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A Simple Identification Technique for Second-Order plus Time-Delay Systems

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Abstract: Second-Order plus Time-Delay (SOPTD) models are commonly used to approximate systems in order to tune PID controllers. Simple models are dominating for control design in industrial applications. Several estimation techniques have been developed and applied to controller design. Most estimation techniques are based on least-squares and excitations such as step, square waves and pseudo-random signals. In order to speed up the experiments, simple and short excitations have also been considered. In this paper alternatives are proposed to robustify the estimates in order to obtain a better model around important frequencies. Simulation results are presented to illustrate the techniques.

Keywords: SOPTD models; parameter estimation; system identification; relay excitation; least-squares.

1. INTRODUCTION

Estimation of First-Order-Plus-Time-Delay (FOPTD) models and Second-Order-Plus-Time-Delay (SOPTD) models have received great attention due to the widespread use of such models in techniques for tuning PID Controllers (Junior et al. (2009)). Identification of dynamic transfer function models from experimental data is essential for model based controller design. Often derivation of rigorous models is difficult due to the complex nature of chemical processes. Hence, system identification is a valuable tool to identify low order models, based on input/output data, for controller design (Ramakrishnan and Chidambaram (2003)).

Simple process models with at most three parameters (Gain K, Time Constant T and Time-Delay L) are dominating for control design in industrial applications (Ljung (2002)). But FOPTD models are limited when the process has under damped behavior or high order dinamics. This way, SOPTD models can better describe these kind of process. Certain higher order models when approximated to a FOPTD model give negative time constant hence second order model is necessary (Ramakrishnan and Chidambaram (2003)).

It should be stressed that the true system may not have the FOPTD or SOPTD structure. This means that a model reduction is performed and the estimated model only captures some aspects of the true system. Time-domain techniques focus on the time response and lack information on the model quality to account for stability and stability margins. Relay techniques match the model to the true system on the high frequency range (the ultimate frequency), so that it may result in a model that is not accurate in important low frequency ranges such as in the vicinity of the cutoff frequency. Simple model reduction techniques use the slowest dynamics to represent the time constant and makes the time delay as the sum of the faster dynamics (Åström and Hägglund (2006)). An interesting model reduction technique is the half-rule proposed in (Skogestad (2003)) which tries to distribute the mid-range dynamics between the the time constant and the time-delay.

There are a few methods to estimate parameters for these models. Among them, one can mention the graphics and the area methods (Åström and Hägglund (1995)). The A-locus method witch uses the exact solutions for the limit cycle frequency and amplitude from the relay feedback system response to estimate the FOPTD and SOPTD models parameters (Kaya and Atherton (2001)). In Majhi (2007), the FOPTD and SOPTD models parameters are determined solving at the most two non-linear equations derived from the symmetrical limit cycle response of the relay feedback experiment.

It is important that the identification experiment does not take the process to a significantly different steady-state operating point (de la Barra and Mossberg (2007)). In this way, pulse testing would be preferred to step and ramp testing. Attending mainly to the "minimal perturbation" requirement, in Hwang and Lai (2004) it is presented a two-stage algorithm to identity continuous-time delay systems with nonzero initial conditions. In Wang et al. (2005) the integration intervals proposed by Hwang and Lai (2004) are manipulated to simplify the regression equations. In both methods, simple time-domain excitations are used to set up identification experiments which disturb process operation as little as possible. More recently, in de la Barra et al. (2008) finite-duration pulses inputs were used to identify FOPTD systems. Exact analytical expressions for the Gain K, Time Constant T and Time-Delay L were obtained from knowledge of two relative

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extrema in the transient pulse response. And de la Barra and Mossberg (2007) proposed a method to identify the parameters of a second-order model without delay based on finite duration pulse response.

The relay feedback experiment have proved to be very useful for system identification. It is possible to build low order models in a relatively simples and fast way. In Åström and Hägglund (1984) the relay feedback test is used to generate sustained oscillations of the controlled variable and to get the ultimate gain (K_u) and ultimate frequency (ω_{180}) directly from the experiment. Based on these values, a PID controller can be tuned. Since only K_u and ω_{180} are available, additional information is required to calculate the three FOPTD model parameters or the four SOPTD model parameters. In Åström and Hägglund (2006), analytical expressions for estimating FOPTD model parameters from gain ratio (κ), which is the ratio of the system at the zero frequency and the ultimate frequency, is presented.

