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Two-point Lyapunov Control of Binary Distillation Columns with four temperature measurements

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Abstract: The problem of controlling binary distillation column effluents by manipulating the reflux and vapor flow rates on the basis of four temperature measurements is addressed. The problem is motivated by the need of understanding and systematizing two-point control industrial schemes driven by four temperature measurements. On the basis of the relative degree structure of the nonlinear passivated model, a four-state linear-decentralized model with reconstructible load inputs is drawn. Then, a passivated by backstepping OF controller is built within a Lyapunov dissipation framework, yielding a linear control scheme with: (i) regulation of distillate (or bottom) composition by adjustment of reflux flow (or heat duty) on the basis of two temperature measurements in the stripping (or rectifying) section, (ii) per-section decentralized structure, (iii) a straightforward construction procedure, and (iv) conventional-like simple tuning guidelines, without the primary-secondary gain separation limitation of the standard cascade TPT controller. The proposed control design is tested with a representative example through simulations, finding a behavior which outperforms the ones of previous TP control schemes.

Keywords: Distillation, cascade control, Lyapunov function, dissipation, temperature measurements.

1. INTRODUCTION

The production of many intermediate and final products in chemical and petrochemical industries depends heavily on energy intensive distillation columns [10], and their efficient (minimum energy consumption) operation requires the regulation of distillate and bottoms compositions [15, 19]. Consequently, the related MIMO feedback control design has been and still is an important industrial problem that has been extensively studied with a diversity of linear and nonlinear approaches. Basically, the related two-input (reflux and vapor flow rate via manipulation of the reboiler heat duty) problem design has been studied with several linear and nonlinear control schemes: (i) two composition measurements (DC) [4, 6, 28], (ii) two point temperature (TPT) schemes [3, 5, 13], and (iii) cascade composition-to-temperature (DCT) controllers on the basis of one (or two) effluent composition primary measurements in conjunction with one (or two) temperature secondary measurements located in the most sensitive tray per section [2, 5, 8, 25, 30]. The DC schemes have the capability of offsetless composition regulation, but yield poor (oscillatory and sluggish) transient response for high-purity columns with measurement dead-times, instrumentation errors as well as feed rate, composition and enthalpy load disturbances. This is so due to the poor load disturbance-to-measured composition output sensitivity, meaning that the control response to unmeasured load disturbances takes place once the entire profile has been upset, and consequently, the feedback loops must be tuned conservatively. These considerations motivate the employment of temperature measurements, in sensitive trays (one per section) [3, 20, 25, 30], to set either a fast two-point temperature (TPT) or a slow-fast dual cascade composition-to-temperature (DCT) control loop scheme. The TPT schemes reject properly feed rate disturbances, but the effluent purities usually have offsets in the presence of persistent feed composition and/or enthalpy disturbances [19].

The improvement of the effluent regulation capability of the TPT control scheme has been pursued in three ways: (i) by adding a feed temperature-driven feedforward component built from the static input-to-output linear gain approximation drawn from a detailed column model [3], (ii) by incorporating two additional temperature measurements, defining two synthetic regulated temperature outputs as combinations of the four temperature measurements, and setting a TPT-like control scheme [14-15] that will be referred to as four-output (4O-TPT), and (iii) by incorporating composition measurements, and setting two cascade control loops, one per section, with primary composition loops and secondary temperature loops, or equivalently, using the above mentioned DCT control scheme [2, 5, 25, 30].

The TPT with feed temperature feedforward component [3] functions adequately with saturated feed stream, otherwise, pressure must be measured and the feedforward component must be redesigned accordingly. Even though improvement of the 4O-TPT control the TPT have been reported on particular column cases, the control functioning depends strongly on the way the four temperature measurements are combined to yield two synthetic outputs, the combination is performed on the basis of experience and column specificities. Finally, it must be pointed out that the behaviour of the DCT cascade schemes [2, 5, 25, 30] is severely limited by the primary-secondary dynamic separation requirement for closed-loop stability [**5**], especially with composition measurement dead-times.

Recently, a unifying control structure-algorithm design framework for binary distillation columns has been established [3-5], on the basis of fundamental connections between passivity, optimality, observability and robustness in nonlinear constructive control theory, showing that the behaviour of exact model-based robust nonlinear SF optimal dual composition (DC) [4], two-point temperature (TPT) [3], and dual cascade composition-to-temperature (DCT) [5] distillation column controllers can be efficiently recovered by means of suitably designed linear-decentralized PI-type component-based control schemes with: (i) PI-type components, (ii) reduced model dependency, and (iii) rather simple conventional-like tuning. On the other hand, in the context of continuous and batch reactor output feedback (OF) control problems [1], the dynamic separation limitation for cascade temperature control loops has been removed on the basis of passivation by backstepping ideas.

