Modulation-Function-Based Data-Driven Design of Fault Detection Systems for Continuous-Time LTI Systems

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Abstract: In this paper, a data-driven or model-free approach is presented to design a fault detection system of continuous-time linear time-invariant (LTI) systems based on input and output data in the time domain. The main idea is to directly identify the subspaces and their related matrices relevant for parity-space-based residual generation based on a modulated output equation by use of modulation functions and their properties. Therefore, the explicit model identification of the process for a model-based approach in a conventional two-step procedure can be avoided saving design effort especially for large-scale systems. A simulation of the resulting fault detection system is provided showing the effectiveness of the design approach.

Keywords: Fault Detection, Modulation Functions, Subspace Methods, Residual Generation, Parity Space, Model-Free, Data-Driven

1. INTRODUCTION

Modern society increasingly depends on the correct functioning of its technical systems which becomes harder as the complexity of these systems increases. To account for this challenge, over the last decades fault detection and isolation (FDI) techniques have been established (Frank, 1990; Frank et al., 2000; Blanke et al., 2015; Chen et al., 2016). Applications can be found in various fields, including process industries, automotive industries and aerospace and aeronautics.

An essential part for the design of fault detection systems is to find fault-indicating signals, which are usually called residual signals. In the context of model-based fault detection, these signals are determined from a model, which needs to be identified from process or test data collected during a data-acquisition stage, *e.g.* by subspace identification methods (Van Overschee and De Moor, 1996; Wang and Qin, 2002). Figure 1 shows the design procedure following this standard two-stage approach. On the other hand, in Ding et al. (2005, 2009) it was shown that the FDI design procedure can be simplified by a parity-space-based residual generation in combination with a subspace-aided identification of the parity space directly from the test data instead of identifying the system first. Figure 2 shows that, by this design procedure, system identification becomes an implicit part of FDI design and implementation.

However, this design procedure was only presented for discrete-time linear time-invariant (LTI) systems and the straight-forward transfer to continuous-time LTI systems poses some challenges as the derivatives of the test data are needed. In order to handle this problem, Zhang (2005) proposed to use frequency domain data instead of time domain data for the subspace identification procedure. Later, she also showed that using a Poisson filter chain, a similar result can be achieved using time domain data (Zhang and Ding, 2007).

In this paper, another approach is proposed to design parity-space-based residual signals using only input and output data of the system acquired during a data-acquisition stage or test stage. Motivated by Enciso et al. (2021) and Jahn and Shardt (2021), the difficulty of differentiating the input and output signals which is inevitable when dealing with a continuous-time LTI system is overcome by filtering the input and output data over a receding horizon using modulation functions before applying subspace identification. Thus, the parity space and its related matrices can be directly obtained, which leaves it to the FDI designer to continue with either a direct residual generation based on the analytic redundancy relations (ARRs) or with an observer-based residual generator.

2. BACKGROUND

Residual signals are used to quantify the amount of mismatch or discrepancy between the expected/nominal and observed behavior of a process or system. Figure 3 shows a FDI system that has two stages: a diagnostic / residual signal generation stage and a decision making or diagnostic classification stage (Chen et al., 2001). A residual signal must satisfy the specific properties given by Definition 1. This means that the generation or construction of a residual signal based on input and output data only is a nontrivial task and will be the focus of this paper. Different residual generation approaches can be divided into three categories: observer-based, parity-space-based and parameter-estimation-based / parameter-identification-based approaches (Frank, 1990). As mentioned in the introduction, parity-space-based residual sig-

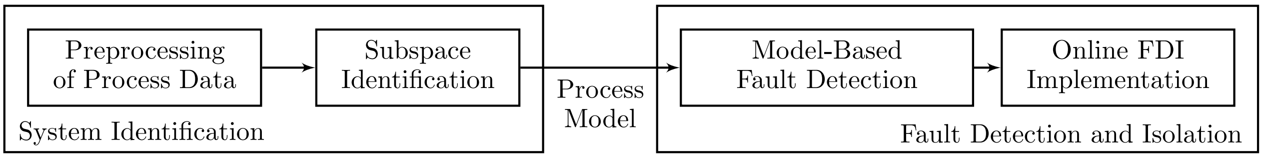

Fig. 1. Model-based FDI system design procedure (after Ding et al. (2009))

