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A Simple Harmonics Based Stiction Detection Method

Monir Ahammad * **M.A.A. Shoukat Choudhury** **

* *Bangladesh University of Engineering and Technology, BUET, Dhaka-1000.(e-mail: mahammad@che.buet.ac.bd)*
** *Bangladesh University of Engineering and Technology, BUET, Dhaka-1000. (e-mail: shoukat@che.buet.ac.bd).*

Abstract: Oscillation or large variability of the control loops results decreased economical advantage to process industries due to inferior quality products, larger rate of rejections, reduced average throughputs and overall increased energy requirements. Stiction is one of the root cause of oscillation. Many studies suggest that 30% of the control loops are oscillatory because of control valve problems such as stiction, deadband and hysteresis. In this study, a simple method has been developed to detect stiction. It has been shown that stiction in control valves produces signals containing odd harmonics. The proposed method estimates frequencies and amplitudes of control error signal using least square method. Then, harmonic relation among the frequencies are examined. The presence of odd harmonics indicates valve stiction. The method has been evaluated successfully using simulated, experimental and industrial data sets.

Keywords: Control Valve, Stiction, Fourier Analysis, Harmonics, Oscillations.

1. INTRODUCTION

Modern process industries are getting increasingly automated due to stringent environmental requirement, high quality product, larger rate of production and higher energy integration. The automation is usually executed through thousands of control loops. Being the only moving part in a control loop, the role of control valve is very important. It implements the controller decision to the process. Hence if the control valve malfunctions, the performance of the loop will deteriorate–no matter how good or expensive the controller is. Commonly encountered control valve problems are: Stiction, Hysteresis, Deadband, Backlash, and Saturation.

The effectiveness of quality control depends on the performance of the control loops. For this reason, the control loop performance monitoring has received much attention among the researchers. The causes of poor performance or oscillation of the control loops can be poor controller tuning, i.e., aggressive controlling, presence of external oscillatory disturbance, sensor or transmitter failure, plant or actuator nonlinearities. Nonlinearities like stiction create not only oscillation in the variables but also shorten the life of the valve. It can upset the plant. Thus it brings the economic loss to the process. Therefore, it is important to detect and identify the valves which are suffering from stiction so that maintenance effort can be directed effectively.

2. PROPOSED METHODOLOGY

2.1 How does stiction cause oscillations?

How stiction causes oscillation is explained by Garcia (2008) and Horch (2000). When a control valve suffers from stiction it doesn't move untill the signal or air pressure which is applied to its actuator is greater than the required with respect to an ideal frictionless valve. The excess control signal is necessary to overcome the stiction and move the stem. As a result, the valve position goes to a point beyond the desired value causing oscillations and variability in the control loop.

2.2 Harmonics as root cause of oscillation

Several studies were undertaken on detection and quantification of valve stiction by inspecting the shape of control error and controlled output signals during sustained oscillations (Choudhury et al. (2008), Rengaswamy et al. (2001), Yamashita (2006), Scali and Ghelardoni (2008), Ruel (2000), Gerry and Ruel (2001), Horch (1999), Jelali and Huang (2009)). They suggested that a sticky control valve produces a rectangular/squared shaped control error signal and a saw toothed/triangular wave type controller output signal, while an aggressive controller produces a sinusoidal signal. However, the signal shapes may change according to the presence of noise and the nature of process.

Figure 1 shows four different type of signals with their power spectra. All these signals have a fundamental frequency of 0.01 in a normalised scale. Harmonics are oscillations whose fundamental frequencies are integer multiple of the fundamental frequency. From Figure 1, it can be observed that the sine curve has only one frequency; the multiple sine has two frequencies; the rectangular signal has the fundamental frequency and its 3^{rd}, 5^{th}, ..., odd harmonics; and the triangular or saw-toothed signal has all harmonics (odd and even) in addition to the fundamental frequency. Therefore it is evident that a 'Squared' signal results in odd harmonics. On the other hand the 'saw-

