Closed Loop System Identification Using Virtual Control Approach

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Abstract: The identification of closed loop systems has played an important role in the current context, since it reduces the operational costs of the identification process in the testing stage, reducing, for instance, off-spec production. However, in order to obtain the models, special care for treating the data is required. In this work, is presented a study on identification of linear models from closed loops operational data, based on the application of a virtual closed loop in the real loop. It consists of the virtual addition and removal of a controller to the analyzed loop, so as to filter the input of the system in a completely off-line procedure. This paper also propose modifications on this methodology resulting in an simplification of the virtual filter and in the ways to recover the open loop model.

Keywords: System identification, closed-loop identification, SISO systems, virtual filter.

1. INTRODUCTION

In the 70's the direct method for closed loop system identification was presented (Ljung et al., 1974) to avoid the usage of knowledge of the controller applied to the loop, treating the system as if it were operating in open loop, without taking any assumptions on the control law. As alternative, indirect methods were proposed based on the idea of finding the closed loop transfer function in first place, and with this estimative and the knowledge of the true controller unravel the open loop transfer function (Lindberger, 1972, 1973).

Later, in the 90's, other alternative to traditional indirect identification method, has been proposed, such as the two-stage approach (Van Den Hof and Schrama, 1993; Van den Hof, 1998), identification via fractional descriptions (Hansen et al., 1989), and identification of the normalized coprime factors (Van den Hof et al., 1995). All of these approaches assume that the controller is linear and some of these methods depend on the controller being perfectly known. A projection method (Forssell and Ljung, 2000) has also been proposed in order to allow closed loop identification for a nonlinear controller.

In this paper an alternative algorithm to identify a system operating in closed loop is studied. This algorithm employs the idea of virtual feedback proposed by Agüero (2005). Single input - single output (SISO) processes were used as example aiming to verify the feasibility of the application of this procedure and propose some simplifications.

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Fig. 1. Closed loop system.



The system considered in this study operates in a closed loop as illustrated in Fig. 1. The corresponding linear model is given by (1), where G_0 and H_0 are linear transfer functions, and e(t) is a sequence of independent random variables with zero mean value.

$$y(t) = G_0 u(t) + H_0 e(t)$$
(1)

3. VIRTUAL CLOSED LOOP METHOD

The virtual closed loop method defines an artificial closed loop which imposes linear constraint by the virtual addition and removal of a controller to the analyzed loop, so as to filter the inputs in a completely off line procedure. This methodology could be equivalent to the parametrization of the noise with a known linear constraint. This approach avoids the unbounded long term predictions in system identification for unstable or a marginally stable open loop process. This methodology can be seen as a technique which is in the middle way between the direct and the indirect methods.



Fig. 2. Virtual closed loop.3.1 The Original Configuration

The original proposal (Agüero, 2004a) requires the virtual filter $C_{virtual}$, defined in (2), with P and L as coprime polynomials, to be added an removed from the original loop without changing the relationship among the variables operating in the loop. To guarantee this issue, all the operations involving the true closed loop variables should be executed in an off line procedure, manipulating previous collected data.

$$C_{virtual} = \frac{P}{L} \tag{2}$$

The virtual closed loop, shown in the Fig. 2, is conceptually constructed introducing a stable filter E, related to P and L by N = E - L. In order to apply the virtual closed loop identification method is necessary to assume that $C_{virtual}$ is linear, perfectly known and if it is applied to the true system, a stable closed loop would be obtained. Note that there need be no relationship between $C_{virtual}$ and the true controller.

The closed loop to be analyzed for identification then is the one limited by the dashed line in Fig. 2. In this figure, we can see that the virtual controller has been added and subtracted so as it does not modify the true loop. All this operations are done off line, and we only use the data generated by the true closed loop in Fig. 2.

Virtual reference signal $\bar{u}(t)$ is taken from the relationship between the output y(t) and the input u(t) and the virtual filter parameters, as shown in (3).

$$\bar{u}(t) = \frac{L}{E} u(t) + \frac{P}{E} y(t) = u(t) - \frac{N}{E} u(t) + \frac{P}{E} y(t) \quad (3)$$

The virtual closed loop transfer function T_0 can be expressed in terms of the output signal y or the input u and the virtual reference signal \bar{u} as follows:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \frac{G_0 \frac{E}{L}}{1 + G_0 C_{virtual}} \\ \frac{E}{L} \\ \frac{E}{1 + G_0 C_{virtual}} \end{bmatrix} \bar{u}(t) = \begin{bmatrix} T_0^y \\ T_0^u \end{bmatrix} \bar{u}(t)$$
(4)

The next step in identification via virtual closed loop is finding a consistent estimate of T_0 in spite of the residual correlation that exists between the virtual reference signal \bar{u} and the process noise. As possible methods to be employed in this case are the Prediction Error Methods (*PEM*) (Ljung, 1999) and Subspace Methods (Ljung and McKelvey, 1996).