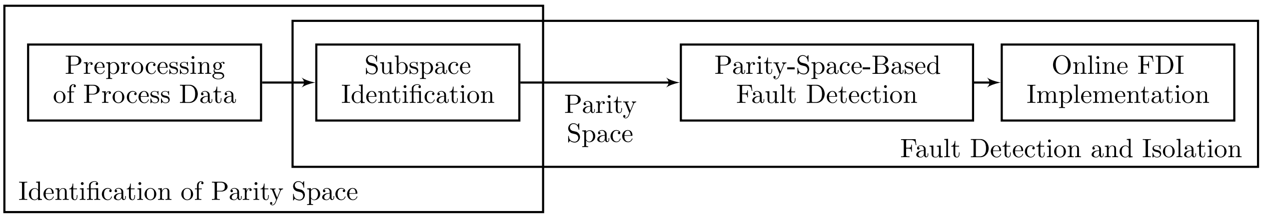

Fig. 2. System identification as an implicit part of FDI system design procedure (after Ding et al. (2009))

nals can be derived from the input and output data in the time domain directly without explicit modeling and identification of model parameters.

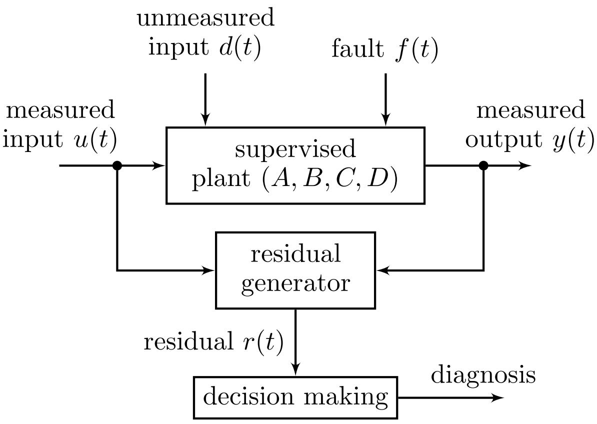

Fig. 3. The two stages in quantitative model-based FDI (Jahn et al., 2020)

Definition 1 (Jahn et al., 2020)**.** *A residual is a signal that is zero when the system under diagnosis is free of faults, and nonzero when particular faults are present in the system. Additionally, a residual must be invariant to any unmeasured, and therefore unknown, input signals (e.g. disturbances) as their influence is not considered to be a fault.*

2.1 Parity-Space-Based Approach for LTI Systems

Consider a nonlinear system that is to be operated only about a nominal operating point which corresponds to a steady-state. Its dynamic behavior can be approximated by a linear time-invariant (LTI) system model of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + E_x d(t) + F_x f(t) \tag{1a}$$

$$y(t) = Cx(t) + Du(t) + E_y d(t) + F_y f(t) \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the vector of known inputs, $y \in \mathbb{R}^q$ is the vector of measured outputs, $d \in \mathbb{R}^{n_d}$ represents unknown disturbances, and $f \in \mathbb{R}^{n_f}$ is the vector of faults which are to be detected. [1]

[1] Although not explicitly indicated, x, u, d, and f represent the deviation from their respective steady-state values.

Definition 2. *A set of n_r linear relations of the known input u and measured output y and their respective derivatives up to a certain order ν*

$$r(t) = \sum_{i=0}^{\nu} W_{y,i} y^{(i)}(t) + \sum_{i=0}^{\nu} W_{u,i} u^{(i)}(t) \tag{2}$$

where $W_{y,i} \in \mathbb{R}^{n_r \times q}$ and $W_{u,i} \in \mathbb{R}^{n_r \times p}$, are called (parity) linear analytic redundancy relations (ARRs) if and only if

$$r(t) \begin{cases} = 0, & \text{in the absence of faults} \\ \neq 0, & \text{in the presence of faults} \end{cases} . \tag{3}$$

They are sometimes synonymous with residual signals, since assuming accessibility of all derivatives needed for their evaluation, they could be directly used for fault detection.