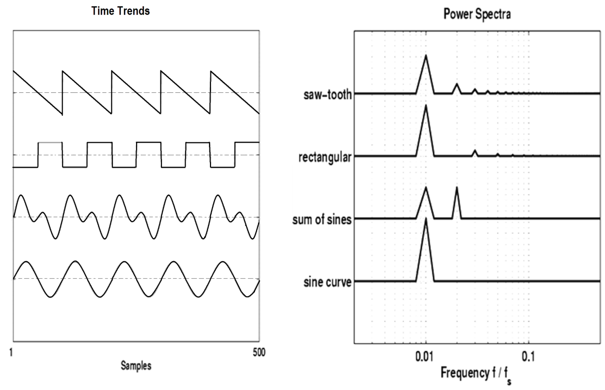

Fig. 1. Time trends and Periodograms of different signals

toothed' or triangular signal produces both odd and even harmonics.

Mathematically a 'Squared' signal can be represented as:

$$y(t) = \begin{cases} -C & -\pi < t < 0 \\ +C & 0 < t < \pi \end{cases} \tag{1}$$

Figure 2 shows it graphically. Since this is an odd function, Euler co-efficients, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\cos(nt)dt = 0$ and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\sin(nt)dt$. Evaluating b_n provides:

$$b_n = \begin{cases} 4C/n\pi & if\ n = 1,3,5,\ldots \\ 0 & if\ n = 2,4,6,\ldots \end{cases} \tag{2}$$

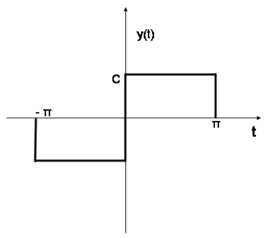

Fig. 2. Control signal in presence of Stiction

The Fourier series Expansion of this signal can be written as:

$$y(t) = \frac{4C}{\pi}\sin(t) + \frac{4C}{3\pi}\sin(3t) + \frac{4C}{5\pi}\sin(5t) + \ldots \tag{3}$$

Therefore, the Fourier series expansion of a rectangular square function shows that the signal has only odd harmonics. Many literature (Horch (1999) Ruel (2000) Choudhury et al. (2005) Jelali (2008)) suggests that a sticky valve also produces a controlled variable signals having rectangular shapes. Hence, odd harmonics in the control/error signal emerges due to the presence of stiction.

Figure 3 shows time trend data and its power spectrum for a flow control loop where stiction was introduced deliberately in a laboratory set-up located in the department of Chemical Engineering, BUET, Dhaka. The power spectrum clearly shows four peaks in Figure 3. The first peak corresponds to the normalized frequency (fundamental) of 0.006 or approximately 167 samples per cycle. The other three peaks are approximately at 0.018, 0.03 and 0.042 frequencies, which are 3^{rd}, 5^{th} and 7^{th} harmonics of the fundamental frequency. Amplitudes and frequencies of first ten components of fourier series expansion of this signal were estimated using least square method to be described in a later section. Table 1 shows the complete result of harmonic analysis(refer to section 2.3). The last column of Table 1 shows that the estimated frequencies are harmonically related and harmonics are odd. Therefore it can be hypothesized that the presence of odd harmonics in control error signal indicates the presence of valve stiction.

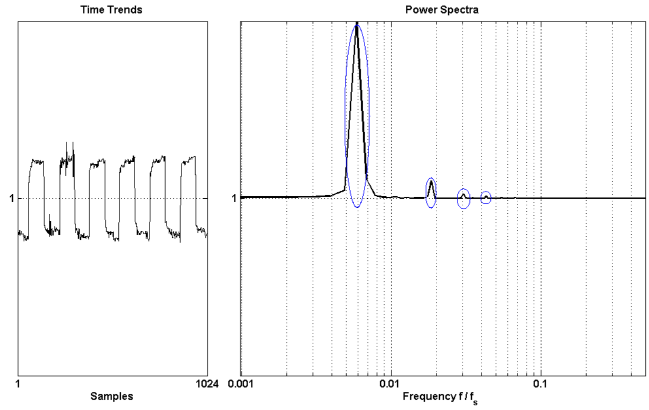

Fig. 3. Time trend (left) and power spectrum (right) of a control loop suffering from stiction

Table 1. Frequency, amplitudes and phases of control error signal of laboratory data.