In Srinivasan and Chidambaram (2003) a modified asymmetrical relay feedback method was proposed to improve estimates of the FOPTD model. Using a single asymmetrical relay, additional equations to evaluate all the FOPTD model parameters are formulated. The asymmetric relay method requires an extra parameter (γ, the displacement in the relay height) and whose value is to be selected appropriately so that the calculation of the process gain and the estimate of ω_{180} are carried out accurately (Shen et al. (1996)). In Ramakrishnan and Chidambaram (2003), a method to identify the SOPTD model parameters based on the asymmetrical relay feedback experiment is proposed. The idea is to combine the FOPTD identification methods to estimate the four parameters of the SOPTD model.

Recently, a combined time/frequency domain FOPTD identification technique based on a single excitation was proposed in Junior et al. (2009). It was derived from a time and a frequency domain methods which were also proposed in Junior et al. (2009). The main feature of the combined identification technique is to recover a model which matches the time response while capturing the true system dynamics around frequencies of interest. In this paper, it is proposed an extension of that technique for SOPTD models using a simple excitation. The existing techniques are also revised. Several simulation results are given to evaluate the techniques.

2. PROBLEM STATEMENT

Consider a SOPTD model characterized by

$$G(s) = \frac{K}{(1+T_1 s)(1+T_2 s)} e^{-Ls} = \frac{K}{as^2+bs+1} e^{-Ls} \quad (1)$$

where $K \neq 0$ is the system gain, $L \geq 0$ is the time-delay, $T_1 > 0$ and $T_2 > 0$ are the time constants and a and b are the corresponding polinomial coeficients. The problem addressed in this paper deals with the estimation of the four parameters that characterize the model in Eq. (1). The aim is to obtain a SOPTD model using a unique excitation and that captures important true system characteristics both in time and in frequency domains.

3. TIME-DOMAIN IDENTIFICATION TECHNIQUES

In this Section pulse based identification techniques (time-domain) are revised.

Simple time-domain excitations are the step (the most common simple excitation), the rectangular pulse and its variants, the double rectangular pulse and the doublet pulse (Åström and Hägglund (2006) and de la Barra et al. (2008)).

Step Signal In many applications step responses are used to estimate models. See Wang and Zhang (2001).The advantages are that it is easy to generate, it gives a robust estimation of the gain and the user can make a graphical evaluation of the delay and time-constant. The limitations are that it may take too long for slow systems and plants with several loops. It may also be sensitive to disturbances which may appear during the step duration.

Rectangular Pulses A rectangular pulse input can be expressed as the difference of two step which are delayed for the pulse duration D, i.e.

$$u_P(t) = A\,[1(t) - 1(t-D)] \quad (2)$$

where $A \neq 0$ is the amplitude, D is the duration and $1(\tau)$ (unit step applied at time $\tau = 0$).

A double rectangular pulse can be used to extract more information as presented in de la Barra and Mossberg (2007) and in de la Barra et al. (2008) for FOPTD and second-order models. In Åström and Hägglund (2006) a doublet pulse, proposed in Shinskey (1996) is employed to estimate the model. In a similar way to the rectangular pulse it can be thought to be formed by a combination of delayed steps and similar equations can be obtained for the FOPTD response.

The advantages of the rectangular pulses over the step signal is that its duration is shorter and is less sensitive to disturbances. The main disadvantage is that the signal excitation is small to apply standard estimation techniques.

4. FREQUENCY-DOMAIN IDENTIFICATION TECHNIQUES

In this Section the identification technique based on the relay experiment presented in Åström and Hägglund (2006) (frequency-domain) is revised.

Ziegler/Nichols frequency domain design techniques use the process information around the ultimate frequency for which the phase is 180^o (ω_{180}), usually generated by a relay. Although a FOPTD or SOPTD model is not needed to design the controller, it would be interesting to obtain a model from such excitation to be used for simulation and controller design. For a pure FOPTD model, the knowledge of a frequency point and the gain uniquely caracterizes the transfer function. In Åström and Hägglund (2006), techniques for FOPTD models are presented which uses a relay experiment plus an estimate of the gain at the zero frequency. Notice that another excitation should be used to obtain the system gain, such as an asymmetrical relay. For the SOPTD models, the relay experiment is combined with the step response to identify the four parameters solving iteratively the equations.

In Ramakrishnan and Chidambaram (2003), the asymmetrical relay feedback experiment is used and a method to identify the SOPTD model parameters is proposed. An extra parameter has to be chosen, the asymmetry of the relay. As the model gain is determined based on the DC output level, three non-linear equations are solved for the three remain parameters.

