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Networked Plantwide Process Control with Asynchronous Communication and Control

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Abstract: The need for economic efficiency and safety has driven the development of large and complex chemical plants. Due to the presence of interactions, controlling such plants are often difficult. This paper aims to address this issue by developing a networked-based process control approach. In this framework, a plantwide process is modeled as network of process units which is controlled by a network of autonomous controllers. The controllers within the network operate with different sampling rates and communicate with each other asynchronously, to allow flexibility and better utilization of communication bandwidth. A key feature of controller network is the connective stability among controllers, which ensures plantwide stability when communication problems such as data packet drop-outs occur. Using the concept of dissipativity, plantwide connective stability and global performance is translated into conditions for which each controller has to satisfy. The controllers are then designed individually to form an autonomous and distributed control system.

Keywords: Process networks, controller networks, dissipativity, asynchronous systems

1. INTRODUCTION

Modern chemical plants are large and complex. Many of these plants consist of more than a hundred process units and require thousands control loops. Due to the wide use of material recycle and heat integration, there are often strong interactions among process units that profoundly complicate the plantwide process dynamics. As a result, the application of conventional control techniques (e.g. centralized and decentralized control) becomes expensive, difficult and inadequate. To address these issues, this paper adopts a plantwide process control framework developed in Xu and Bao (2009). A complex plant is modeled as a network of interconnected process units (known as process network), which is controlled by a network of controllers (known as distributed controllers). Within the controller network, each controller not only exchanges information with other controllers, but also provides local control action based on its local measurements and information received from other controllers. This form of control network representation is different to the existing work in NCS (Networked Control Systems), which is focused on the communications between sensors, actuators and controllers. Detailed discussions on NCS can be found in Baillieul and Antsaklis (2007); Tian and Levy (2008). The proposed controller network can have arbitrary topologies and is very flexible where a variety of control network configurations can be implemented.

The work in this paper extends the above framework and develops an asynchronous control and communication network approach that ensures connective stability. The communication network between controllers often experiences issues such as data packets drop-out that may affect the stability of plantwide process. To overcome these problems, the condition known as *plantwide connective stability* is derived and incorporated in the control design framework. This condition ensures the stability of plantwide systems when one or more communications links between controllers is/are lost. The development of an asynchronous communication network is particularly useful in network-based control applications where controllers are able communicate at different rates. This allows faster information broadcast rates to be assigned to controllers for process units with smaller time constants and/or larger interaction effects. In doing so, it ensures better utilization of the network bandwidth. Apart from the asynchronous communication, each controller within the network provides control action to its respective unit at different rates. This feature is important to large scale chemical plants where each process unit possesses very different dynamics. As such, it would be ideal and economically beneficial if these units are asynchronously controlled. For example, a reactor often has faster dynamics compared to a distillation column and should be sampled and controlled at a higher rate than the latter process. Through this formulation, the control system becomes more scalable, autonomous and reliable.

It has been shown (Xu and Bao (2009)) that interactions between process units are best captured by the physical mass and energy flow. Therefore a similar network development is used in this paper. Due to the different rates used for control and communication, the overall plant model is generally time-varying. To accommodate this asynchronous nature of the plant, a time invariant reformulation is required where a common "time-window" (e.g. the least common multiple, **lcm**, of all control/sampling and communication time periods) is selected to allow the overall plant model to be *T-periodic*. This reformulation is known as *lifting* and has been described in Colaneri et al. (1990); Chen and Qiu (1994); Chen and Francis (1995). Under each T time period, the plantwide process is described as a network of sub-models where each model describes the dynamics of its respective process unit at a particular time instant within T. Time-varying local controllers in the network will provide the control action to each sub-model at different time instant. The networked control problem is then converted into one that can be solved in a decentralized fashion.

The paper is organized as follows. Section 2 presents detailed modeling of process and control networks of processes with asynchronous control and communication. The framework for controller network design based on discrete-time dissipativity is described in Section 3, followed by a conclusion in Section 4.

Notations

For any kT where T is a sampling period, denote a lifted signal $\underline{u}^i(k) = \left[u^i(kT), \ldots, u^i(kT + \mathcal{N}_i T_i)\right]$ where $T = (\mathcal{N}_i + 1)T_i$, $\forall i = 1, \ldots, N$. Then denote $\underline{\mathbf{u}}(k) = \left[\underline{u}^1(k), \ldots, \underline{u}^N(k)\right]$. Denote the i-th element of a block diagonal constant matrix $\mathcal{K}$ and the (i, ℓ)-th constant block matrix of $\mathcal{G}$ associated at j-th sampling instant e.g. $\mathcal{K}^i(jT_i)$ and $\mathcal{G}^{i\ell}(jT_i)$, as $\mathcal{K}^{i \cdot j}$ and $\mathcal{G}^{i\ell \cdot j}$ respectively. On the other hand, the i-th element of a block diagonal system P and the (i, ℓ)-th block of $\tilde{\mathcal{P}}$ at j-th sampling instant are denoted as $P^{i \cdot j}$ and $\tilde{\mathcal{P}}^{i\ell \cdot j}$ respectively. A diagonal matrix made up of N_i repeating G^i is represented by $\underset{1,\ldots,N_i}{\mathbf{rep}}\ G^i$. We denote $G^{(i)}$ corresponding to the m_i block rows of G where G is a $m \times n$ matrix and $m = [m_1^T, m_2^T, \ldots, m_N^T]^T$.

2. PROCESS AND CONTROL NETWORKS WITH ASYNCHRONOUS CONTROL AND COMMUNICATION

In this networked process control approach, each process unit is modeled as a two-port system that describes the relationship between the input and output *physical* and *information* flow (Xu and Bao (2009)). Physical flows, represented by extensive variables such as energy, mass or molar flowrate, are flows that interconnect process units to form a process network. These flows are unique features of chemical processes and is often neglected in most process control approaches. Information flows, on the other hand, are made up of measured and manipulated variables that are used to form the control loops. The process network is controlled by a controller network made up of individual local controllers that are similarly modeled as a two-port system. These controllers not only responds to its local sensor output but also the information received from other controllers. In this section, the above representations of process and control networks are extended to a form suitable for discrete time control network synthesis.