For the purpose of fault detection and isolation, analytic redundancy relations (ARRs), as introduced by Definition 2, are of great importance in the context of residual generation. The subsequent determination of ARRs for LTI systems of the form (1) is taken from Kinnaert (2003). Therefore, let us consider the successive derivatives of the output $y(t)$ with respect to time up to order ν

$$\begin{aligned} y &= Cx + Du + E_y d + F_y f \\ \dot{y} &= C(Ax + Bu + E_x d + F_x f) + D\dot{u} + E_y \dot{d} + F_y \dot{f} \\ &\vdots \\ y^{(\nu)} &= CA^{\nu}x + \\ &\quad CA^{\nu-1}Bu + \cdots + CBu^{(\nu-1)} + Du^{(\nu)} + \\ &\quad CA^{\nu-1}E_x d + \cdots + CE_x d^{(\nu-1)} + E_y d^{(\nu)} + \\ &\quad CA^{\nu-1}F_x f + \cdots + CF_x f^{(\nu-1)} + F_y f^{(\nu)} \ . \end{aligned} \tag{4}$$

Stacking y, u, d, f and their derivatives (*i.e.* $Y = [y^\top, \dot{y}^\top, \ldots, y^{(\nu)\top}]^\top \in \mathbb{R}^{(\nu+1)q}$ and correspondingly for $U \in \mathbb{R}^{(\nu+1)p}, D \in \mathbb{R}^{(\nu+1)n_d}$ and $F \in \mathbb{R}^{(\nu+1)n_f}$) allows us to write the equations in a more compact form as

$$Y(t) = \mathcal{O}x(t) + \mathcal{T}_u U(t) + \mathcal{T}_d D(t) + \mathcal{T}_f F(t) \tag{5}$$

where $\mathcal{O} = [C^\top, (CA)^\top, \ldots, (CA^\nu)^\top]^\top \in \mathbb{R}^{(\nu+1)q \times n}$ is the well-known observability matrix and the lower triangular block Toeplitz matrix $\mathcal{T}_u \in \mathbb{R}^{(\nu+1)q \times (\nu+1)p}$ is

$$\mathcal{T}_u = \mathcal{T}(B,D) = \begin{bmatrix} D & 0 & 0 & \dots & 0 \\ CB & D & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & & \\ CA^{\nu-1}B & \dots & \dots & CB & D \end{bmatrix} . \quad (6)$$

A similar definition holds for the lower block triangular Toeplitz matrices $\mathcal{T}_d = \mathcal{T}(E_x, E_y) \in \mathbb{R}^{(\nu+1)q\times(\nu+1)n_d}$ and $\mathcal{T}_f = \mathcal{T}(F_x, F_y) \in \mathbb{R}^{(\nu+1)q\times(\nu+1)n_f}$.

As $x(t)$ and $D(t)$ are the only unknown quantities in Equation (5), consider the extended matrix $M = [\mathcal{O},\ \mathcal{T}_d] \in \mathbb{R}^{(\nu+1)q\times(n+(\nu+1)n_d)}$. According to the fundamental theorem of linear algebra, if there exists a value ν such that the dimension of its column space $\mathrm{col}(M)$ is less than its number of rows $(\nu+1)q$, then the left nullspace $\mathrm{null}(M^\top)$ is nonempty. As the left nullspace is orthogonal to the column space, its dimension is given by $n_r = (\nu+1)q - \mathrm{rank}[\mathcal{O},\ \mathcal{T}_d]$. As all vectors of $\mathrm{col}(\mathcal{O})$ and $\mathrm{col}(\mathcal{T}_d)$ are in this left nullspace, we can annihilate $x(t)$ and $D(t)$ from Equation (5) as follows

$$M^\perp Y(t) - \underbrace{M^\perp \mathcal{O} x(t)}_{=0} - M^\perp \mathcal{T}_u U(t) - \underbrace{M^\perp \mathcal{T}_d D(t)}_{=0} = M^\perp \mathcal{T}_f F(t) \quad (7)$$

where $M^\perp \in \mathbb{R}^{n_r\times(\nu+1)q}$ denotes a basis for the left nullspace of M. Note that in the absence of faults the right-hand side equals zero. It is different from zero in the presence of faults. Therefore, the set of all (parity) linear analytic redundancy relations (for a given number of considered output derivatives) of the form (2) is given by

$$r(t) = M^\perp Y(t) - M^\perp \mathcal{T}_u U(t) \ . \quad (8)$$

This is equivalent to the form of Equation (2), as it is only a concatenated version, *i.e.* $M^\perp = [W_{y,0}, W_{y,1}, \dots, W_{y,\nu}]$ and $M^\perp \mathcal{T}_u = [W_{u,0}, W_{u,1}, \dots, W_{u,\nu}]$.

As the basis of the left nullspace $M^\perp$ is nonunique, there exists an infinite number of parity vectors r. The method presented for LTI systems gets its name from the space comprising these parity vectors, the parity space.