5. A SIMPLE IDENTIFICATION TECHNIQUE

In order to get a better model around lower frequencies a different excitation is used. In this section a frequency domain method is proposed. The main idea is to estimate the model using an excitation that has contents at a couple of frequencies.

5.1 The Excitation

Consider the excitation shown in Figure (1). The excitation is assumed to be generated by a single relay from which the critical time T is obtained. The relay is applied for $(N_1+0.5)T$. This part characterizes the high frequency part of the excitation. After a short interval N_2T with zero output a rectangular pulse of width $N_3T/2$ is applied followed by another interval $N_3T/2$ with zero output. This characterizes the low frequency part of the excitation.

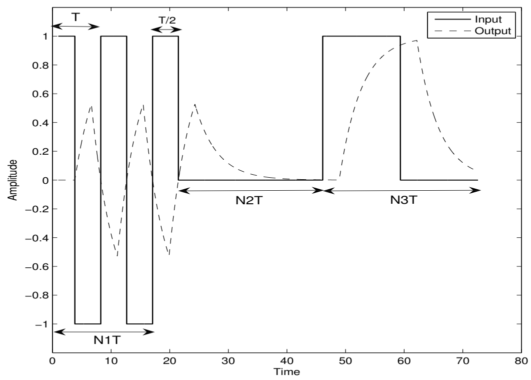

Fig. 1. Proposed excitation - SOPTD response

The excitation can be used to match the model at the desired frequencies related to the critical frequency ($\frac{\pi}{T}$). This way, the identified model better match the true system in a wider frequency region than the one based only in the critical frequency.

It is necessary to determine the parameters N_1, N_2 and N_3. The parameter N_1 is the number of periods of the relay. Choosing N_1 small makes the excitation shorter but the power contribution at the high frequency point is also small. This results in a poor model parameters estimations at this point. Increasing N_1 makes the excitation longer but also increase the power contribution. This results in a better model parameters estimations at this point. So, there is a tradeoff between excitation duration and quality of the model parameters estimations. The parameter N_2 is the interval between the two parts, it is suggested to choose a value that the output almost return to the operation point before the experiment. The parameter N_3 is the pulse period. This parameter is a multiple of the critical period T, that is, the period of the first part of the excitation. It is suggested to choose $N_3 = 3$, so the low frequency point is in the frequency range of interest for control purposes. This frequency range is where the process fase is between 90° and 180°.

5.2 The Proposed Technique

The proposed technique uses more than one frequency point to match the identified SOPTD model in a wider frequency region of the true system. This is possible because the excitation excites the system in a range of frequency points. The data is collected for a time duration t_b long enough for the signal to return to the initial state.

Proposition 1. Consider a SOPTD model $G(s)$. Define $|G(j\omega_i)|$ and $\phi(\omega_i)$ as the system gain and phase at ω_i, respectively. Assume that $G(0)$ and $G(j\omega_i)$ are estimated. Define the relative gain at the frequency ω_i

$$\kappa(\omega_i) = \frac{|G(j\omega_i)|}{|G(0)|}. \tag{3}$$

Then, the SOPTD parameters can be computed as

$$\hat{a} = \sqrt{\alpha_1} \tag{4}$$

$$\hat{b} = \sqrt{\alpha_2 + 2\sqrt{\alpha_1}} \tag{5}$$

$$\hat{L}(\omega_i) = -\frac{1}{\omega_i}\left[\phi(\omega_i) + \arctan\left(\frac{\hat{b}\omega}{1-\hat{a}\omega^2}\right)\right] \tag{6}$$

$$\hat{K} = G(0) \tag{7}$$

where $\alpha_1 = a^2$ e $\alpha_2 = b^2 - 2a$ the solutions of the system equation given by

$$\begin{cases} \omega_1^4\alpha_1 + \omega_1^2\alpha_2 = \frac{1-\kappa^2(\omega_1)}{\kappa^2(\omega_1)} \\ \omega_2^4\alpha_1 + \omega_2^2\alpha_2 = \frac{1-\kappa^2(\omega_2)}{\kappa^2(\omega_2)} \end{cases} \tag{8}$$

It is importante to point out that this technique can be applied to all the kinds of SOPDT models classified based on the damping factor, say critically, under and over damped. There is no restriction on the estimation values of the polinomial coeficients a and b.