The dynamics of each process unit P^i with a sampling period of T_i, where T_i $(i = 1, \ldots, N)$ are positive integers is represented as follows:

$$\begin{bmatrix} x^i(jT_i + T_i) \\ y_p^i(jT_i) \\ y_c^i(jT_i) \end{bmatrix} = \begin{bmatrix} A^i & B_1^i & B_2^i \\ C_1^i & D_{11}^i & D_{12}^i \\ C_2^i & D_{21}^i & D_{22}^i \end{bmatrix} \begin{bmatrix} x^i(jT_i) \\ u_p^i(jT_i) \\ u_c^i(jT_i) \end{bmatrix} \tag{1}$$

where $y_p^i(jT_i) \in \mathbb{R}^{n_{ypi}}$, $u_p^i(jT_i) \in \mathbb{R}^{n_{upi}}$ represent the respective physical mass/energy flow leaving from and coming into P^i, $y_c^i(jT_i) \in \mathbb{R}^{n_{yci}}$ and $u_c^i(jT_i) \in \mathbb{R}^{n_{uci}}$ describe the information flow coming to and leaving P^i respectively and $x^i(jT_i) \in \mathbb{R}^{m_i}$ is the state vector. In a first principle model, $y_p^i(jT_i)$ and $u_p^i(jT_i)$ are extensive variables and $y_c^i(jT_i)$ and $u_c^i(jT_i)$ are intensive variables.

In this paper, we study the problem of state feedback distributed control where $C_2^i = I$, $D_{21}^i = D_{22}^i = 0$ in (1). Each controller $\mathcal{C}^i$ at jth sampling instant is represented as follows:

$$\mathcal{C}^i : \begin{cases} \begin{bmatrix} \hat{u}_c^i(jT_i) \\ \tilde{u}_c^i(jT_i) \end{bmatrix} = \begin{bmatrix} \mathcal{K}_1^i \\ \mathcal{K}_2^i \end{bmatrix} y_c^i(jT_i) \\ u_c^i(jT_i) = \hat{u}_c^i(jT_i) + \sum_{\bar{n}=\ell_1}^{\ell_2} \tilde{u}_c^{\bar{n}}(jT_{\bar{n}}), \\ \qquad \forall i \neq \ell_1, \ell_2 = 1, \ldots, N \end{cases} \tag{2}$$

where $\mathcal{K}_1^i \in \mathbb{R}^{n_{uci} \times m_i}$ and $\mathcal{K}_2^i \in \mathbb{R}^{n_{\tilde{u}ci} \times m_i}$ are controller gain matrices. Vectors $\hat{u}_c^i(jT_i) \in \mathbb{R}^{n_{uci}}$ and $\tilde{u}_c^i(jT_i) \in \mathbb{R}^{n_{\tilde{u}ci}}$ represent the intermediate control signal and information sent out to other controllers respectively. Variables u_c^i and y_c^i denote the local sensor output and controller output respectively. The local controller output, $u_c^i(jT_i)$ is calculated based on information received from its sensor output ($y_c^i(jT_i)$) and information from other controllers.

Using the above the representations, the following issues will occur in networked plantwide process control:

- *Asynchronous control* - Due to the different dynamics each process P^i possesses, it is beneficial for the different units within the plant to be controlled at different rates.
- *Asynchronous communication* - To ensure the better utilization of the bandwidth within communication networks, it is useful that each controller, $\mathcal{C}^i$, within a controller network communicates information at different rates

To facilitate the development of an asynchronous control and communication process and controller network, a new time window T (defined as *plantwide system period*) is selected to account for the use of different T_i and $\tilde{T}_i$. This ensures that the dynamics of each process and controller at different time instants within is T is captured and representative at every T. A suitable T can be found as follows:

$$T := \mathbf{lcm}\{T_1,\ \tilde{T}_1, \ldots, T_N, \tilde{T}_N\} \tag{3}$$

Using the plantwide system period T, the process and controller networks are defined as follows.

Process Network. Within every T, each i-th process in (1) sends out (i.e. y_p^i and y_c^i) and receives (i.e. u_p^i and u_c^i) signals $(\mathcal{N}_i + 1) = \frac{T}{T_i}$ times. Using constant matrices $H_p^{i\ell}$, the input-output relationship of physical flow between between the i-th process and N processes at a every T (i.e. $T \leq T + \epsilon T_i < 2T, \forall \epsilon = 0, 1, 2, \ldots, \mathcal{N}_i$), can be defined as

$$u_p^i(\epsilon T_i) = u_e^i(\epsilon T_i) + d_p^i(\epsilon T_i) - \sum_{\substack{\ell \\ \ell \neq i}}^{N} \sum_{j=0}^{\mathcal{N}_\ell} H_{p_\epsilon}^{i\ell \cdot j} y_p^\ell(jT_\ell) \qquad (4)$$

where

$$H_{p_\epsilon}^{i\ell \cdot j} := \begin{cases} \mathcal{H}_p^{i\ell} & T_\ell \geq (j+1)T_\ell - \epsilon T_i > 0, \\ 0 & \text{otherwise} \end{cases}$$

Note that $H_{p_\epsilon}^{i\ell} = \left[H_{p_\epsilon}^{i\ell \cdot 0}, \ldots, H_{p_\epsilon}^{i\ell \cdot \bar{N}_\ell}\right]$ and $H_p^{i\ell} = \left[H_{p_0}^{i\ell^T}, \ldots, H_{p_{\bar{N}_i}}^{i\ell^T}\right]^T$. The elements of $\mathcal{H}_p^{i\ell}$ with appropriate dimensions are between 0 and 1(inclusive), where $\mathcal{H}_p^{i\ell} = I$ implies that the ℓ-th process is connected to the i-th process while $\mathcal{H}_p^{i\ell} = \mathbf{0}$ implies no physical connection between the them. When the values of $\mathcal{H}_p^{i\ell}$ is between 0 and 1, it implies that the physical output of the i-th process is split prior to flowing into the ℓ-th process and the value in $\mathcal{H}_p^{i\ell}$ represents the split ratio of the i-th unit. Here $u_e^i \in \mathbb{R}^{n_{upi}}$ and $d_p^i \in \mathbb{R}^{n_{upi}}$ represent the external physical flow and input disturbance in the physical flow into the i-th process.

