are collected in closed-loop, under the output feedback $u_k = r_k - y_k$. The reference $\{r_k\}$ and the innovation $\{e_k\}$ are Gaussian random variables with covariance of I and $0.20I$, respectively. For all SID methods we use past and future horizons of $p = 10$ [1] and $f = 5$, respectively, and $n = 3$. We present in Fig. 1 the RME function $\epsilon(\omega)$ obtained with the five SID methods using 2000 data-points, whereas the average RME $\bar{\epsilon}$ *vs.* the data sample size is shown in Fig. 2.

Fig. 1 shows that PARSIM-S and N4SID identify erroneous models when data are collected in closed loop. On the other hand, as expected, PARSIM-E, PARSIM-K and PBSID_OPT deliver models with much lower error at any frequency. Fig. 2 also shows that as the data sample size increases, these methods provide consistent model estimates. On the same plot, we can also see that PARSIM-S and N4SID deliver significantly biased model estimates even if the data sample size is increased. These results are in agreement with the theoretical properties of PARSIM-S and N4SID which are guaranteed to be consistent only for open-loop data, whereas PARSIM-E, PARSIM-K and PBSID_OPT are proved to be consistent for closed-loop data. A closer analysis of Figs. 1 and 2 shows that PARSIM-K performs slightly better than PARSIM-E and PBSID_OPT.

4.3 Wood-Berry Distillation Column

As second example, we consider the Wood-Berry distillation column model (Wood and Berry, 1973):

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-9.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{19}$$

in which time constants and delays are in minutes. Using a sampling time of 1 minute, a minimal model has 14 states.

Data are collect in closed-loop for 24 hours (i.e. 1440 samples) using the (decentralized) feedback $u_k = r_k - Fy_k$, with $F = \begin{bmatrix} 0.235 & 0 \\ 0 & -0.155 \end{bmatrix}$. The reference $\{r_k\}$ is a Gaussian random variable with covariance $0.25^2 I$, while the innovation $\{e_k\}$ is also a Gaussian random variable with covariance chosen in way that specific noise-to-signal ratios are attained in each output, as detailed later on. In all methods we use: $p = 30$, $f = 20$, and $n = 14$ (except for the results of Fig 4, as later detailed).

The performance of five MPCs, using same parameters but different models, is compared in closed loop with the *true* process (19). The common MPC parameters are:

$$B_d = B,\ C_d = 0,\ Q_x = 10^{-3}I,\ Q_d = I,\ R_v = 10^{-2}I$$
$$\bar{Q} = Q = I,\ \bar{R} = 10^{-6}I,\ S = 0.5I,\ u_{\max} = -u_{\min} = [2\ 2]'$$

We denote with MPC-ℓ, $\ell \in \{\text{N4S, S, E, K, PB}\}$, the MPC based on model ℓ identified by N4SID, PARSIM-S, PARSIM-E, PARSIM-K or PBSID_OPT, respectively. As performance index we use: $J_\ell = \frac{1}{2}\sum_{k=0}^{\infty}(y_k - \bar{y}_k)'Q(y_k - \bar{y}_k) + (u_k - u_{k-1})'S(u_k - u_{k-1})$ in which we observe that J_ℓ is equivalent to the objective of the MPC problem (18), although (y_k, u_k) are *actual* outputs and inputs. If we denote with J_0 the value of J_ℓ obtained when the MPC model equals the true process (19), we can define a relative sub-optimality index for MPC–ℓ as: $R_\ell = \frac{J_\ell - J_0}{J_0}$.