Proof. Define the relative gain κ as

$$\kappa(\omega_i) = \frac{|G(j\omega_i)|}{G(0)}. \tag{9}$$

The system gain is

$$|G(j\omega)| = \frac{K}{\sqrt{(1-a\omega^2)^2 + (b\omega)^2}}. \tag{10}$$

Dividing by $|G(0)|$ the Eq. (10) and taking the square, we have

$$\frac{|G(j\omega)|^2}{|G(0)|^2} = \frac{1}{(1-a\omega^2)^2 + (b\omega)^2} = \kappa^2(\omega). \tag{11}$$

Separating the terms

$$\kappa^2(\omega)((1-a\omega^2)^2+(b\omega)^2)=1 \quad (12)$$

$$\kappa^2(\omega)(1-2a\omega^2+a^2\omega^4+(b\omega)^2))=1 \quad (13)$$

$$a^2\omega^4+(b^2-2a)\omega^2+1=\frac{1}{\kappa^2(\omega)} \quad (14)$$

$$a^2\omega^4+(b^2-2a)\omega^2=\frac{1-\kappa^2(\omega)}{\kappa^2(\omega)}. \quad (15)$$

Using Eq. (15) and two frequency points information, we can do

$$\omega_1^4\alpha_1+\omega_1^2\alpha_2=\frac{1-\kappa^2(\omega_1)}{\kappa^2(\omega_1)} \quad (16)$$

$$\omega_2^4\alpha_1+\omega_2^2\alpha_2=\frac{1-\kappa^2(\omega_2)}{\kappa^2(\omega_2)} \quad (17)$$

where $\alpha_1 = a^2$ e $\alpha_2 = b^2 - 2a$. Solving this system of equations it is possible to find α_1 and α_2. Now write the equation for the system fase defined as

$$\phi(\omega) = -\omega L - \arctan\left(\frac{b\omega}{1-a\omega^2}\right). \quad (18)$$

Based on Eq. (18), the two frequency points, the system fase at this two points and the polinomial coeficientes of the system SOPDT model, the time delay is obtained by

$$\hat{L}(\omega_i) = -\frac{1}{\omega_i}\left[\phi(\omega_i)+\arctan\left(\frac{\hat{b}\omega}{1-\hat{a}\omega^2}\right)\right]. \quad (19)$$

The system gain can also be computed as the ratio between the integral of the deviations of the output and input given by

$$K=\frac{\int_0^{t_b} y(t)}{\int_0^{t_b} u(t)} \quad (20)$$

but this also an alternative because the system gain is estimated using Eq. (7).

6. SIMULATION EXAMPLES

In this section the simple identification techniques are applied to three systems listed in Table (1).

Table 1. Simulated Processes

System 1	$G_1(s) = \frac{1}{(10s+1)(s+1)}e^{-2s} = \frac{1}{10s^2+11s+1}e^{-2s}$
System 2	$G_2(s) = \frac{1}{(0.5s+1)^3}e^{-4s} = \frac{1}{0.125s^3+0.75s^2+1.5s+1}e^{-4s}$
System 3	$G_3(s) = \frac{1}{(2s-1)(0.5s+1)}e^{-0.5s} = \frac{1}{s^2+1.5s+1}e^{-0.5s}$

The cost function used to compare the estimates in time domain is

$$\varepsilon = \frac{1}{N}\sum_{k=0}^{N-1}\left[y\left(kT_s\right)-\hat{y}\left(kT_s\right)\right]^2$$

where $y\left(kT_s\right)$ and $\hat{y}\left(kT_s\right)$ is the actual and estimated process output, respectively. In frequency domain, the cost function used to compare the estimates is

$$E=\frac{1}{N_\omega}\sum_{k=0}^{N_\omega-1}\left|G(j\omega_k)-\hat{G}(j\omega_k)\right|$$

where N_ω is the number of frequency points, $G(j\omega_k)$ is the true system frequency response and $\hat{G}(j\omega_k)$ is the identified model frequency response.

6.1 System 1

The first system is a pure SOPTD model. The transfer function estimates are shown in Table (2).

Table 2. Identification results for system 1

True system	$G_1(s) = \frac{1}{10s^2+11s+1}e^{-2s}$
Chindambaram2003	$G_c(s) = \frac{1.05}{12.412s^2+11.037s+1}e^{-1.814s}$
Frequency method	$G_f(s) = \frac{0.9968}{10.64s^2+10.75s+1}e^{-2.065s}$

Applied the excitation, the system input and output used in the estimation are shown in Fig. (2).