Motivated by the above mentioned idea of using two additional temperature measurements to improve the effluent regulation capability of the TPT scheme, and by the need of assessing the general-purpose feasibility of the idea and of systematizing its control design, in this work the problem of controlling the effluent compositions by manipulating reflux flow and heat duty on the basis of four temperature measurements, two per column section, is addressed. First, in a way that is analogous to the development of a linear-decentralized DCT cascade control scheme [5], a four-state linear-decentralized model with reconstructible load inputs, built on the basis of the column relative degree structure, is set. Then, following the chemical reactor constructive control ideas [1], a passivated by backstepping OF controller is built within a Lyapunov dissipation framework, yielding a linear TPT-like distillation column control scheme with: (i) regulation of distillate (or bottom) composition by adjustment of reflux flow (or heat duty) on the basis of two temperature measurements in the column's rectifying (or stripping) section, (ii) per-section decentralized structure, (iii) a straightforward construction procedure, and (iv) conventional-like simple tuning guidelines. The proposed approach is applied to a representative example through simulations, yielding behaviours that outperform the ones obtained with a TPT control scheme with setpoint compensation, and resembles the ones obtained with previous control schemes based on non-delayed temperature and composition measurements.

2. CONTROL PROBLEM

Consider the N-tray distillation column (depicted in Figure 1), where a binary mixture with molar flow F and composition c_F is fed at the tray n_F, yielding effluent flows B and D with compositions c_B and c_D respectively. The control inputs are the vapour V and the reflux R flow rates. The effluent composition pair (c_B, c_D) must be indirectly regulated by manipulating the reflux-vapour flow rate pair (R, V) on the basis of four temperature measurements at locations to be determined (see Figure 1): two measurements T_1^s and T_2^s (or T_1^r and T_2^r) in the stripping (or rectifying) section, with superindex s (or r) denoting the stripping (or rectifying) section, and subindex 1 (or 2) denoting the secondary (or primary) measurement. Following the TPT control design [3, 5], the secondary measurement~~s~~ T_2^s (or T_2^r) is located at the trays with the largest temperature gradient in the stripping (or rectifying) column's section [3, 5, 20, 29]. The primary measurement T_1^s (or T_1^r) is located as close as possible to the reboiler (or top) tray, with a sufficiently large stage-to-stage temperature gradient. For the sake of illustration, here we take the reboiler and top tray temperatures as the primary outputs, in the understanding that the proposed methodology can be applied for other primary locations, depending on and a suitable tradeoff between sensitivity and product offsets.

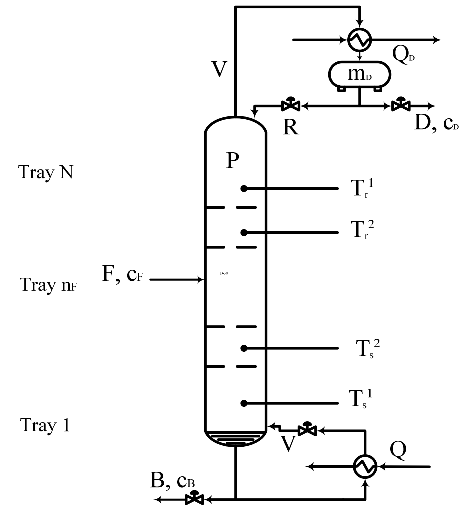

Figure 1. Schematic of a distillation column

From standard assumptions [17] (constant pressure; equilibrium in all trays, perfect level control, equimolal overflows), the column dynamics are given by:

$$\dot{c}_i = [L(m_{i+1})\Delta^+ c_i - V\Delta^- E(c_i) + \delta_{i,nF}\, F(c_F - c_i)]/m_i, \quad 0 \le i \le N-1$$

$$\dot{c}_N = [R\Delta^+ c_N - V\Delta^- E(c_N)]/m_N, \quad \dot{c}_{N+1} = V[E(c_N) - c_{N+1}]/m_D$$

$$\dot{m}_i = L(m_{i+1}) - L(m_i) + \delta_{i,nF} F, \quad 1 \le i \le N-1;$$

$$\dot{m}_N = R - L(m_N) \qquad (1a\text{-}e)$$

where: $\Delta^+ c_i := c_{i+1} - c_i$, $\Delta^- E(c_i) := E(c_i) - E(c_{i-1})$

$E(c_{-1}) := c_0$, $c_{N+1} = c_D$, $c_B = c_0$

$T_1^s = \beta(c_1^s)$, $T_2^s = \beta(c_2^s)$, $T_1^r = \beta(c_1^r), T_2^r = \beta(c_2^r)$