We compare in Fig. 3 the closed-loop performance of the five MPCs for different values of the noise-to-signal ratio considered in the outputs used for identification. As expected for all SID methods we observe that the lower the noise-to-signal ratio, the better the performance. We can also observe that for all noise-to-signal ratios, the MPC designed on a PARSIM-K model outperforms the MPCs designed on the other models. As a matter of fact, the closed-loop performance of MPC-K appears to be close to *optimal* for any noise-to-signal ratio. In particular when the noise-to-signal ratio on each output equal to 0.20, the relative sub-optimality indices for all MPCs are: $R_{\text{N4S}} = 1.52$, $R_{\text{S}} = 0.0797$, $R_{\text{E}} = 0.0785$, $R_{\text{K}} = 0.0151$ and $R_{\text{PB}} = 0.0193$. We observe that MPC-K and MPC-PB perform almost identically, MPC-E and MPC-S display larger sub-optimality, and MPC-N4S is inadequate. In Fig. 4 we present the relative sub-optimality of the five MPCs as a function of the chosen model order n (with $p = 30$ and $f = 20$). Again, we notice that PARSIM-K outperforms the other methods irrespectively of the considered model order.

5. CONCLUSIONS

We presented a novel subspace identification algorithm with the following main features. It is based on the predictor form

[1] It must be noticed that A_K^p becomes sufficiently small for p, at least, greater than 50. Nonetheless, the results obtained with $p = 50$ and those obtained with $p = 10$ are almost identical.

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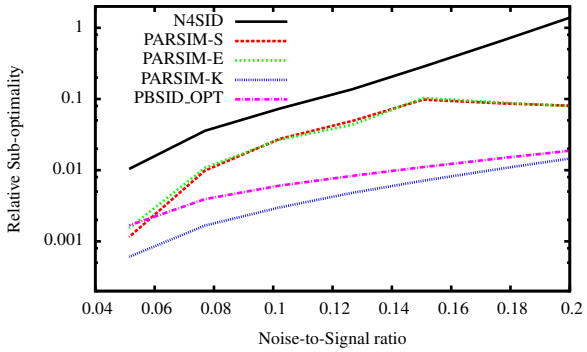

Fig. 3. Wood-Berry model. Relative sub-optimality index vs. output noise-to-signal ratio for MPC-N4S, MPC-S, MPC-E, MPC-K and MPC-PB.

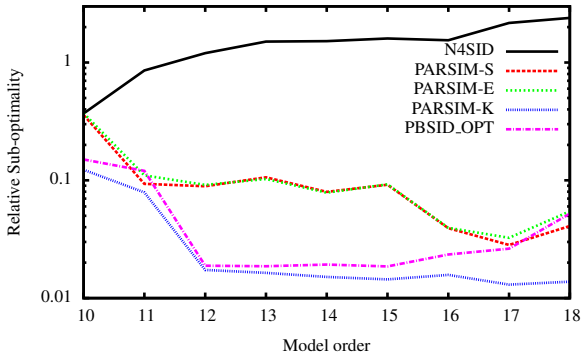

Fig. 4. Wood-Berry model. Relative sub-optimality vs. model order for MPC-N4S, MPC-S, MPC-E, MPC-K and MPC-PB. Output noise-to-signal ratio equal to 0.20.

of the state-space equation, it exploits completely the Toeplitz nature of the Markov parameters matrices in the spirit of the PARSIMonious methods (Qin et al., 2005), and it is applicable to closed-loop data. The method consists of two major steps. In the first one, small dimensional LS problems are solved sequentially to compute the Markov parameters, and the extended observability matrix (in predictor form) is obtained from a truncated SVD. We proved consistency of this step both for open-loop and closed-loop data. In the second step, after $A_K = A - KC$ and C are estimated from the observability matrix, a single LS problem is solved to estimate $(B_K = B - KD, D, K, x_0)$.

We reported numerical simulations to show the consistency properties of the proposed method with data collected in feedback conditions. Furthermore, we compared the proposed method against other subspace methods, and from the reported results it appears that the proposed method outperforms the other PARSIM methods in closed-loop conditions, and performs similarly to (actually slightly better than) PBSID_OPT, which is proved to compare favorably with many SID algorithms (Chiuso, 2007b). Furthermore, thanks to the predictor form model parameterization, the proposed method does not suffer from ill-conditioning in the of case unstable systems. We finally compared the performance of MPCs designed on models identified from closed-loop data by the different subspace methods, and we observed that the proposed SID algorithm appears suitable for closed-loop identification in MPC design.

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