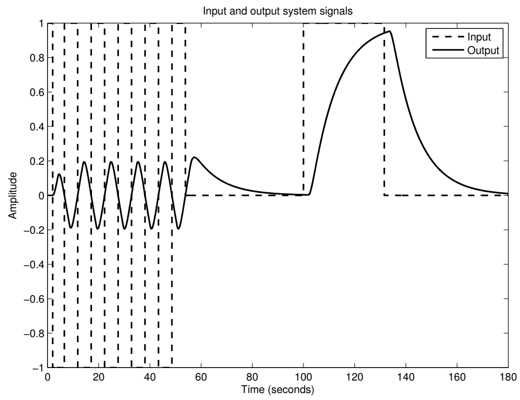

Fig. 2. Proposed excitation applied for system 1

The relay oscillates with a period of 10.52 seconds, which corresponds to a frequency of $0.596 rad/s$ and a phase of $2.919 rad$ or 167,32 degrees. The pulse width is 31.56 seconds, which corresponds to a frequency of $0.198 rad/s$ and a phase of $1.697 rad$ or 97.28 degrees.

The frequency method uses the two points listed above to fit the response from the low frequency region up to high frequency region. The estimates for the time delay L_f for each two frequency points is shown in Table (3). In this Table, ω_L is the low frequency point and ω_H is the high frequency point.

Table 3. Parameter estimates for system 1

	ω_L	ω_H
L_f	2.01	2.12

In table (4), the cost function values used to compare the estimates in time and frequency domain (ε and E) are presented.

Table 4. ε and E for system 1

	$G_c(s)$	$G_f(s)$
ε	0.0022	0.000029
E	0.0268	0.0107

The Nyquist diagrams for $G_c(s)$ and $G_f(s)$ are shown in Fig. (3). The step responses for $G_c(s)$ and $G_f(s)$ are shown in Fig. (4)

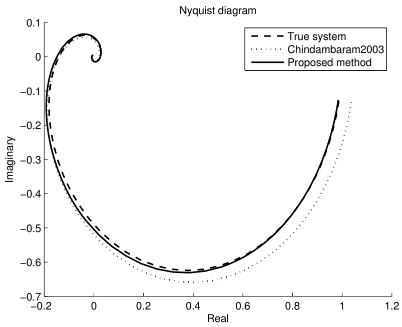

Fig. 3. Nyquist diagram for system 1

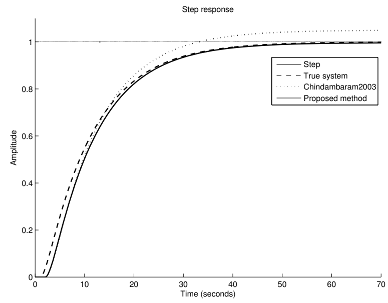

Fig. 4. Step response for system 1

6.2 System 2

The second system is a high order model. The transfer function estimates are shown in Table (5).

Table 5. Identification results for system 2

True system	$G_2(s) = \frac{1}{0.125s^3+0.75s^2+1.5s+1}e^{-4s}$
Chindambaram2003	$G_c(s) = \frac{1.005}{0.202s^2+1.113s+1}e^{-4.42s}$
Frequency method	$G_f(s) = \frac{0.9997}{0.279s^2+1.16s+1}e^{-4.35s}$

The relay oscillates with a period of 10.68 seconds, which corresponds to a frequency of $0.588 rad/s$ and a phase of $3.141 rad$ or 180 degrees. The pulse width is 32.04 seconds, which corresponds to a frequency of $0.196 rad/s$ and a phase of $1.579 rad$ or 90.51 degrees.

The estimates for the time delay L_f for each two frequency points is shown in Table (6). In this Table, ω_L is the low frequency point and ω_H is the high frequency point.

Table 6. Parameter estimates for system 2

	ω_L	ω_H
L_f	4.345	4.351

In table (7), the cost function values used to compare the estimates in time and frequency domain (ε and E) are presented.