$\delta_{i,nF}$ is Kronecker's delta, c_i (or m_i) is the light component mole fraction (or holdup) at the i-th tray, E, β and L are the nonlinear liquid-vapor equilibrium, bubble point and hydraulic functions, respectively. Regulating the top tray temperature $T_N = \beta(c_N)$ (i.e., regulating c_N) is equivalent to regulating the distillate composition c_D because the steady-state values of c_N and T_N are bijectively related via the equilibrium function (i.e., $\bar{c}_N = E^{-1}(\bar{c}_D)$, [4, 6]). In compact notation, the dynamics (1) are written as follows:

$$\dot{c} = F_c(c, m, \delta, \upsilon), \quad \dot{m} = F_m(m, \delta, \upsilon), \quad \psi = h(c) \quad \text{(2a-b)}$$

Where $c = (c_0,..., c_{N+1})'$, $m = (m_1,..., m_N)'$

$\delta = (F, c_F)'$, $\upsilon = (V, R)'$

$\psi = (T_1^s, T_2^s, T_1^r, T_2^r)'$, $h(c) = [\beta(c_1^s), \beta(c_2^s), \beta(c_1^r), \beta(c_2^r)]'$

$0 = F_c(\bar{c}, \bar{m}, \bar{\delta}, \bar{\upsilon})$, $0 = F_m(\bar{m}, \bar{\delta}, \bar{\upsilon})$, $\bar{\psi} = h(\bar{c})$

$(\cdot)'$ denotes the transposition of $(\cdot)$, and $(\bar{\cdot})$ denotes the nominal steady-state value of $(\cdot)$.

Our *problem* consists in designing a linear two-input four output (2I-4O) temperature controller that, driven by four temperature measurements, regulates the effluent compositions pair about its prescribed setpoint. We are interested in a design methodology with: (i) per-section linear-decentralized controllers, (ii) a systematic construction with reduced model dependency, and (iii) a simple tuning procedure without the primary-secondary gain separation limitation of the standard cascade control designs. The proposed control scheme must be compared with its previous TPT counterpart [3].

3. CONTROL DESIGN

From the nonlinear control theory we know that optimal nonlinear SF controllers [7, 24]: (i) are inherently robust and passive (i.e., minimum phase with relative degrees equal to one), (ii) cannot in general be constructed in analytic form via direct optimality, (iii) can be constructed in analytic form via inverse optimality by starting with a passive controller and verifying for which objective function the controller is optimal, and (iv) can be constructed, on the basis of non-passive models, with a passivation via backstepping procedure. From previous distillation column [3-5] and chemical reactor [1] control studies, we know that: (i) the behaviour of a passive nonlinear SF controller can be recovered with a linear OF controller made of conventional proportional and integral components, and (ii) that a linear-decentralized model for control can be set according to the system relative-degree structure and observability property.

3.1 Nonlinear passive model

Here, a passive model for control design purposes is drawn. As it is known in distillation column control, the hydraulic dynamics are faster than composition dynamics [12], so that they can be assumed in quasi-steady state in the design stage, and their effect must be accounted for in the tuning stage [3-5, 7, 11]. Then, Eq. (2b) can be set as (3) with liquid flows given by (4):

$$\dot{m} = F_m(m, \delta, \upsilon) \approx 0 \quad (3)$$

$$L(m_i) \approx R + F, \quad 1 \le i \le n_F \quad \text{Stripping section} \quad (4a)$$

$$L(m_i) \approx R, \quad n_F + 1 \le i \le N \quad \text{Rectifying section} \quad (4b)$$