Controller Network. Within each T time period, each $\mathcal{C}^i$ in (2) receives information from both its local sensor output and other controllers and sends out a controller output to its respective process every T_i period. The controller, $\mathcal{C}^i$, then sends out information to other controllers every $\tilde{T}_i$ period, which is equivalent to $(\tilde{\mathcal{N}}_i + 1) = \frac{T}{\tilde{T}_i}$ times within every T. Introduce block matrices $H_c^{i\ell},\ i \neq \ell, \forall i, \ell = 1, \ldots, N$ to describe the information exchange paths between controllers, the controller output u_c^i of $\mathcal{C}^i$ at a time instant ϵT_i within T can be represented as follows:

$$u_c^i(\epsilon T_i) = \hat{u}_c^i(\epsilon T_i) + \overbrace{\sum_{\substack{\ell \\ \ell \neq i}}^{N} \sum_{j=0}^{\tilde{N}_\ell} H_{c_\epsilon}^{i\ell \cdot j} \tilde{u}_c^\ell(j\tilde{T}_\ell)}^{\text{information received from other controllers}} \qquad (5)$$

where

$$H_c^{i\ell \cdot j} := \begin{cases} \mathcal{H}_c^{i\ell} & \tilde{T}_\ell \geq (j+1)\tilde{T}_\ell - \epsilon T_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

Similar to each $H_p^{i\ell}$, $H_c^{i\ell \cdot j} = \left[H_{c_0}^{i\ell \cdot j^T}, H_{c_1}^{i\ell \cdot j^T}, \ldots, H_{c_{\tilde{\mathcal{N}}_i}}^{i\ell \cdot j^T}\right]^T$ where $H_{c_\epsilon}^{i\ell \cdot j} = \left[H_{c_\epsilon}^{i\ell \cdot 0}, H_{c_\epsilon}^{i\ell \cdot 1}, \ldots, H_{c_\epsilon}^{i\ell \cdot \tilde{\mathcal{N}}_\ell}\right]$. Note that $H_c^{ii} = 0, \forall i = 1, \ldots, N$. Matrices $\mathcal{H}_c^{i\ell}$ which have elements of 1 or/and 0 is partitioned vertically into N_ℓ sub-matrices, where N_ℓ are the number of controllers $\mathcal{C}^\ell$ sends information to. If the $\mathcal{C}^i$ receives information from the ℓ-th controller($i \neq \ell$), then the sub-matrix of $\mathcal{H}_c^{i\ell}$ corresponding to $\tilde{u}_c^{\ell i}$ is matrix I. Otherwise this sub-matrix is a zero matrix. Matrices $\mathcal{H}_c^{i\ell}$ is used to describe the topology of the controller network while matrices $H_c^{i\ell}$ is used to represent the network topology within every T based on the effects of asynchronous communication among controllers.

Through this formulation, the parameters (i.e. controller gains) for each $\mathcal{C}^i$ are time varying within T but they remain the same for every T period.

The above descriptions of the process and controller networks have the following key features:

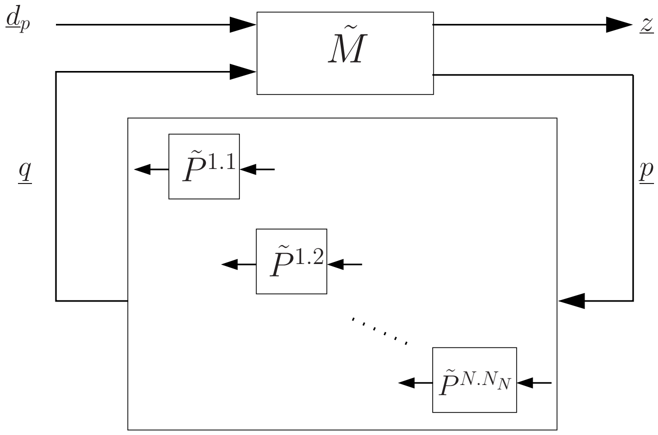

Fig. 1. $\tilde{M}$- $\tilde{\mathcal{P}}$ system

- From the process network description, the network topology is separated from the models of individual process units. This makes plantwide modeling simpler and less complex since individual plant models can always be derived using first principle modeling approaches.
- The process network topology can be directly obtained from a process flow diagram.
- An arbitrary topology for controller network can be used in the proposed approach. This allows flexibility in choosing a controller network that offers a good balance between complexity of the network and control performance.
- For given model of each process unit and controller in (1) and (2) and the network topology in $\mathcal{H}_p^{i\ell}$ and $\mathcal{H}_c^{i\ell}$, the formulation of matrices $H_p^{i\ell}$ and $H_c^{i\ell}$ in (4) and (5) can be easily implemented on any programming platforms. This made the description of such networks more systematic and computer friendly.

3. CONTROLLER NETWORK DESIGN

Given the representations for both the processes and controllers, the proposed controller network design can be implemented. The control design framework is implemented in two steps. First, the plantwide connective stabilizability problem is solved. The second step then involves designing each controller independently. This framework requires the controller network problem to be solved at every T period using the lifted signals.