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3.2 Recovering the Open Loop Transfer Function

After estimating the virtual closed loop T is necessary then unraveling the open loop transfer function G by using its known relationship with the closed loop parameters in (4). Some of the alternatives for recovering G from T are described here, and were detailed by Agüero and Goodwin (2004) and Agüero (2004b).

Direct Calculation Using T^y or T^u . An estimation for G can be calculated from T^y or T^u given by:

$$G = \frac{T^y \frac{L}{E}}{1 - T^y \frac{P}{E}} \tag{5}$$

or

$$G = \frac{1 - T^u \frac{L}{E}}{T^u \frac{P}{E}} \tag{6}$$

Joint Approach. A second alternative to recover G is to use the estimates of both T^y and T^u via

$$G = \frac{T^y}{T^u} \tag{7}$$

Fractional Representation. We can introduce parameters in the system G as follows:

$$G_0 = \frac{U + \frac{L}{E} R_0}{V - \frac{P}{E} R_0} \tag{8}$$

where $G_{nom} \equiv \frac{U}{V}$ is any plant which is stabilized by the known controller $C_{virtual}$, where $U, V, \frac{P}{E} \in \frac{L}{E}$ satisfy the Bezout identity $V \frac{L}{E} + U \frac{P}{E} = 1$. Then, an estimate of T_0 is given by

$$[U-V] T = R_0 \tag{9}$$

where $T = [T^y \ T^u]^T$.

Solving a System of Equations. A final alternative for estimating G arises from the relationship between the parameters of $T^y(\Theta)$ and the open loop parameters θ , i.e $\Theta = M\theta - rho$. Once the nominal closed loop parameters Θ are obtained we can simply solve the system of equations by Least Squares to yield:

$$\theta = M[\Theta + \rho] \tag{10}$$

The squared matrix M and the vector ρ are obtained from the application of Bias Elimination Least Squares (BELS) method (Zheng and Feng, 1995) for estimating the closed loop model T. The obtainment of the equations and the usage of this approach are detailed by Agüero (2004b).

4. CHANGES IN VIRTUAL CLOSED LOOP METHOD

Aiming to reduce the complexity in practical application of the virtual closed loop method, two modifications were proposed. A simplification for application of the virtual controller approach, and a new strategy of recovering the closed loop transfer function.



Fig. 3. Virtual closed loop modified.

4.1 Virtual Controller

The virtual filter $C_{virtual}$ to be chosen has to be linear and stabilize the system without being related to the true controller. At the same time, a stable filter E associated to the application of the virtual controller to assure the quality of the filtered signal. In this work we propose the elimination of the polynomial E, using no specific parametrization for the virtual controller, only applying lead-lag filters. Figure 3 shows the new configuration of the virtual closed loop to be identified, which is mathematically represented by (11).

$$T_0^y = \frac{G_0}{1 + G_0 C_{virtual}}$$
(11)

4.2 Frequency Domain to Recover G

In the manipulation of the T^y estimated without any constraint there is no warranty that the obtained system will be attainable in a practical situation. Numerator order is likely to be greater than the order of the denominator, and/or the resulting system has a high order.

Assuming the identified model T^y has all the analyzed information about the system in the frequency domain, it is possible to use it to obtain the model G, using the basic idea of the frequency domain approach (Trierweiler et al., 2000), without restriction.

The developed strategy is based on minimizing the difference between the identified model T^y and the true closed loop model responses $\frac{G_0}{1+G_0C_{virtual}}$ weighted by a signal $\kappa(s)$ evaluated in a frequency vector w for a set of Ndiscrete points, as in (12).

$$\min_{\alpha \in \Re} \sum_{s=j\omega_1}^{s=j\omega_N} \left\| \left(T(s) - \frac{G_0(s)}{1 + G_0(s) \ C_{virtual}(s)} \right) \ \kappa(s) \right\|$$
(12)

Equation (13) generalizes the true open loop transfer function, where there are two vector containing the transfer function coefficients, one for the numerator β_n and another to the denominator α_d , as well, two vectors for the *s* terms, γ_n and γ_d . This generalization and assuming the signal $\kappa(s)$ as a step signal allow us to rewrite the objective function in (14).

$$G_0(s) = \frac{\gamma_n(s) \,\beta_n}{\gamma_d(s) \,\alpha_d} \tag{13}$$

$$\min_{\beta_n,\alpha_d} \left\| \left(T \, \gamma_d \, \alpha_d + T \, \gamma_n \, \beta_n \, C_{virtual} - \gamma_n \, \beta_n \right) \, \frac{1}{s} \right\| \qquad (14)$$

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The optimization procedure is based on an iterative least squares technique, and performs a simple optimization problem is written in (14). The main objective is to find the coefficient vectors β_n and α_d that minimize the difference between the true virtual closed loop and the identified one.