2.2 Subspace Identification from Input and Output Data

For a data-driven or model-free approach only input and output data during fault-free operation is available. Additionally, during fault-free data collection, disturbances are supposed to be avoided as they cannot be measured. Therefore, all disturbances acting on the systems during data collection will be recognized as part of the process and are not considered explicitly by the data-driven approach. Accordingly, output Equation (5) simplifies to

$$Y(t) = \mathcal{O}x(t) + \mathcal{T}_u U(t), \quad (9)$$

while $M = \mathcal{O}$ reduces to the observability matrix and only a basis for its left nullspace $\mathcal{O}^\perp$ is needed for residual design, that is,

$$r(t) = \mathcal{O}^\perp Y(t) - \mathcal{O}^\perp \mathcal{T}_u U(t) \ . \quad (10)$$

Subspace identification methods based on singular value decomposition (SVD) (Van Overschee and De Moor, 1996; Wang and Qin, 2002) have been used in Ding et al. (2005, 2009) to identify the corresponding matrices $\mathcal{O}^\perp$ and $\mathcal{O}^\perp\mathcal{T}_u$ for direct design of fault detection systems based on input and output data of discrete-time LTI systems. This approach can be transferred to the continuous-time LTI systems in a straight-forward manner. Therefore, for the sake of convenience, we denote $y^{(i)}(k) = y^{(i)}(t)|_{t=t_k}, u^{(i)}(k) = u^{(i)}(t)|_{t=t_k}, x^{(i)}(k) = x^{(i)}(t)|_{t=t_k}$ for $i = 0,\dots,\nu$ and $k = 1,\dots,N$ with N being the number of measurements. Then, the output Equation (9) for a collection of N measurements can be represented as

$$Y_N = \mathcal{O}X_N + \mathcal{T}_u U_N, \quad (11)$$

with

$$\begin{aligned} Y_N &= [Y(1), Y(2), \dots, Y(N)] \in \mathbb{R}^{(\nu+1)q\times N} \\ U_N &= [U(1), U(2), \dots, U(N)] \in \mathbb{R}^{(\nu+1)p\times N} \\ X_N &= [x(1), x(2), \dots, x(N)] \in \mathbb{R}^{n\times N} \ . \end{aligned} \quad (12)$$

Following the general approach of Zhang and Ding (2007), the output equation for the collection of measurements (11) can be reformulated as

$$Z_N = \underbrace{\begin{bmatrix} \mathcal{O} & \mathcal{T}_u \\ 0_{(\nu+1)p\times n} & I_{(\nu+1)p\times(\nu+1)p} \end{bmatrix}}_{H} \begin{bmatrix} X_N \\ U_N \end{bmatrix}, \quad (13)$$

where $Z_N = [Y_N^\top, U_N^\top]^\top \in \mathbb{R}^{(\nu+1)(q+p)\times N}$.

Under the assumption of persistently exciting input and for a sufficient large number of measurements N, the matrix $[X_N^\top, U_N^\top]^\top$ is of full row rank, *i.e.* its column space is $\mathbb{R}^{(\nu+1)(q+p)}$, and has therefore no left nullspace. Subsequently, according to Equation (13) Z_N and H must have the same left nullspace, which allows for the extraction of the parity-space relevant subspaces using singular value decomposition (SVD) from the input and output data matrix Z_N alone as follows

$$Z_N = U\Sigma V^\top, \quad (14)$$

where

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \in \mathbb{R}^{(\nu+1)(q+p)\times(\nu+1)(q+p)} \quad (15)$$

$$\Sigma = \begin{bmatrix} \Sigma_H & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(\nu+1)(q+p)\times N} \ , \ V^\top \in \mathbb{R}^{N\times N} \quad (16)$$

$$\Sigma_H \in \mathbb{R}_+^{((\nu+1)p+n)\times((\nu+1)p+n)} \quad (17)$$

and U, V are orthogonal matrices and Σ is a rectangular diagonal matrix with non-negative real numbers on the diagonal, *i.e.* the singular values of Z_N.