3.1 Framework

Prior to designing each controllers, the process and control networks described in the previous sections are first converted into a form suitable for stability and performance analysis. A typical form is shown in Fig. 1, which represents a linear fractional transformation (LFT) of systems $\tilde{M}$ and $\tilde{\mathcal{P}}$. The control performance is represented by the H_∞ norm from $\underline{\mathbf{d_p}}$ to $\underline{\mathbf{z}}$, which are lifted signals. In Fig. 1, $\tilde{M}$ includes the topologies of both process and controller networks captured by matrices $H_p^{i\ell}$ and $H_c^{i\ell}$ and the performance specifications while system $\tilde{\mathcal{P}} = \underset{i=1,\ldots,N}{\mathbf{diag}} \left\{ \underset{j=0,\ldots,\mathcal{N}_i}{\mathbf{diag}} \tilde{\mathcal{P}}^{i \cdot j} \right\}$ where each $\tilde{\mathcal{P}}^{i \cdot j}$ is a LFT of the

i-th "virtual" process unit $P_v^{i \cdot j}$ and its "virtual" controller $\mathcal{C}_v^{i \cdot j}$ at $kT + jT_i$ time instant. The input to $\tilde{\mathcal{P}}^{i \cdot j}$ carries the information related to physical flow from other process units to the i-th process unit and information received by the i-th controller from other controllers at different time instants within T. The output of $\tilde{\mathcal{P}}^{i \cdot j}$ carries information on the state variables of the local process and the information sent from its controller at jT_i sampling instant. System $\tilde{M}$ has the following state space representation:

$$\begin{bmatrix} \mathbf{x_m}(kT+T) \\ \underline{\mathbf{z}}_w(kT) \\ \underline{\mathbf{p}}(kT) \end{bmatrix} = \underbrace{\begin{bmatrix} A_m & B_{m_1} & B_{m_2} \\ C_{m1} & D_{zw} & D_{zp} \\ C_p & D_{pw} & D_{pq} \end{bmatrix}}_{M} \begin{bmatrix} \mathbf{x_m}(kT) \\ \underline{\mathbf{d}}_\mathbf{p}(kT) \\ \underline{\mathbf{q}}(kT) \end{bmatrix} \tag{6}$$

where $\underline{\mathbf{q}}(kT)$ and $\underline{\mathbf{p}}(kT)$ are interconnection signals for systems $\tilde{M}$ and $\tilde{\mathcal{P}}$. The state space representation of each $\tilde{\mathcal{P}}^{i \cdot j}$ within T is as follows:

$$\begin{bmatrix} x_p^i(jT_i + T_i) \\ q_i(jT_i) \end{bmatrix} = \begin{bmatrix} A_{c\ell}^{i \cdot j} & B_{c\ell}^{i \cdot j} \\ C_{c\ell}^{i \cdot j} & D_{c\ell}^{i \cdot j} \end{bmatrix} \begin{bmatrix} x_p^i(jT_i) \\ p_i(jT_i) \end{bmatrix} \tag{7}$$

with system $P_v^{i \cdot j}$ described by:

$$\begin{bmatrix} x_p^i(jT_i + T_i) \\ q^i(jT_i) \\ y^i(jT_i) \end{bmatrix} = \underbrace{\begin{bmatrix} A_p^{i \cdot j} & B_p^{i \cdot j} & B_u^{i \cdot j} \\ C_q^{i \cdot j} & D_{qp}^{i \cdot j} & D_{qu}^{i \cdot j} \\ C_y^{i \cdot j} & D_{yp}^{i \cdot j} & D_{yu}^{i \cdot j} \end{bmatrix}}_{\mathcal{P}^{i \cdot j}} \begin{bmatrix} x_p^i(jT_i) \\ p^i(jT_i) \\ u^i(jT_i) \end{bmatrix} \tag{8}$$

and $\mathcal{C}_v^{i \cdot j}$:

$$u^i(jT_i) = \mathcal{K}_p^{i \cdot j} y^i(jT_i) \tag{9}$$

Consider the following performance specifications:

(1) *Disturbance attenuation.* This relates to the system $T_{yd}(z)$ from input disturbance to the physical flow $\underline{\mathbf{d}}_\mathbf{p}$ to measured process output $\underline{\mathbf{y}}_\mathbf{c}$.
(2) *Control gain constraint.* This relates to the system $T_{ud}(z)$ from $\underline{\mathbf{d}}_\mathbf{p}$ to the controller outputs $\underline{\mathbf{u}}_\mathbf{c}$.

Both specifications can be represented by the upper bound $\gamma > 0$ of the following weighted H_∞ norm:

$$\left\| \begin{matrix} \mathbf{W}_1 T_{yd} \\ \mathbf{W}_2 T_{ud} \end{matrix} \right\|_\infty < \gamma. \tag{10}$$

where $\mathbf{W}_1 = \underset{i=1,\ldots,N}{\mathbf{diag}} \left\{ \underset{j=0,1,\ldots,\mathcal{N}_i}{\mathbf{diag}} W_1^{i \cdot j} \right\}$ and $\mathbf{W}_2 = \underset{i=1,\ldots,N}{\mathbf{diag}} \left\{ \underset{j=0,1,\ldots,\mathcal{N}_i}{\mathbf{diag}} W_2^{i \cdot j} \right\}$.

Theorem 1. Assume constant productions rates (i.e. $\underline{\mathbf{u}}_\mathbf{e} = \mathbf{0}$). For an asynchronous control and communication network of process systems, P_i $(i = 1, \ldots, N)$, and state-feedback controllers, $\mathcal{C}_i$ $(i = 1, \ldots, N)$, with matrices $H_p^{i\ell}$ and $H_c^{i\ell}$ and weighted performance specifications described in P1 and P2, it can be represented by the system shown in Fig. 1. Then the state-space representation of system $\tilde{M}$ is as follows:

$$\begin{aligned}
\underline{\mathbf{x}}_{\mathbf{w_1}}(kT+T) &= \mathbf{A_{w_1}} \underline{\mathbf{x}}_{\mathbf{w_1}}(kT) + \mathbf{B_{w_1}} \underline{\mathbf{x}}(kT) \\
\underline{\mathbf{x}}_{\mathbf{w_2}}(kT+T) &= \mathbf{A_{w_2}} \underline{\mathbf{x}}_{\mathbf{w_2}}(kT) \\
&\quad + \mathbf{B_{w_2}} \left(\underline{\hat{\mathbf{u}}}_\mathbf{c}(kT) + \mathbf{H_c} \underline{\tilde{\mathbf{u}}}_\mathbf{c}(kT) \right) \\
\underline{\mathbf{z}}_{\mathbf{w_1}}(kT) &= \mathbf{C_{w_1}} \underline{\mathbf{x}}_{\mathbf{w_1}}(kT) + \mathbf{D_{w_1}} \underline{\mathbf{x}}(kT) \\
\underline{\mathbf{z}}_{\mathbf{w_2}}(kT) &= \mathbf{C_{w_2}} \underline{\mathbf{x}}_{\mathbf{w_2}}(kT) \\
&\quad + \mathbf{D_{w_2}} \left(\underline{\hat{\mathbf{u}}}_\mathbf{c}(kT) + \mathbf{H_c} \underline{\tilde{\mathbf{u}}}_\mathbf{c}(kT) \right) \\
\underline{\mathbf{y}}_\mathbf{pp}(kT) &= \mathbf{H_c} \underline{\tilde{\mathbf{u}}}_\mathbf{c}(kT) \\
\underline{\mathbf{u}}_\mathbf{pp}(kT) &= \Theta \underline{\mathbf{d}}_\mathbf{p}(kT) - \phi_b \underline{\mathbf{x}}(kT) + \xi_b \left(\underline{\hat{\mathbf{u}}}_\mathbf{c}(kT) + \mathcal{H} \right)
\end{aligned}$$

with $\mathbf{W}_1 := (\mathbf{A_{w_1}}, \mathbf{B_{w_1}}, \mathbf{C_{w_1}}, \mathbf{D_{w_1}})$, $\mathbf{W}_2 := (\mathbf{A_{w_2}}, \mathbf{B_{w_2}}, \mathbf{C_{w_2}}, \mathbf{D_{w_2}})$ and

$$\begin{aligned}
\Theta &= \mathbf{B_1} - \mathbf{B_1} \mathbf{H_p} (I + \mathbf{D_{11}} \mathbf{H_p})^{-1} \mathbf{D_{11}}, \\
\phi &= \mathbf{B_1} \mathbf{H_p} (I + \mathbf{D_{11}} \mathbf{H_p})^{-1} \mathbf{C_1}, \\
\xi &= \mathbf{B_1} \mathbf{H_p} (I + \mathbf{D_{11}} \mathbf{H_p})^{-1} \mathbf{D_{12}}
\end{aligned}$$

which both ξ and ϕ can be similarly reduced to

$$\xi = \underbrace{\underset{i=1,\ldots,N}{\mathbf{diag}} \left\{ \underset{j=1,\ldots,N_i}{\mathbf{diag}} \xi^{ii}(j,j) \right\}}_{\xi_a} + \underbrace{(\xi - \xi_a)}_{\xi_b} \tag{11}$$

and where $\xi^{ii}(j,j)$ is described as the (j,j)-th component in ξ^{ii}. Lifted signals $\underline{\mathbf{x}}_{\mathbf{w_1}}(k)$ and $\underline{\mathbf{x}}_{\mathbf{w_2}}(k)$ represent the states that are associated with weighting function $\mathbf{W}_1$ and $\mathbf{W}_2$ respectively. Lifted $\underline{\mathbf{z}}_{\mathbf{w_1}}(k)$ and $\underline{\mathbf{z}}_{\mathbf{w_2}}(k)$ represents the measured process output and the controller output respectively. Furthermore, we define new variables $\underline{\mathbf{y}}_\mathbf{pp}(k)$ and $\underline{\mathbf{u}}_\mathbf{pp}(k)$ as new variables. Constant matrices Θ, ϕ and ξ are introduced to simplify the representation of the theorem. Matrix $\mathcal{H}$ vertical concatenate the information received by the i-th controller from all ℓ-th controllers $(\forall \ell = 1, \ldots, N)$. By assigning $\mathbf{x_m} = \underline{\mathbf{x}}_\mathbf{w}$, $p^i(kT + jT_i) = \left[y_{pp}^{i^T}(kT + jT_i),\ u_{pp}^{i^T}(kT + jT_i) \right]^T$ and $q^i(kT + jT_i) = \left[x^{i^T}(kT + jT_i),\ \bar{u}_c^{i^T}(kT + jT_i),\ \tilde{u}_c^{i^T}(kT + jT_i) \right]^T$ when $jT_i = \jmath\tilde{T}_i$, $\forall \jmath = 0, 1, \ldots, \tilde{\mathcal{N}}_i$, else $q^i(kT + jT_i) = \left[x^{i^T}(kT + jT_i),\ \bar{u}_c^{i^T}(kT + jT_i) \right]^T$ $\forall j = 0, 1, \ldots, N_i$ then (11) can be represented by in the form given in (6). The state-space representations of each $P_v^{i \cdot j}$ within T described by $\mathcal{P}^{i \cdot j}$ is as follows:

$$\left[\begin{array}{c|cc|cc}
\left(A^i - \phi^{ii}(j,j) \right) & (B_2^i - \xi^{ii}(j,j)) & I & (B_2^i - \xi^{ii}(j,j)) & \mathbf{0} \\
\hline
I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
 & & & & \\
\mathbf{0} & \vdots & \vdots & I & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \\
\hline
I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array} \right] \tag{12}$$

where $x_p^i = x^i$, $y^i = y_c^i$ and $u^i = \left[\hat{u}_c^{i^T} \ \tilde{u}_c^{i^T} \right]^T$. Note that when $jT_i \neq \jmath\tilde{T}_i$, the last column and third row of (12) are removed. Each $\mathcal{C}_v^{i \cdot j}$ is described as follows:

$$\mathcal{C}_v^{i \cdot j} : \begin{cases} \begin{bmatrix} \hat{u}_c^i(jT_i) \\ \tilde{u}_c^i(jT_i) \end{bmatrix} = \begin{bmatrix} \mathcal{K}_1^{i \cdot j} \mathcal{K}_2^{i \cdot j} \end{bmatrix} y_c^i(jT_i), & jT_i = \jmath\tilde{T}_i \\ \\ \hat{u}_c^i(jT_i) = \mathcal{K}_1^{i \cdot j} y_c^i(jT_i), & jT_i \neq \jmath\tilde{T}_i \end{cases} \tag{13}$$

for all $i = 1, \ldots, N$, $j = 0, 1, \ldots, \mathcal{N}_i$ and $\jmath = 0, 1, \ldots, \tilde{\mathcal{N}}_i$.