The unique root of (4) around the SS $(\bar{c}, \bar{m}, \bar{\delta}, \bar{\upsilon})$ is given by:

$$m_i^* = G_i(\delta, \upsilon) = (R + F - \bar{R} - \bar{F})\tau_i + \bar{m}_i, \quad 1 \le i \le n_F ; \quad (5a)$$

$$m_i^* = G_i(\delta, \upsilon) = (R - \bar{R})\tau_i + \bar{m}_i, \quad n_F + 1 \le i \le N \quad (5b)$$

where τ_i is the tray hydraulic time constant. Substituting Eq. (4) and (5) in Eq. (1a) yields the *reduced–order passive model*:

$$\dot{c}_0 = [(R + F)(c_1 - c_0) - V\, \Delta^- E(c_0)]/\bar{m}_B := f_0(c, \upsilon, \delta) \quad (6a)$$

$$\dot{c}_i = [(R + F)\Delta^+ c_i - V\Delta^- E(c_i)]/[(R + F - \bar{R} - \bar{F})\tau_h + \bar{m}_i] := f_i(c, \upsilon, \delta), \quad 1 \le i \le n_F - 1 \quad (6b)$$

$$\dot{c}_{n_F} = [R\, \Delta^+ c_{n_F} + F(c_F - c_{n_F}) - V\Delta^- E(c_i)] / [(R + F - \bar{R} - \bar{F})\tau_h + \bar{m}_{n_F}] := f_{n_F}(c, \upsilon, \delta) \quad (6c)$$

$$\dot{c}_i = [R\, \Delta^+ c_i - V\, \Delta^- E(c_i)]/[(R - \bar{R})\tau_h + \bar{m}_i] := f_i(c, \upsilon, \delta), \quad n_F + 1 \le i \le N \quad (6d)$$

$$\dot{c}_{N+1} = V[E(c_N) - c_{N+1}] / \bar{m}_D := f_{N+1}(c, \upsilon, \delta) \quad (6e)$$

$$\psi = h(c) = [\beta(c_1^s), \beta(c_2^s), \beta(c_1^r), \beta(c_2^r)]' \quad (6f)$$

In compact deviation-form about the prescribed nominal SS, the reduced passive model (6) is rewritten as follows:

$$\dot{x}_I = f_I(x_I, x_{II}, u, d), \quad \dot{x}_{II} = f_{II}(x_I, x_{II}, u, d), \quad y = x_I \quad (7)$$

where

$$x = (x_I', x_{II}')', \quad x_I = (x_1^s, x_2^s, x_1^r, x_2^r)', \quad x_{II} = c_{II} - \bar{c}_{II} \quad (8)$$

$x_1^s = \beta(c_1^s) - \beta(\bar{c}_1^s)$; $x_2^s = \beta(c_2^s) - \beta(\bar{c}_2^s)$, $u_R := R - \bar{R}$

$x_1^r = \beta(c_1^r) - \beta(\bar{c}_1^r)$, $x_2^e = \beta(c_2^e) - \beta(\bar{c}_2^e)$, $u_V = V - \bar{V}$

$u = (u_V, u_R)'$; $f_I = (f_1^s, f_2^s, f_1^r, f_2^r)'$

$c_{II} = (c_1, c_2,\ldots, c_{s-1}, c_{s+1},\ldots, c_{e-1}, c_{e+1},\ldots,c_{N-1}, c_D)'$

$f_{II} = (f_1, f_2,\ldots, f_{s-1}, f_{s+1},\ldots, f_{e-1}, f_{e+1},\ldots, f_{N-1}, f_D)'$

x_I are the four temperature deviations at the measurement trays after a bubble point function-based coordinate change, and x_{II} are the remaining deviated compositions, and u_v (or u_R) is the deviation vapor (or reflux) flow control input, and d accounts for deviations in feed flow and composition.

Observe that the model (6) or (7) has relative degrees (RD´s) equal to one for both inputs and any choice of measured temperatures, excepting the distillate. Knowing that the CL column forces a unique material balance [3-4], then the zero dynamics of the resulting system are stable. Thus, system (6) or (7) is *passive*, implying that related robust nonlinear SF control problem is solvable [7, 24].

3.2 Linear-decentralized model for OF Control

Next, a linear-decentralized model with reconstructible load inputs is set for OF control design purposes. Following previous developments in two point temperature and composition-temperature cascade control designs [3-5], on the basis of the preceding RD structure and the linearitydecentralization feature specifications for our OF control design rewrite the passive model (7) as follows:

$$\dot{x}_1^s = a_1^s u_V + b_1^s, \quad b_1^s = \varphi_1^s(x_I, x_{II}, q, u), \quad y_1^s = x_1^s \tag{9}$$

$$\dot{x}_2^s = a_2^s u_V + b_2^s, \quad b_2^s = \varphi_2^s(x_I, x_{II}, q, u), \quad y_2^s = x_2^s$$

$$\dot{x}_1^r = a_1^r u_R + b_1^r, \quad b_1^r = \varphi_1^r(x_I, x_{II}, q, u), \quad y_1^r = x_1^r$$