Due to the two zero matrices in the last block row of Σ we have

$$[U_{12}^\top, U_{22}^\top]\, Z_N = [U_{12}^\top, U_{22}^\top]\, U\Sigma V^\top = 0 \quad (18)$$

which implies that

$$[U_{12}^\top, U_{22}^\top] \begin{bmatrix} \mathcal{O} & \mathcal{T}_u \\ 0_{(\nu+1)p\times n} & I_{(\nu+1)p\times(\nu+1)p} \end{bmatrix} \begin{bmatrix} X_N \\ U_N \end{bmatrix} = 0 \ . \quad (19)$$

Since we have ensured that $[X_N^\top, U_N^\top]^\top$ has no left nullspace we get

$$[U_{12}^\top, U_{22}^\top] \begin{bmatrix} \mathcal{O} & \mathcal{T}_u \\ 0_{(\nu+1)p\times n} & I_{(\nu+1)p\times(\nu+1)p} \end{bmatrix} = 0 \quad (20)$$

which results in the relations

$$U_{12}^\top \mathcal{O} = 0 \quad (21)$$

$$U_{12}^\top \mathcal{T}_u + U_{22}^\top = 0 \quad (22)$$

from which we can identify the needed subspaces for the analytic redundancy relations as follows

$$\mathcal{O}^\perp = U_{12}^\top \quad (23)$$

$$\mathcal{O}^\perp \mathcal{T}_u = -U_{22}^\top \; . \quad (24)$$

Since any linear combination of rows $\alpha U_{12}^\top$ and $-\alpha U_{22}^\top$ can be used for the final residual, the number of derivatives needed can be reduced by a QL decomposition of $U_{12}^\top = U_Q U_L$ and selection of α as the first row of $U_Q^\top$.

In order to ensure the existence of the left nullspace of Z_N and H, the number of derivatives considered ν needs to be large enough. This already indicates the problem of such straight-forward extension to continuous-time LTI systems as the calculation of estimates of these derivatives based on the measured input and output signals is highly sensitive to noise. The problem becomes even worse as the order of derivatives increases. In the context of subspace identification methods, Poisson filter chains (Johansson et al., 1997) and Laguerre filter chains (Chou et al., 1999) have been proposed to circumvent this problem. This allows subspace identification methods to be applied to filtered input and output data avoiding the need for derivatives. Thus, the next section presents a different approach to circumvent the need of input and output signal derivatives using modulation functions that can be directly used for the design of fault detection systems.

3. DESIGN APPROACH

The basic idea of this paper is motivated by Ding et al. (2009) and Zhang and Ding (2007) for a subspace-aided residual generation, and the use of modulation functions to apply these ideas to input / output data of a continuous-time LTI system.

Modulation functions have been classically used in parameter estimation of dynamical systems to avoid the computation of derivatives of noisy input and output signals (Shinbrot, 1957; Pearson, 1992; Preisig and Rippin, 1993; Unbehauen and Rao, 1998) but can also be used for state estimation as shown by Jouffroy and Reger (2015).

Definition 3. *A function $\varphi : [0, T] \mapsto \mathbb{R}$ is called a modulation function of order m if it is sufficiently smooth and if, for some fixed T, one has*

$$\varphi^{(i)}(0) = \varphi^{(i)}(T) = 0 \quad (25)$$

for all $i \in \{0, 1, \ldots, m-1\}$.

Multiplication of an unknown derivative signal $y^{(i)}$ of arbitrary order i of the base signal y with a modulation function φ of Definition 3 yields by integration by parts in combination with the boundary conditions (25)

$$\int_0^T \varphi(\tau) y^{(i)}(\tau) \mathrm{d}\tau = \int_0^T (-1)^i \varphi^{(i)}(\tau) y(\tau) \mathrm{d}\tau \quad (26)$$

This fundamental result of modulation functions allows us to avoid the need to compute derivatives of the measured base signal and to eliminate unknown initial and final conditions of the integration which otherwise have to be considered. Over the last decades, various modulation functions have been proposed and used such as trigonometric functions $\varphi(t) = \sin(^{m\pi t}/_{T})^m$ (Shinbrot, 1957) and polynomial functions $\varphi(t) = (T-t)^m t^m$ (Loeb and Cahen, 1965), where m is the order of the modulation function.

Similar to Enciso et al. (2021), the modulation function operator $l_i\{\cdot\} : y(t) \mapsto l_i\{y(t)\}$ is introduced based on the modulation function shifted by $t - T$ as

$$l_i\{y(t)\} = \int_{t-T}^{t} (-1)^i \varphi^{(i)}(\tau - (t-T)) y(\tau) \mathrm{d}\tau \; . \quad (27)$$

which acts over the receding time horizon $[t - T, t]$.