Proof. Due to space constraints, a simplified version of the proof is presented in this paper. By introducing weighted performance specifications 1 and 2, the overall model for the plant can be rewritten as follows:

$$\underline{\mathbf{x}}(kT+T) = (\mathbf{A}-\phi)\underline{\mathbf{x}}(kT) + (\mathbf{B_2}-\xi)\underline{\mathbf{u}}_\mathbf{c}(kT) + \Theta\underline{\mathbf{d}}_\mathbf{p}(kT)$$
$$\underline{\mathbf{x}}_{\mathbf{w_1}}(kT+T) = \mathbf{A_{w_1}}(kT)\mathbf{x_{w_1}}(kT) + \mathbf{B_{w_1}}(k)\mathbf{x}(kT)$$
$$\underline{\mathbf{x}}_{\mathbf{w_2}}(kT+T) = \mathbf{A_{w_2}}(kT)\mathbf{x_{w_2}}(kT) + \mathbf{B_{w_2}}(k)\mathbf{u_c}(kT)$$
$$\underline{\mathbf{z}}_{\mathbf{w_1}}(kT) = \mathbf{C_{w_1}}(kT)\mathbf{x_{w_1}}(kT) + \mathbf{D_{w_1}}\mathbf{x}(kT)$$
$$\underline{\mathbf{z}}_{\mathbf{w_2}}(kT) = \mathbf{C_{w_2}}(kT)\mathbf{x_{w_2}}(kT) + \mathbf{D_{w_2}}\mathbf{u_c}(kT)$$
$$\underline{\mathbf{y}}_\mathbf{c}(kT) = \underline{\mathbf{x}}(kT) \tag{14}$$

where $\underline{\mathbf{z}}_{\mathbf{w_1}}$, $\underline{\mathbf{z}}_{\mathbf{w_2}}$, $\underline{\mathbf{x}}_{\mathbf{w_1}}$ and $\underline{\mathbf{x}}_{\mathbf{w_2}}$ are the measured process output, controller output and states of $\mathbf{W}_1$ and $\mathbf{W}_2$ respectively. By shifting $\underline{\mathbf{x}}_{\mathbf{w_1}}$, $\underline{\mathbf{x}}_{\mathbf{w_2}}$, $\underline{\mathbf{z}}_{\mathbf{w_1}}$ and $\underline{\mathbf{z}}_{\mathbf{w_2}}$ into system $\tilde{M}$, the rest of system in (14) together with the controller network can be place into system $\tilde{\mathcal{P}}$ if there exists connecting signals $\underline{\mathbf{p}}(kT)$ and $\underline{\mathbf{q}}(kT)$. Define

$$p^i(kT+jT_i) = \left[y_{pp}^{i^T}(kT+jT_i),\ u_{pp}^{i^T}(kT+jT_i)\right]^T \tag{15}$$

and $q^i(kT+jT_i)$ when $jT_i = \jmath\tilde{T}_i$ as

$$q^i(kT+jT_i) = \Big[x^{i^T}(kT+jT_i),\ \bar{u}_c^{i^T}(kT+jT_i),\ \tilde{u}_c^{i^T}(kT+jT_i)\Big]^T \tag{16}$$

and for $jT_i \neq \jmath\tilde{T}_i$ as

$$q^i(kT+jT_i) = \left[x^{i^T}(kT+jT_i),\ \bar{u}_c^{i^T}(kT+jT_i)\right]^T \tag{17}$$

$\forall i = 1,\ldots,N,\ j = 0,1,\ldots,\mathcal{N}_i$ and $\jmath = 0,1,\ldots,\tilde{\mathcal{N}_i}$. Let (15) - (17) be the augmented input and output of each $\tilde{\mathcal{P}}^{i\cdot j}$. The rest of system in (14)and the controller network representations in (5) can be rearranged to form each pair of $P_v^{i\cdot j}$ and $C_v^{i\cdot j}$ in (12) - (13). This completes the proof.

3.2 Plantwide connective stability design

The use of communication in a controller network makes it prone to issues such as data packet drop-outs. As such, it is important for the controller network to still maintain stability of the plant when the communication links between controller are lost. A controller network that possesses this feature leads to a connectively stable plant. Based on the concept of dissipativity, a plantwide connective stability condition is developed and incorporated into the design framework. Dissipativity is an effective tool for analyzing the stability of large -scale interconnected systems based on its input-output property.

Definition 1. (Byrnes and Lin (1994)). A dynamic system Σ is said to be dissipative if there exists a nonnegative function $\mathcal{V}:\mathbb{X}\to\mathbb{R}$ with $\mathcal{V}(0)=0$ called storage function, such that for all $u(k)\in\mathbb{U}$, $x(0)\in\mathbb{X}$ and $\tau\in\mathbb{Z}_+ := \{0,1,2,\ldots\}$,

$$\mathcal{V}(x(\tau+1)) - \mathcal{V}(x(0)) \leq \sum_{k=0}^{\tau} w(u(k),y(k)) \tag{18}$$

where $w(u(k),y(k)):\mathbb{U}\times\mathbb{Y}\to\mathbb{R}$ is called the supply rate.