	Frequency ω_i	Amplitude A_i	Phase ϕ_i	Harmonics ω_i/ω_1
1	0.0383	1.6841	2.3745	**1**
2	0.1152	0.5434	-2.2759	**3.0079**
3	0.1922	0.2901	-0.6785	**5.0159**
4	0.2687	0.1917	1.1606	**7.0148**
5	0.3456	0.1293	2.8778	**9.0209**
6	0.4225	0.0991	-1.866	11.0282
7	0.0678	0.0573	-0.1883	1.7704
8	0.4988	0.0691	0.172	13.021
9	0.1644	0.0511	-2.2712	4.2925
10	0.0624	0.0517	1.1144	1.6282

2.3 Fourier Series Analysis

Fourier series states that any time signal can be represented as a summation of sinusoids. A time series $y(t)$ defined as:

$$y(t) = \sum_{t=0}^{\infty} A_i \cos(\omega_i t + \phi_i) \tag{4}$$

where, ω is the fundamental frequency, which has the largest amplitude; A_i's are amplitudes of sinusoids having frequencies ω_i's. The basic idea is to estimate amplitudes, frequencies, and phases for each term of Equation (4) and examine the relationships among the frequencies to find whether harmonics are present in the signal. Since it is practically impossible to estimate amplitudes, frequencies and phases for infinite number of terms, only 'm' number of terms are estimated. Therefore, Equation (4) can be rewritten as:

$$y(t) = A_0 + A_1\cos(\omega t + \phi_1) + A_2\cos(2\omega t + \phi_2) + .. \\ ..A_m\cos(m\omega t + \phi_m) + \epsilon(t) \tag{5}$$

$\epsilon(t)$ is the error due to omission of terms after m^{th} term. As the chemical process units acts as a filter for higher order frequencies, Choudhury (2008) suggested that it suffices to write the equation upto tenth term, i.e., m=10. Iterative ARMA technique with Least Squares Linear Regression Method has been employed to estimate the frequencies, amplitudes and phases of Equation (5).

2.4 Estimation of frequency by iterative ARMA technique

By assuming a simple sinusoidal model of the form given in Equation (6), Quinn and Hannan (2001) estimated the frequencies by using an Iterative ARMA technique.

$$y(t) = A_0 + \alpha \cos(\omega t) + \beta \sin(\omega t) + \epsilon(t) \tag{6}$$

Now, if a time trend $y(t)$ satisfies Equation 5, it also satisfies

$$\begin{aligned} &y(t) - 2\cos\omega y(t-1) + y(t-2) \\ &= \epsilon(t) - 2\cos\omega\epsilon(t-1) + \epsilon(t-2) \end{aligned} \tag{7}$$

This representation suggests that ω may be estimated by iterative ARMA-based techniques. Suppose that we wish to estimate α and β in

$$y(t) - \beta y(t-1) + y(t-2) = \epsilon(t) - \alpha\epsilon(t-1) + \epsilon(t-2) \tag{8}$$

while preserving $\alpha = \beta$. If α is known, and the $\epsilon(t)$ are independent and identically distributed, then β can be estimated by Gaussian maximum likelihood, that is, by minimizing

$$\begin{aligned} &\sum_{t=0}^{T-1} \epsilon^2_{\alpha,\beta}(t) \\ &= \sum_{t=0}^{T-1} \{\xi(t) - \beta\xi(t-1) + \xi(t-2)\}^2 \end{aligned} \tag{9}$$

with respect to β, where $\xi(t) = y(t)+\alpha\xi(t-1)+\xi(t-2)$ and $\xi(t) = 0, t < 0$. As this is quadratic in β, the minimizing value is the regression coefficient of $\xi(t)+\xi(t-2)$ on $\xi(t-1)$, viz.,

$$\begin{aligned} \frac{\sum_{t=0}^{T-1} \{\xi(t) + \xi(t-2)\}\,\xi(t-1)}{\sum_{t=0}^{T-1} \xi^2(t-1)} &= \alpha + \frac{\sum_{t=0}^{T-1} y(t)\xi(t-1)}{\sum_{t=0}^{T-1} \xi^2(t-1)} \\ &= \alpha + h_T(\alpha) \end{aligned} \tag{10}$$

We then put α equal to this value and re-estimate β using Equation (10) and continue until α and β are sufficiently close. Then, estimate ω from the equation $\alpha = 2\cos\omega$. The factor 2 is introduced for rapid convergence.