$$\dot{x}_2^r = a_2^r u_R + b_2^r, \quad b_2^r = \varphi_2^r(x_I, x_{II}, q, u), \quad y_2^r = x_2^r$$

$$\varphi_1^s(x_I, x_{II}, q, u) = f_1^s(x_I, x_{II}, u, d) - a_1^s u_V, \quad a_1^s = -(\Delta^+\bar{T}_1^s/\bar{m}_1^s)p_s$$

$$\varphi_2^s(x_I, x_{II}, q, u) = f_2^s(x_I, x_{II}, u, d) - a_2^s u_V, \quad a_2^s = -(\Delta^+\bar{T}_2^s/\bar{m}_2^s)p_s$$

$$\varphi_1^r(x_I, x_{II}, q, u) = f_1^r(x_I, x_{II}, u, d) - a_1^r u_R, \quad a_1^r = (\Delta^+\bar{T}_1^r/\bar{m}_1^r)$$

$$\varphi_2^r(x_I, x_{II}, q, u) = f_2^r(x_I, x_{II}, u, d) - a_2^r u_R, \quad a_2^r = (\Delta^+\bar{T}_2^r/\bar{m}_2^r)$$

where $\Delta^+\bar{T}_k$ is the temperature gradient at the k-th stage, and p_s is the nominal operating line slope of the stripping section in the column's McCabe-Thiele design diagram [3-5,21]. The inputs $(b_1^s, b_2^s, b_1^r, b_2^r)$ satisfy the matching condition [7], as they enter in the same channels as the control inputs. Since the temperature state x_I is measured, the load disturbances $(b_1^s, b_2^s, b_1^r, b_2^r)$ are instantaneously observable [9], as they can be reconstructed from the inputs and the measured output derivatives, according to the expressions:

$$b_1^s = \dot{y}_1^s - a_1^s u_V, \; b_2^s = \dot{y}_2^s - a_2^s u_V, \; b_1^r = \dot{y}_1^r - a_1^r u_R, \; b_2^r = \dot{y}_2^r - a_2^r u_R \tag{10}$$

This in turn implies that each input can be quickly reconstructed with a linear-decentralized reduced-order observer, one per measurement-load pair. Accordingly, our *model for OF control design* is given by [23]:

$$\dot{x}_1^s = a_1^s u_V + b_1^s, \quad \dot{b}_1^s \approx 0, \quad y_1^s = x_1^s \tag{11}$$

$$\dot{x}_2^s = a_2^s u_V + b_2^s, \quad \dot{b}_2^s \approx 0, \quad y_2^s = x_2^s$$

$$\dot{x}_1^r = a_1^r u_R + b_1^r, \quad \dot{b}_1^r \approx 0, \quad y_1^r = x_1^r$$

$$\dot{x}_2^r = a_2^r u_R + b_2^r, \quad \dot{b}_2^r \approx 0, \quad y_2^r = x_2^r$$

where $(b_1^s, b_2^s, b_1^r, b_2^r)$ are unknown-reconstructible load inputs.

3.3 Feedforward-State feedback Lyapunov Control

In this section, assuming the load disturbances $(b_1^s, b_2^s, b_1^r, b_2^r)$ are known, a stabilizing FF-SF controller for the column is built with the Lyapunov-based passivation by backstepping temperature control design employed in a polymer reactor control study [1]. Let x_2^{s*} (or x_2^{r*}) denote the secondary temperature set point (to be determined) in the stripping (or rectifying) section, and u_V^* (or u_R^*) be the corresponding vapour (or reflux) flow, according to the expressions:

$$\dot{x}_2^{s*} = a_2^s u_V^* + b_2^s, \quad u_V^* + \tilde{u}_V = u_V \tag{12a}$$

$$\dot{x}_2^{r*} = a_2^r u_R^* + b_2^r, \quad u_R^* + \tilde{u}_R = u_R \tag{12b}$$

Regard the candidate Lyapunov function (13):

$$V = [(x_1^s)^2 + (e_2^s)^2 + (x_1^r)^2 + (e_2^r)^2]/2 > 0, \tag{13}$$
$$e_2^s = x_2^s - x_2^{s*}, \qquad e_2^r = x_2^r - x_2^{r*}$$

write its dissipation (14) along the column motion:

$$\dot{V} = x_1^s\,[a_1^s\,(u_V^* + \tilde{u}_V) + b_1^s] + a_2^s\,e_2^s\,\tilde{u}_V +$$
$$+ x_1^r\,[a_1^r\,(u_R^* + \tilde{u}_R) + b_1^r] + a_2^r\,e_2^r\,\tilde{u}_V \tag{14}$$