In this paper, a quadratic supply rate of

$$w(u(k),y(k)) = y(k)^TQy(k) + 2y(k)^TSu(k) + u(k)^TRu(k). \tag{19}$$

is used, where $Q = Q^T$, S, $R = R^T$ are constant matrices of appropriate dimensions. Systems that are dissipative with respect to the above supply rate are said to be (Q,S,R)-dissipative. For linear systems, the (Q,S,R)-dissipativity condition can be represented using Linear Matrix Inequalities (LMIs) as shown in Tan et al. (1999). Using the dissipativity of system $\tilde{M}$ and $\tilde{\mathcal{P}}$, the connective stability of the plantwide process system is given in following theorem:

Theorem 2. Suppose a stable system $\tilde{\mathcal{P}}$ is $(\mathbf{Q},\mathbf{S},\mathbf{R})$-dissipative with $\mathbf{Q} < 0$ and $\mathbf{R} > 0$. Then system $\tilde{M}$-$\tilde{\mathcal{P}}$ shown in Fig. 1 is plantwide connectively stable if $\tilde{M}$ is $(\mathbf{diag}(-\gamma^{-2}I,-\mathbf{R}),\ \mathbf{diag}(0,-\mathbf{S}^T),\ \mathbf{diag}(I,-\mathbf{Q}))$-dissipative where $\mathbf{Q} = \underset{i=1,\ldots,N}{\mathbf{diag}}\left\{\underset{j=0,\ldots,\mathcal{N}_i}{\mathbf{diag}} Q^{i\cdot j}\right\}$ with $\mathbf{R}$ and $\mathbf{S}$ having similar structure.

Due to space constraints, the proof to Theorem 2 is omitted in this paper. From Theorem 1 any existing network of processes and controllers with asynchronous communication and control can be represented by a $\tilde{M}$-$\tilde{\mathcal{P}}$ system. With this representation and Theorem 2, the existence of each state feedback controller within the controller network that ensures connective stability of plantwide process is formulated into the following stabilizability problem.

Theorem 3. Given a network of N processes and N controllers with asynchronous control and communication that is represented by system $\tilde{M}$-$\tilde{\mathcal{P}}$ (as shown in Theorem 1). Then there exists controller gains, $\mathcal{K}_1^{i\cdot j}$ and $\mathcal{K}_2^{i\cdot \jmath}$ ($\forall i = 1,\ldots,N,\ j = 0,1,\ldots,\mathcal{N}_i$, and $\jmath = 0,1,\ldots,\tilde{\mathcal{N}_i}$) that ensure

1: the plantwide connective stability and the system from $\underline{\mathbf{d}}_\mathbf{p}$ to $\underline{\mathbf{z}}$ at each time period T to have an H_∞ norm less than γ,
2: the local stability of each $\tilde{\mathcal{P}}^{i\cdot j}$ at each j-th time instant within T,

if symmetric matrices $\tilde{X}^m > 0$, $\tilde{X}^{i\cdot j} > 0$, $\tilde{Q}^{i\cdot j} < 0$, $\tilde{R}^{i\cdot j} > 0$ and $\tilde{S}^{i\cdot j}$ with matrices $\tilde{\mathbf{S}}$, $\tilde{\mathbf{R}}$ and $\tilde{\mathbf{Q}}$ represented as $\tilde{\mathbf{Q}} = \underset{i=1,\ldots,N}{\mathbf{diag}}\left\{\underset{j=1,\ldots,\mathcal{N}_i}{\mathbf{diag}} \tilde{Q}^{i\cdot j}\right\}$ and $\begin{bmatrix}\mathcal{N}_{Bu}^{i\cdot j}\\ \mathcal{N}_{Dqu}^{i\cdot j}\end{bmatrix} = \mathbf{Ker}\left(\begin{bmatrix}B_u^{i\cdot j^T} & D_{qu}^{i\cdot j^T}\end{bmatrix}\right)$ can be found such that

$$\begin{bmatrix} M^T \\ I \end{bmatrix}^T \left[\begin{array}{ccc|ccc} \tilde{X}^m & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\mathbf{R}} & 0 & 0 & \tilde{\mathbf{S}}^T \\ \hline 0 & 0 & 0 & -\tilde{X}^m & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & \tilde{\mathbf{S}} & 0 & 0 & \tilde{\mathbf{Q}} \end{array}\right] \begin{bmatrix} M^T \\ I \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix}\mathcal{N}_{Bu}^{i\cdot j}\\ \mathcal{N}_{Dqu}^{i\cdot j}\end{bmatrix}^T \begin{bmatrix} A_p^{i\cdot j}\tilde{X}^{i\cdot j}A_p^{i\cdot j^T} - \tilde{X}^{i\cdot j} - B_p^{i\cdot j}\tilde{Q}^{i\cdot j}B_p^{i\cdot j^T} & A_p^{i\cdot j}\tilde{X}^{i\cdot j}C_q^{i\cdot j^T} - B_p^{i\cdot j}\left(\tilde{S}^{i\cdot j} + \tilde{Q}^{i\cdot j}D_{qp}^{i\cdot j^T}\right) \\ C_q^{i\cdot j}\tilde{X}^{i\cdot j}A_p^{i\cdot j^T} - \left(\tilde{S}_i^{i\cdot j^T} + D_{qp}^{i\cdot j}\tilde{Q}^{i\cdot j}\right)B_p^{i\cdot j^T} & \left(C_q^{i\cdot j}\tilde{X}^{i\cdot j}C_q^{i\cdot j^T} - \tilde{R}^{i\cdot j} - \tilde{S}_i^{i\cdot jT}D_{qp}^{i\cdot j^T} - D_{qp}^{i\cdot j}\tilde{S}^{i\cdot j} - D_{qp}^{i\cdot j}\tilde{Q}^{i\cdot j}D_{qp}^{i\cdot j^T}\right) \end{bmatrix} \times \begin{bmatrix}\mathcal{N}_{Bu}^{i\cdot j}\\ \mathcal{N}_{Dqu}^{i\cdot j}\end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} -\tilde{X}^{i \cdot j} - B_p^{i \cdot j}\tilde{Q}^{i \cdot j}B_p^{i \cdot j^T} \\ -D_{qp}^{i \cdot j}\tilde{Q}^{i \cdot j}B_p^{i \cdot j^T} - \tilde{S}^{i \cdot j^T}B_p^{i \cdot j^T} \end{bmatrix.$$
$$\left. \begin{matrix} -B_p^{i \cdot j}\tilde{Q}^{i \cdot j}D_{qp}^{i \cdot j^T} - B_p^{i \cdot j}\tilde{S}^{i \cdot j} \\ -\tilde{R}^{i \cdot j} - D_{qp}^{i \cdot j}\tilde{S}^{i \cdot j} - \tilde{S}^{i \cdot j^T}D_{qp}^{i \cdot j^T} - D_{qp}^{i \cdot j}\tilde{Q}^{i \cdot j}D_{qp}^{i \cdot j^T} \end{matrix} \right] < 0 \tag{22}$$

for all $i = 1, \ldots, N$, $j = 1, \ldots, \mathcal{N}_i$.