This algorithm can be summarized as below:

(1) Put $\alpha_1 = 2\cos\hat{\omega_1}$, where $\hat{\omega_1}$ is an initial estimator of the true value ω_0. This can be estimated from power spectrum.
(2) For $j > 0$, put $\xi(t) = y(t) + \alpha_j\xi(t-1) - \xi(t-2)$, $t = 0, ..., T-1$ where $\xi(t) = 0, t < 0$.
(3) Put $\beta_j = \alpha_j + 2\frac{\sum_{t=0}^{T-1} y(t)\xi(t-1)}{\sum_{t=0}^{T-1} \xi^2(t-1)}$
(4) If $|\beta_j - \alpha_j|$ is suitably small, estimate $\hat{\omega} = \cos^{-1}(\beta_j/2)$. Otherwise, let $\alpha_{j+1} = \beta_j$ and go to step 2.

Once the frequency is estimated, the amplitudes and phases can be estimated using least-square regression method.

2.5 Least squares linear regression method for estimating amplitudes and phases

Data are available as time series sampled at a fixed interval of time. Least-square regression technique is used to estimate each component of any time series data $y(t)$, shown in Equation (11).

$$y(t) = \sum_{t=0}^{m} A_i \cos(\omega_i t + \phi_i) + \epsilon(t) \tag{11}$$

For example, if y is the time series data, $y_1 = A_1\cos(\omega_1 t + \phi_1)$ will be first estimated. Therefore, let us write,

$$\begin{aligned} y &= A_0 + A_1\cos(\omega_1 t + \phi_1) + e_1 \\ &= A_0 + \alpha\cos(\omega_1 t) + \beta\sin(\omega_1 t) + e_1 \end{aligned} \tag{12}$$

where, $\alpha = A_1\cos(\phi_1)$ and $\beta = -A_1\sin(\phi_1)$. Equation (12) contains four unknowns namely A_0, α, ω_1 and β. The frequency ω_1 will be estimated first by using Quinn-Hannan's techniques discussed in the last section. If ω_1 is known, parameters of Equation (12) can be calculated using simple linear regression techniques. Predictions of y can be made from the regression model,

$$\hat{y} = \hat{A_0} + \hat{\alpha}\cos(\omega t) + \hat{\beta}\cos(\omega t) \tag{13}$$

where $\hat{A_0}, \hat{\alpha}$ and $\hat{\beta}$ denote the esimated values of A_0, α and β, $\hat{y}$ denotes the predicted value of y. Each observation or sample of y will satisfy

$$y_i = A_0 + \alpha\cos(\omega_1 t_i) + \beta\sin(\omega_1 t_i) + e_i$$

The least square method calculates values of A_0, α and β, that minimizes the sum of the squares of the errors S for an arbitrary number of data points, T:

$$S = \sum_{i=1}^{T} e_i^2$$

After some calculations, it can be shown that least-squares estimates of A_0, α and β is as follows:

$$\begin{bmatrix} \widehat{A}_0 \\ \widehat{\alpha} \\ \widehat{\beta} \end{bmatrix} = D^{-1}(\omega_1)E(\omega_1)$$

where

$$D(\omega_1) = \begin{bmatrix} T & \sum_{t=0}^{T-1}\cos(\omega_1 t) & \sum_{t=0}^{T-1}\sin(\omega_1 t) \\ \sum_{t=0}^{T-1}\cos(\omega_1 t) & \sum_{t=0}^{T-1}\cos^2(\omega_1 t) & \sum_{t=0}^{T-1}\sin(\omega_1 t)\cos(\omega_1 t) \\ \sum_{t=0}^{T-1}\sin(\omega_1 t) & \sum_{t=0}^{T-1}\sin(\omega_1 t)\cos(\omega_1 t) & \sum_{t=0}^{T-1}\sin^2(\omega_1 t) \end{bmatrix} \tag{14}$$

$$E(\omega_1) = \begin{bmatrix} T \\ \sum_{t=0}^{T-1} y(t)\cos(\omega_1 t) \\ \sum_{t=0}^{T-1} y(t)\sin(\omega_1 t) \end{bmatrix} \tag{15}$$

Thus, $\hat{A_1}, \omega_1$ and ϕ_1 of first term of Fourier series expansion are estimated. Similarly, all m–terms can be estimated.