perform *backstepping* by transfering the term $x_1^s a_1^s \tilde{u}_V$ (or $x_1^r a_1^r \tilde{u}_R$) from the first (or third) to the second (or fourth) term of (14) [1],

$$\dot{V} = x_1^s\,(a_1^s u_V^* + b_1^s) + (a_1^s x_1^s + a_2^s e_2^s)\tilde{u}_V$$
$$+ x_1^r\,(a_1^r u_R^* + b_1^r) + (a_1^r x_1^r + a_2^r e_2^r)\tilde{u}_R \tag{15}$$

and enforce the (implicit) control expressions

$$a_1^s u_V^* + b_1^s = -k_1^s x_1^s, \quad \tilde{u}_v = -k_2^s\,(a_1^s x_1^s + a_2^s e_2^s) \tag{16a}$$
$$a_1^r u_R^* + b_1^r = -k_1^r x_1^r, \quad \tilde{u}_R = -k_2^r\,(a_1^r x_1^r + a_2^r e_2^r) \tag{16b}$$

to enforce the negative dissipation (k_1^s, k_2^s, k_1^r and k_2^r are control gains):

$$\dot{V} = -k_1^s\,(x_1^s)^2 - k_2^s\,(a_1^s x_1^s + a_2^s e_2^s)^2 - k_1^r (x_1^r)^2 - k_2^r\,(a_1^r x_1^r + a_2^r e_2^r)^2 < 0 \tag{17}$$

with the *closed-loop output temperature stable dynamics*:

$$\dot{x}_1^s = -k_1^s x_1^s + a_1^s e_2^s, \quad \dot{e}_2^s = -k_2^s e_2^s - a_1^s x_1^s \tag{18a-d}$$

$$\dot{x}_1^r = -k_1^r x_1^r + a_1^r e_1^r, \quad \dot{e}_2^r = -k_2^r e_2^r - a_1^r x_1^r \tag{18a-d}$$

Finally, the combination of the FF-SF controller with the setpoint generator (12a) (or (12b)) yields the dynamic FF-SF controllers for each column section:

Stripping section

$$u_V^* = -k_1^s\,(x_1^s + b_1^s)/a_1^s, \quad \dot{x}_2^{s*} = a_2^s u_V^* + b_2^s \quad \text{(primary)} \tag{19a}$$
$$u_v = u_V^* - k_2^s\,[a_1^s x_1^s + a_2^s\,(x_2^s - x_2^{s*})] \quad \text{(secondary)} \tag{19b}$$

Rectifying section

$$u_R^* = -k_1^r\,(x_1^r + b_1^r)/a_1^r, \quad \dot{x}_2^{r*} = a_2^r u_R^* + b_2^r \quad \text{(primary)} \tag{20a}$$

$$u_R = u_R^* - k_2^r[a_1^r x_1^r + a_2^r\,(x_2^r - x_2^{r*})] \quad \text{(secondary)} \tag{20b}$$

Observe that: (i) the primary control component (19a) [or (20a)] generates the setpoint-control pair (x_2^{s*}, u_V^*) [or (x_2^{r*}, u_R^*)], (ii) the secondary controller component (19b) [or (20b)] decides the actual vapor (or reflux) rate u_V (or u_R), and (iii) the "primary" (k_1^s and k_1^r) and "secondary" (k_1^r and k_2^r) gain pair does not have to be dynamically separated.

3.4 OF control

The combination of the FF-SF (19-20) with the battery of the reduced-order observers associated to model (11) yields the OF controller in IMC form [23] (ω is the filter gain):

Stripping section

$$\dot{\chi}_1^s = -\omega\chi_1^s - \omega\,(\omega y_1^s + a_1^s u_V), \quad u_V^* = -k_1^s\,(y_1^s + \chi_1^s + \omega y_1^s)/a_1^s$$
$$\dot{\chi}_2^s = -\omega\chi_2^s - \omega\,(\omega y_2^s + a_2^s u_V), \quad \dot{x}_2^{s*} = a_2^s u_V^* + \chi_2^s + \omega y_2^s$$
$$u_V = u_V^* - k_2^s\,[a_1^s y_1^s + a_2^s\,(y_2^s - x_2^{s*})] \tag{21a-e}$$