Due to space constraints, the proof is omitted.

Control design. If the conditions in Theorem 3 are satisfied, then there exist state feedback controllers that ensure the connective stability and specified control performance. These controllers can then be designed individually and independently: At each jT_i time instant within T, $\mathcal{P}^{i \cdot j}$ in (7) can be rewritten as

$$\mathcal{P}^{i \cdot j} = \begin{bmatrix} A_p^{i \cdot j} + B_u^{i \cdot j}\mathbf{K}_p^{i \cdot j} & B_p^{i \cdot j} \\ C_q^{i \cdot j} + D_{qu}^{i \cdot j}\mathbf{K}_p^{i \cdot j} & D_{qp}^{i \cdot j} \end{bmatrix} \tag{23}$$

where for all $jT_i = \jmath\tilde{T}_i$

$$\mathbf{K}_p^{i \cdot j} = \begin{bmatrix} \mathcal{K}_1^{i \cdot j} \\ \hline \mathcal{K}_2^{i\ell_1 \cdot \jmath} \\ \vdots \\ \mathcal{K}_2^{i\ell_2 \cdot \jmath} \end{bmatrix}_{\ell_1, \ell_2 \neq i}$$

otherwise $\mathbf{K}_p^{i \cdot j} = \mathcal{K}_1^{i \cdot j}$, $\forall i, \ell_1, \ell_2 = 1, \ldots, N$, $j = 0, 1, \ldots, \mathcal{N}_i$ and $\jmath = 0, 1, \ldots, \tilde{\mathcal{N}}_i$. Matrices $Q^{i \cdot j}$, $S^{i \cdot j}$, $R^{i \cdot j}$ are obtained by observing

$$\begin{bmatrix} Q^{i \cdot j} & S^{i \cdot j} \\ S^{i \cdot j^T} & R^{i \cdot j} \end{bmatrix} = \begin{bmatrix} -\tilde{R}^{i \cdot j} & \tilde{S}^{i \cdot j^T} \\ \tilde{S}^{i \cdot j} & -\tilde{Q}^{i \cdot j} \end{bmatrix}^{-1} \tag{24}$$

and $\tilde{X}^{i \cdot j} = (X)^{i \cdot j^{-1}}$. With these matrices, the controllers ($\mathbf{K}_p^{i \cdot j}, \forall i = 1, \ldots, N$ $j = 0, 1, \ldots, \mathcal{N}_i$) are designed by solving by the (Q,S,R)-dissipativity LMI (described in Tan et al. (1999)) where matrices $A_{c\ell}^{i \cdot j}$ and $C_{c\ell}^{i \cdot j}$ are replaced by matrices $(A_p^{i \cdot j} + B_u^{i \cdot j}\mathbf{K}_p^{i \cdot j})$ and $(C_q^{i \cdot j} + D_{qu}^{i \cdot j}\mathbf{K}_p^{i \cdot j})$ respectively.

The design of a controller network with asynchronous control and communication is summarized in the following procedure:

Procedure 1.

(1) For any given plantwide process, develop the matrices $\mathcal{H}_p^{i\ell}$ based on the process flow diagram and generate matrices $\mathcal{H}_c^{i\ell}$ to describe the desired control network topology.
(2) Determine plantwide system period T from T_i and $\tilde{T}_i$, for all $i = 1, \ldots, N$. Construct both matrices $H_p^{i\ell}$ and $H_c^{i\ell}$ using (4) and (5).
(3) Using models of both process unit P^i and controllers $\mathcal{C}^i$ and matrices $H_p^{i\ell}$ and $H_c^{i\ell}$, the process and controller network with asynchronous control and communication is converted into system $\tilde{M}$-$\tilde{\mathcal{P}}$ (as shown in Theorem 1). This step also allows performance specifications in P 1 and P 2 to be incorporated into system $\tilde{M}$-$\tilde{\mathcal{P}}$.
(4) Proceed with Theorem 2 to establish the existence of each state feedback controller at different jT_i time instants within T. If no solutions are found, go back to Step 1 and redesign the control network topology.
(5) Obtain Q^i, S^i, R^i through the relationship shown in Equation (24) and $\tilde{X}^{i \cdot j} = (X)^{i \cdot j^{-1}}$.
(6) Each controllers are designed independently by solving the dissipativity LMI in Tan et al. (1999).

The above framework can be extended to output feedback control. This extension is particulary useful in process control where full state information is usually not available. Similar steps in the above procedure can be undertaken to design each output feedback controller.

A case study has been conducted using the proposed approach. The results, while cannot be presented here due to space limitation, have shown the proposal approach effective.

4. CONCLUSION

A new discrete-time networked process control approach is presented. In this framework, plantwide process with asynchronous control and communication is described by a process and controller network. Due to the asynchronous nature of the network, lifted process and control models are used to represent the dynamics of the plant at each time instant within a plantwide-system period, T. The design of each controller is implemented in two steps. A plantwide connective stabilizability condition and the conditions that ensure plantwide performance are formulated in the form of LMIs. The solutions to these LMIs translate the performance and stability specifications into constraints for which controllers at each time instant within T has to satisfy. The controllers are then independently designed. This approach leads to a more scalable, autonomous and fault tolerant plantwide control strategy.

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