2.6 Summary of the proposed method

The proposed method can be summarized as follows:

(1) Get routine operating data for control loops.
(2) Remove outliers from the data.
(3) Estimate all m–terms of Fourier series expansion for control error signal.
(4) Examine whether, the frequencies are harmonically related. The presence of odd harmonics indicates stiction.

3. SIMULATION

The objective of this part is to demonstrate the effectiveness of the proposed method in different situations.

A simple single-input, single-output (SISO) linear system with a feedback-control configuration has been used to generate the control error signal. A first order plus time delay process bearing the following transfer function with an PI controller ($K_C = 0.44$, and $\tau_I = 65$) with a sampling time of 1 s is considered for the process simulations:

$$G(s) = \frac{4e^{-2.1s}}{6.5s+1} \tag{16}$$

Oscillations resulting from stiction (in control valve), external oscillatory disturbances and/or aggressive controller tuning are simulated and the proposed detection method are employed. Simulations are carried out for 4000 sampling instants in each case, at the sampling interval of 1 s. Only 1024 sampling data from 1501 to 2524 were analyzed. First 1500 data were discarded to avoid the transient nature of the error signal at the beginning. The cases involving stiction in control valve are simulated using Choudhury et al. (2005) valve-stiction model with varying amount of slip jump (J)and deadband plus stickband (S). The cases, where oscillatory external disturbance is one of the root-cause for oscillations in control loop, a sinusoidal signal, $y(t) = 5\sin(0.02t)$ is added to the process output in closed loop. For simulating the cases involving aggressive controller tuning, the controller integral gain has been changed to 1.59. The effectiveness of this method is also determined for varying amount of noise.

Figure 4 shows time trends and power spectra of the error signals for four simulated cases. Power spectra shows the presence of oscillation for these cases. For well tuned controller, there is no well defined oscillation. For aggressively tuned controller, oscillations are present, but regularity of oscillation is low. For External oscillatory cases, the normalised frequency of oscillation is 0.003 samples/cycle, i.e., 0.02 rad/sec. For stiction case, there is clear limit cycle oscillation at frequency 0.025 cycles/samples which is the largest oscillation among the cases shown here and it takes a period of 40 seconds.

Table 2 shows the results of the harmonic analysis. It represents two cases- one for noise free and another with noise. The followings are the main observations:

(1) For a well tuned controller, no harmonics were found either for noise free or for noise corrupted case.
(2) For a controller with an excessive integral action, no harmonics were found for both noise free and noise corrupted cases.

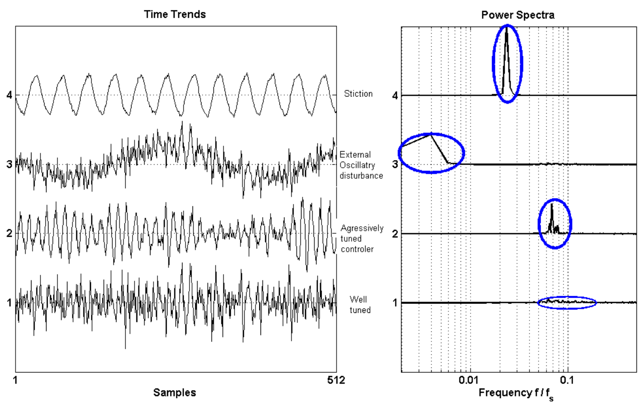

Fig. 4. Time trend and power spectra of controller error (1) well tuned controller (2) controller with excessive integral action (3) presence of external oscillatory disturbance (4) presence of stiction in the control valve.

(3) A sinusoid of amplitude 2 and frequency 0.01 was added to the process output in order to feed an external oscillatory disturbance to the process. For this case, also no harmonics were found.
(4) Several simulations were performed with varying amount of stiction. For S=1and J=1, Table 2 row 4 shows that there are odd harmonics with respect to the fundamental frequency of 0.1664 (normalized). Row 5, 9 and 10 show that odd harmonics can also be found for other amount of stiction. Hence odd harmonics indicates that stiction as non-linearities is present in these control loops and causes variability of controlled variable.