Rectifying section

$$\dot{\chi}_1^r = -\omega\chi_1^r - \omega\,(\omega y_1^r + a_1^r u_R), u_R^* = -k_1^r\,(y_1^r + \chi_1^r + \omega y_1^r)/a_1^r$$

$$\dot{\chi}_2^r = -\omega\chi_2^r - \omega(\omega y_2^r + a_2^r u_R), \dot{x}_2^{r*} = a_2^r u_R^* + \chi_2^r + \omega y_2^r$$
$$u_R = u_R^* - k_2^r [a_1^r y_1^r + a_2^r (y_2^r - x_2^{r*})] \qquad (22a\text{-}e)$$

The implementation of this controller requires only a (possibly rough) approximation of the four-constant set (a_1^s, a_2^s, a_1^r, a_2^r) determined by the SS gradients, holdups and operating line slope in the stripping section. The per-section controllers (19 and 20) are linear and decentralized. Comparing with its TPT counterpart [3], each section controller (19 and 20) is a standard PI cascade controller plus a secondary-primary interconnecting component. Differently from cascade control schemes, here the "primary" (k_1^s and k_1^r) and "secondary" (k_1^r and k_2^r) gain pair does not need to be dynamically separated. From the TPT control approach [3] (supported by the same controller-observer realization approach) in conjunction with the resulting close-loop regulation-estimation error dynamics, the next tuning guidelines follow:

1. Let the triplet (κ_ω, κ_p, κ_s) denotes that: (i) the observer is set κ_ω times faster than the open-loop characteristic period λ_c of the output responses, (ii) the primary controller is κ_p times faster than λ_c, and (iii) the secondary controller is κ_s times faster than the primary one. This is:

$$\omega = \kappa_\omega\lambda_n, \qquad (k_1^s, k_1^r) = \kappa_p\lambda_n(1, 1), \qquad (k_2^s, k_2^r) = \kappa_s(k_1^s, k_1^r)$$

2. Set (κ_ω, κ_p, κ_s) ≈ (10, 1, 3), gradually increase the observer gain κ_ω until oscillatory behaviour due to measurement-model error propagation is obtained at κ_ω^*, and backoff to $\kappa_\omega = \kappa_p^*/(2\text{-to-}3)$.

3. Gradually increase the primary control gain κ_p until oscillatory response is obtained at κ_p^*, and backoff to $\kappa_p = \kappa_p^*/(2\text{-to-}3)$.

4. Adjust the secondary control gain κ_s to draw a suitable compromise between effluent composition regulation and control effort.

4. APPLICATION EXAMPLE

To test the proposed methodology, a high-purity column (studied before with TPT [3] and DCT [5] approaches), was considered: a 12-tray column where an equimolal methanol-water mixture is separated to a 99 % mol (or 1%) distillate (or bottoms) product purity. The column model includes non-ideal thermodynamics, i.e., an energy balance per tray must be accounted for. Following Shinskey's recommendation [26], the comparison of response times will be performed in terms of the natural (open-loop) settling time units (N_{stu}). This column has an N_{stu} = 160 min (i.e., $\lambda_n \approx 1/40$ min^{-1}), and (1, 4) as the sensitive tray location pair.

The column was set at its nominal SS and then subjected to a sequence of input step disturbances: (i) At t = 0, the saturated feed rate (F) increases from 10 to 11 mol/sec, (ii) at t = 300 min, F decreases to 9 mol/sec, (iii) at t = 600 min, the feed composition changes from 0.5 to 0.55, and (iv) at t = 900 min, the feed composition decreases to 0.45. For the sake of comparison, the CL column behaviour with the proposed scheme is matched against the ones of two previous TPT control schemes with setpoint compensation (based on a feedforward scheme driven by feed temperature measurements [3]), and the same TPT with constant setpoints. The resulting behaviours are depicted in Figure 2, showing that, with the proposed approach: (i) both effluent compositions are regulated (with reduced offsets) in about 30 min, (≈ 0.20 N_{stu}), with reasonable and smooth control efforts, (ii) the disturbances are well compensated for, and (iii) the control efforts are smooth, i.e., without overshoots, (iv) there are product offsets when no setpoint compensation is made, and (v) the previous scheme [3] with setpoint compensation can eliminate the offset. Observe that, with smooth and coordinated control action, the proposed controller yields smaller effluent composition deviations and shorter response times than the ones of the temperature measurement-based previous schemes.