Table 7. ε and E for system 2

	$G_c(s)$	$G_f(s)$
ε	$4.071e^{-5}$	$1.350e^{-5}$
E	0.0173	0.0122

The Nyquist diagrams for $G_c(s)$ and $G_f(s)$ are shown in Fig. (5). The step responses for $G_c(s)$ and $G_f(s)$ are shown in Fig. (6)

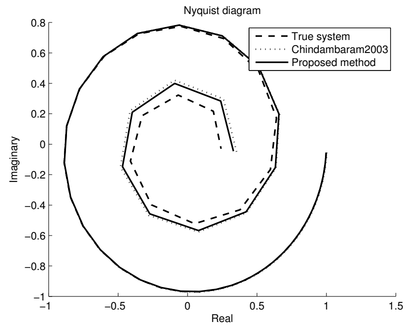

Fig. 5. Nyquist diagram for system 2

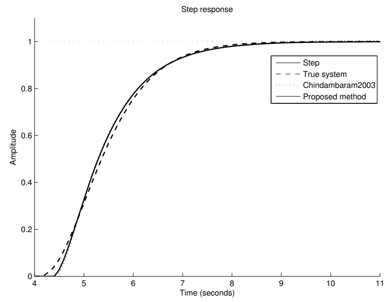

Fig. 6. Step response for system 2

6.3 System 3

The third system is an unstable open loop system. The transfer function estimates are shown in Table (8).

Table 8. Identification results for system 3

True system	$G_3(s) = \frac{1}{s^2+1.5s+1}e^{-0.5s}$
Chindambaram2003	$G_c(s) = \frac{1}{0.9673s^2+1.51s+1}e^{-0.52s}$
Frequency method	$G_f(s) = \frac{0.9991}{1.007s^2+1.488s+1}e^{-0.504s}$

In table (9), the cost function values used to compare the estimates in time and frequency domain (ε and E) are presented.

Table 9. ε and E for system 3

	$G_c(s)$	$G_f(s)$
ε	$1.571e^{-6}$	$1.011e^{-6}$
E	0.0043	0.0026

The Nyquist diagrams for $G_c(s)$ and $G_f(s)$ are shown in Fig. (7).

6.4 System 4

The fourth system is the same of system 1 but adding noise at the process output. The noise is Gaussian with zero mean and variance of 0.005. Note that in this case the relay histerese need to be configured. The transfer function estimates are shown in Table (10). The values are close to that obtained without noise. The method is robust to the measurement noise.

Applied the excitation, the system input and output used in the estimation are shown in Fig. (8).

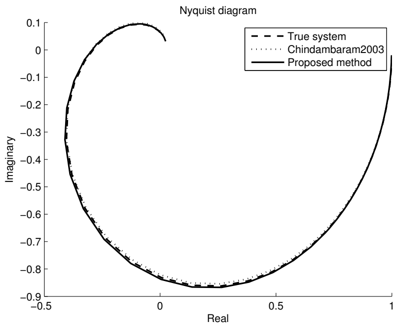

Fig. 7. Nyquist diagram for system 3

Table 10. Identification results for system 4

True system	$G_1(s) = \frac{1}{10s^2+11s+1}e^{-2s}$
Chindambaram2003	$G_c(s) = \frac{1.06}{9.871s^2+11.65s+1}e^{-2.04s}$
Frequency method	$G_f(s) = \frac{0.9987}{10.09s^2+10.97s+1}e^{-2.01s}$

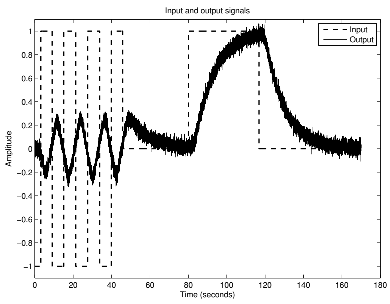

Fig. 8. Proposed excitation applied for system 4

In Table (11), the cost function values used to compare the estimates in time and frequency domain (ε and E) are presented.

Table 11. ε and E for system 4

	$G_c(s)$	$G_f(s)$
ε	0.0024	0.00011
E	0.0288	0.0133

The Nyquist diagrams for $G_c(s)$ and $G_f(s)$ are shown in Fig. (9).

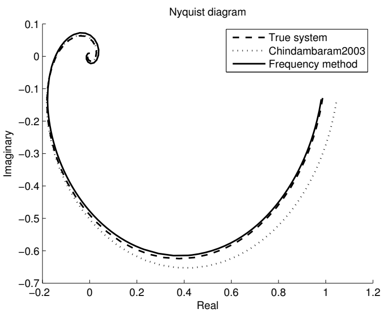

Fig. 9. Nyquist diagram for system 4

7. CONCLUSION

In this paper a proposed identification technique in the frequency domain for SOPTD models was presented. The excitation that was used has content around important frequencies. The idea is to estimate a model which capture the true systems dynamics around frequencies of interest. The main disadvantage of the proposed excitation when compared with other excitations such as relay is that the experiment period is longer. But, in the other side, the model is more accurate around the frequency range. Simulation examples illustrated the capabilities of the proposed technique.

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