4. INDUSTRIAL CASE STUDIES

The proposed stiction detection method was used to diagnose the cause of oscillations in different types of selected loops from various process industries. These loops include Flow Control (FC), Pressure Control (PC), Level Control (LC), Concentration Control (CC), Thickness Control (ThC) and Analyzer Control (AC). Error signal have been generated for each loop by subtracting controlled output (PV) from set point (SP).

For the sake of brevity, results of six representative loops are tabulated in Table 5. In this table 'Other' corresponds to the causes like excessive integral action, faults in the sensor and external oscillatory disturbances.

Power spectra plot of Figure 5 shows that substantial amount of oscillation were present in the error signals of industrial data set. Harmonic analysis of these loops confirms whether the valve was sticky or not.

4.1 Loop 1

This is a Pressure control loop. When the error signal of this loop has been analyzed by the proposed method, the result corresponds to the second row of Table 3. Odd harmonics dictates that the root cause of oscillation in this loop was stiction. Later it was confirmed that the loop had indeed stiction problem.

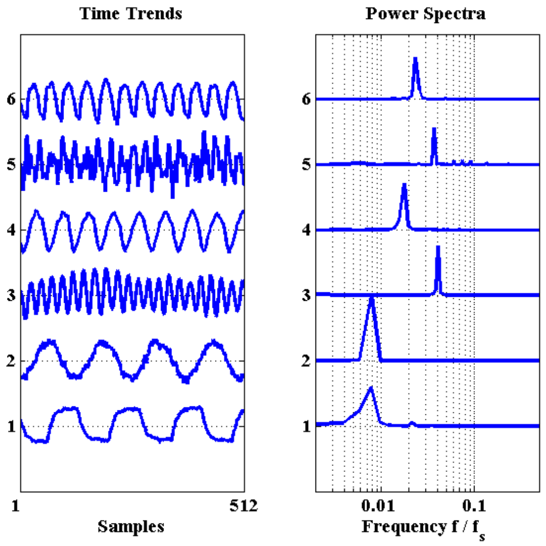

Fig. 5. Time trend and power spectrum of six different industrial data sets. (1) PC (2) FC (3) AC (4) LC (5) ThC (6) CC

4.2 Loop 2

This signal corresponds to a flow control loop in a chemical industry. It was also affected by stiction. When analyzed, odd harmonics results,so this validates that the proposed method works.

4.3 Loop 3

This error signal is from an analyzer control loop of a refinery separation unit. When this signal is analyzed no harmonics found. Thus no sticky valve was in this loop.

4.4 Loop 4

This is a level control loop of a power plant. The proposed method results both odd and even harmonics of the fundamental frequency as shown in the row 5 of table 3. Thus presence of stiction is confirmed by odd harmonics. Even harmonics might state the presence of any other types of nonlinearity along with stiction.

4.5 Loop 5

The thicknesses of metal sheet have been controlled in a metal processing plant by this loop. Prior to the analyses of this signal it was known that this loop suffered from external disturbance. The analysis results only even harmonics. Thus even harmonics dictates the nonlinearity is something else other than stiction.

4.6 Loop 6

This signal is from a concentration control loop of a pulp and paper industry. Prior to the processing, it was known that this loop is affected with dead zone and tight tuning. Similar to loop 4, analyses of this signal also results odd and even harmonics. Thus, the method detects stiction and the possible presence of other type of nonlinearity.

5. CONCLUSION

This paper described a novel method based on harmonic analysis to detect stiction. First, the control error signal is decomposed using Fourier series. Then, amplitude, frequency and phases of each term of Fourier series expansion have been estimated using least square regression technique. The relationship among the frequencies are examined. The presence of odd harmonics indicates the presence of stiction in control valves. This method has been successfully evaluated using simulated and industrial data sets.