The procedure can be seen as a way to weigh the most important frequencies to the system. The virtual controller is calculated by a weighed least squares problem.

5. EXAMPLES

In this section, two examples are presented in order to show the feasibility of virtual closed loop in system identification by performing the proposed technique.

Two third order transfer functions where they both have same poles and different zeros, like in (15), were used.

$$G(s) = \frac{\beta s + 1}{(5s+1)(3s+1)(1s+1)}$$
(15)

The considered systems are simulated in closed loop with the same proportional-integral controller (16), whose parameters are $K_c = 1.1$ and $\tau_i = 8.8$.

$$\Delta U(s) = K_c \left\{ [Y_{set}(s) - Y(s)] + \frac{1}{\tau_I [Y_{set}(s) - Y(s)]} \right\} (16)$$

The first analyzed system is characterized by an inverse response, and the second by an overshoot response. These systems represent a very common set of models usually find in chemical industries.

Virtual closed loop method is applied to the examples in both configurations, original and changed. For the first one we choose to use the direct calculation (VCL CD) for recovering the model G from the virtual closed loop model. Our proposed model procedure based on the frequency domain approach will be designed by VCL FR.

Both examples simulations are performed in a noisefree environment. In Racoski (2009) master thesis other simulation scenarios, including noise and dither signals, were discussed.

5.1 Choosing the Virtual Filter

The original configuration of the virtual closed loop does not lead us to a perfect way of estimating the best filter to be applied. Intuitively, the first trial for obtaining a stable filter that stabilizes the system is a lead-lag compensator with constants identical to the dominant time constant of the system under discussion. Then, for virtual system identification procedure for all cases in example section we use the lag compensator given in (17).

$$C_{LL} = \frac{s + 0.2}{s + 0.02} \tag{17}$$

5.2 Third Order with Inverse Response

Consider the transfer function presented in (15), with $\beta = -3$, in closed loop with controller in (16) under the configuration shown in Fig. 1. Fig. 4 presents the system output y responding to the excitation signal w inserted. Data presented here are noisefree.



Fig. 4. Third order with inverse response. Excitation signal and system response under closed loop condition.



Fig. 5. Third order with inverse response. Step response in identified models.

Closed loop system identification is executed using MAT-LAB SYSID Toolbox. Figure 5 shows the step response for the identified models, performing the virtual closed loop method recovering the open loop model G in the frequency domain (*VCL FR*), the virtual closed loop using direct calculation for unravel G (*VCL CD*), and the standard direct method (*Direct*) in comparison with the true model.

Frequency domain comparison among the true model and the identified models using standard direct method, virtual closed loop in the original format (VCL CD) and frequency domain approach (VCL FR) are shown in Fig. 6. Clearly the VCL FR model is closer to the true model than the others, in the frequency interest range. Integrated Square Error (ISE) results for the models related to the true model are shown in table 1, confirming the quality of VCL FR identified model in time domain.

5.3 Third order with overshoot

In this example, we use the transfer function (15) with $\beta = 10$. The operation conditions are equal to earlier example. The virtual filter used is (17).

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Fig. 6. Third order with inverse response. Frequency response comparison.



Fig. 7. Third order with overshoot. Excitation signal and system response under closed loop condition.

Sampled data used in the system identification procedures is shown in Fig. 7. Figure 8 shows the step response for the true third order model with overshoot and the correspondent identified models. Their frequency domain response can be seen in Fig. 9.

All the identified models are very close. Step response for true and identified models illustrated in Fig. 8 reveal offset in model final value and mismatch in system dynamics for all models. Although there are differences in dynamics, the identified models are very close in time domain. The results of ISE in table 1 confirm this similarity.

The most important evidence of the quality of the VCL FR is not only the time domain response but also the frequency domain response, for both examples showed here.

Table 1. ISE for identified models.

Example	VCL CD	VCL FR	Direct
inverse response	0.038	0.013	0.187
overshoot	0.238	0.208	0.202



Fig. 8. Third order with overshoot. Step response in identified models.



Fig. 9. Third order with overshoot. Frequency response comparison.

6. CONCLUSION

The feasibility of using virtual filters in closed loop system identification has been discussed in this paper. The presented method for closed loop identification is performed using two third order transfer function as example. The original virtual closed loop methodology and its changed form were applied and compared with the standard direct methodology for closed loop system identification.

This work shows the application of virtual closed loop approach in simple cases. Virtual filters employed in identification do not necessarily have to be related to the true controller. In this case, the true controller parameters could even be unknown. Moreover, utilizing virtual loops in association with frequency domain approach prevents numerical problems in identification procedure and ensures realistic models for practical application.

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