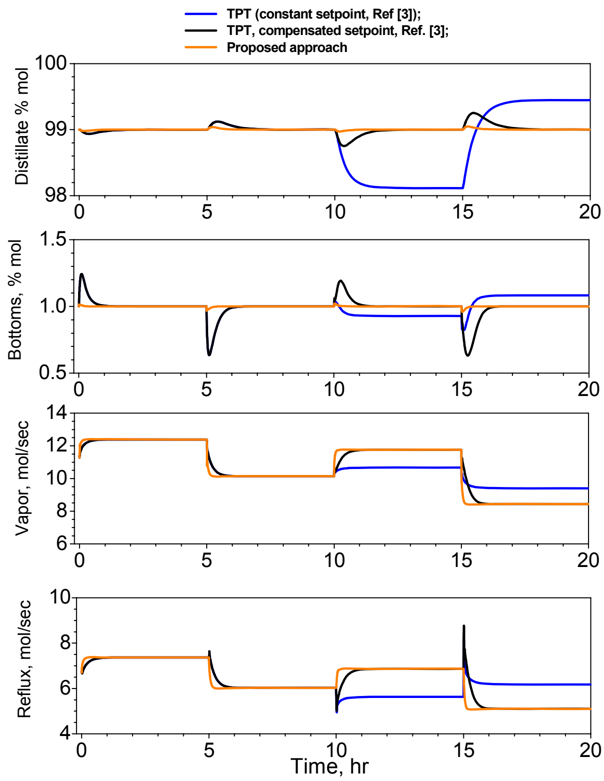

Fig. 2. Closed-loop response of the column to a sequence of feed flow rate and composition disturbances.

To assess under a more severe (realistic) test the functioning of the proposed controller against the one of previous temperature based controller [3], the column was fed with a -2° C subcooled stream, and subjected to a more aggressive sequence of input step disturbances: (i) At t = 0, the feed rate (F) increases from 10 to 12 mol/sec; (ii) at t = 300 min, F decreases to 8 mol/sec; (iii) at t = 600 min, the feed composition changes from 0.5 to 0.60; and finally (iv) at t = 900 min, the feed composition decreases to 0.40. The resulting closed-loop behaviours are presented in Figure 3, showing that: (i) the proposed scheme yields appreciably faster and smoother responses with smaller offsets than the ones of the previous approach [3]Ŧ, and (ii) the proposed approach has smooth and coordinated control efforts, with more efficient actions than the ones of the previous scheme. In other words, the proposed controller yields tighter and faster effluent purity-pair regulation with smooth efficient control efforts.

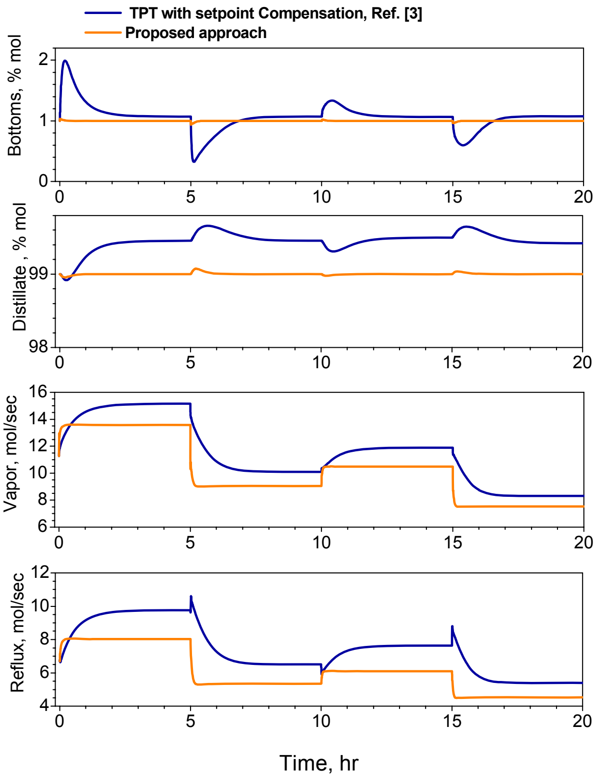

Fig. 3. Closed-loop response of the column to a sequence of feed flow rate and composition disturbances.

5. CONCLUSIONS

The problem of controlling binary distillation column effluents by manipulating the reflux and vapor flow rates on the basis of four temperature measurements has been addressed. The problem was regarded within a constructive control framework where: (i) the system´s relative degree structure and observability properties were exploited to set a linear-decentralized model for OF control design, and (ii) the OF control design was performed via passivation by backstepping according to a Lyapunov approach. The result was a pair of linear-decentralized controllers: (i) the heat duty (or reflux) is adjusted according to a temperature measurement pair in the stripping (or rectifying) section, and (ii) the Lyapunov design enables tuning without primary-secondary dynamic separation. The proposed methodology was tested with a representative example through simulations, finding that the behavior of the proposed schemes outperforms the ones of previous control schemes.

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