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Table 2. Harmonic analysis results of noise free and noisy simulated data of four different cases

Case		$\omega 1$	$\omega 2$	$\omega 3$	$\omega 4$	$\omega 5$	A1	A2	A3	A4	A5	$\omega 1/\omega 1$	$\omega 2/\omega 1$	$\omega 3/\omega 1$	$\omega 4/\omega 1$	$\omega 5/\omega 1$	Comments
No noise	excessive Integral action	0.3686	0.4602	0.466	0.4923	0.6988	0.0869	0.07	0.0686	0.068	0.0588	**1**	1.24	1.26	1.33	1.89	
	external Oscillation	0.0099	0.0052	0.0141	0.0032	0.0098	0.0847	0.001	0.0007	0.0003	0.0002	**1**	0.5253	1.4242	0.32323	0.9899	
	Stiction (S=1, J=1)	0.1664	0.49933	0.8322	1.165099	0.25118	0.59	0.0463	0.0146	0.0073	0.0063	**1**	**3.001**	**5.001**	**7.0016**	1.5094	Odd harmonics
	Stiction (S=2, J=1)	0.1428	0.42835	0.7139	0.226822	0.99948	0.91209	0.069	0.0167	0.0092	0.0083	**1**	**3**	**5.001**	1.58873	**7.001**	Odd harmonics
Noise variance 0.05	Well tune controller	0.6991	0.3687	0.8046	0.6885	0.4923	0.0441	0.032	0.0326	0.0341	0.0315	**1**	0.5274	1.1509	0.98484	0.7042	
	Excessive Integral action	0.3686	0.4603	0.4922	0.466	0.6989	0.0618	0.0491	0.0499	0.0475	0.0455	**1**	1.2488	1.3353	1.26424	1.8961	
	External Oscillation	0.01	0.6991	0.3687	0.8046	0.6885	0.0845	0.0441	0.032	0.0326	0.0341	**1**	69.91	36.87	80.46	68.85	
	Stiction (S=4, J=3)	0.1555	0.46658	0.1649	0.14839	0.77773	2.13421	0.1446	0.0788	0.0546	0.0523	**1**	**3**	1.0605	0.95421	**5.001**	Odd harmonics
	Stiction (S=4, J=4)	0.1661	0.49854	0.1742	0.149535	0.18268	2.33418	0.1797	0.0789	0.0758	0.0689	**1**	**3.001**	1.0489	0.90027	1.099	Odd harmonics

Table 3. Harmonic analysis results of six industrial data sets

Case	$\omega 1$	$\omega 2$	$\omega 3$	$\omega 4$	$\omega 5$	$\omega 6$	$\omega 7$	$\omega 8$	$\omega 9$	$\omega 10$	$\omega 1/\omega 1$	$\omega 2/\omega 1$	$\omega 3/\omega 1$	$\omega 4/\omega 1$	$\omega 5/\omega 1$	$\omega 6/\omega 1$	$\omega 7/\omega 1$	$\omega 8/\omega 1$	$\omega 9/\omega 1$	$\omega 10/\omega 1$	Comments
loop 1	0.044	0.134	0.221	0.037	0.0492	0.062	0.0825	0.308	0.03	0.171	**1**	**3.027**	**5.01**	0.837	1.114	1.4156	1.869	**6.97**	0.62	3.88	Stiction
loop 2	0.05	0.148	0.242	0.217	0.1945	0.083	0.1328	0.112	0.54	0.688	**1**	**2.949**	4.83	4.336	3.886	1.6666	2.652	2.23	10.83	13.73	Stiction
loop 3	0.262	0.272	0.256	0.251	0.2399	0.295	0.2985	0.29	0.32	0.221	**1**	1.037	0.98	0.957	0.915	1.1237	1.138	1.11	1.218	0.84	No stiction
loop 4	0.106	0.212	0.086	0.318	0.1249	0.099	0.4244	0.141	0.11	0.232	**1**	**1.999**	0.82	**2.998**	1.179	0.9346	**4.004**	1.33	1.084	2.19	Stiction + Other
loop 5	0.234	0.467	0.572	0.381	0.8512	0.04	0.7602	0.098	0.26	0.121	**1**	**1.998**	2.44	1.629	3.64	0.1716	3.251	0.42	1.109	0.51	Other
loop 6	0.149	0.142	0.156	0.299	0.1333	0.164	0.5883	0.125	0.29	0.449	**1**	0.953	1.04	**2.001**	0.893	1.0964	3.943	0.84	1.948	**3.007**	Stiction + Other