In order to yield the output equation for a data collection similar to Equation (11) but without any input and output derivatives, a modulation function as defined in Definition 3 is applied to the subsequent output derivatives (4). Making use of the modulation function operator notation gives

$$\begin{aligned}
l_0\{y\} &= C l_0\{x\} + D l_0\{u\} + E_y l_0\{d\} + F_y l_0\{f\} \\
l_1\{y\} &= C(A l_0\{x\} + B l_0\{u\} + E_x l_0\{d\} + F_x l_0\{f\}) + \\
&\quad D l_1\{u\} + E_y l_1\{d\} + F_y l_1\{f\} \\
&\;\vdots \\
l_\nu\{y\} &= C A^\nu l_0\{x\} + \\
&\quad C A^{\nu-1} B l_0\{u\} + \cdots + D l_\nu\{u\} + \\
&\quad C A^{\nu-1} E_x l_0\{d\} + \cdots + E_y l_\nu\{d\} + \\
&\quad C A^{\nu-1} F_x l_0\{f\} + \cdots + F_y l_\nu\{f\}. \quad (28)
\end{aligned}$$

Stacking all the shifted derivatives of the output signal y as follows $L\{y\} = [l_0\{y\}^\top, l_1\{y\}^\top, \ldots, l_\nu\{y\}^\top]^\top$ as well as for the input u, disturbances d and faults f, yields the compact form of the modulated output equation in comparison to the original one (5)

$$L\{y\} = \mathcal{O} l_0\{x\} + \mathcal{T}_u L\{u\} + \mathcal{T}_d L\{d\} + \mathcal{T}_f L\{f\} \; . \quad (29)$$

Note that the matrices $\mathcal{O}, \mathcal{T}_u$ are preserved under the modulation operation. Therefore, the relevant subspaces can be directly identified without intermediate step depending on filter parameters as needed in Zhang and Ding (2007).

Similar to the straight-forward extension to continuous-time LTI systems presented in the last section, for a model-free approach input and output data is acquired in fault-free operation avoiding unknown external disturbances analog to Equation (9)

$$L\{y\} = \mathcal{O} l_0\{x\} + \mathcal{T}_u L\{u\} \; . \quad (30)$$

However, the main difference is that modulation function filtered input and output data is acquired. A simple implementation of such filtering based on discrete-time measurements is presented in Appendix A.

Considering a collection of N samples (*i.e.* $L\{y\}(k) = L\{y\}(t)|_{t=t_k}$)

$$\begin{aligned}
Y_N &= [L\{y\}(1), L\{y\}(2), \ldots, L\{y\}(N)] \in \mathbb{R}^{(\nu+1)q \times N} \\
U_N &= [L\{u\}(1), L\{u\}(2), \ldots, L\{u\}(N)] \in \mathbb{R}^{(\nu+1)p \times N} \\
X_N &= [l_0\{x\}(1), l_0\{x\}(2), \ldots, l_0\{x\}(N)] \in \mathbb{R}^{n \times N} \quad (31)
\end{aligned}$$

the aggregated output equation has the same form as for the straight-forward extension (11). Therefore, we can use the same subspace aided method in order to identify $\mathcal{O}^\perp$

and $\mathcal{O}^\perp \mathcal{T}_u$ for residual generation directly without explicit system identification from the time domain input and output data.

The following algorithm summarizes the major steps in the modulation-function-based design of a data-driven residual generator:

S1: Set ν, φ, T and N.
S2: Filter input and output signal over the receding horizon $[t-T, T]$ according to (27).
S3: Build the matrix Y_N and U_N by (31).
S4: Perform an SVD of $Z_N = [Y_N^\top, U_N^\top]^\top$ to get matrix U and its partitions U_{12} and U_{22}.
S5: Using U_{12} and U_{22}, calculate $\mathcal{O}^\perp$ and $\mathcal{O}^\perp \mathcal{T}_u$ according to Equation (24).
S6: (Optional) Reduce order by QL decomposition.
S7a: Design a residual generator based on the (reduced) ARRs (10) directly. During fault detection operation, the derivatives of the input and output data are not easily available as well. Modulation functions can be used again to circumvent this problem as presented in Enciso et al. (2021) and as applied to permanent magnet synchronous motors (PMSM) in Jahn and Shardt (2021).
S7b: Design a (reduced) observer-based residual generator (Frank et al., 2000) as done by Zhang (2005).

4. SIMULATION EXAMPLE

To show the effectiveness of the proposed algorithm, it is applied to the same example system considered in Zhang (2005) in order to make the results comparable

$$\dot{x} = \begin{bmatrix} 0 & 0.5 & 1 \\ -1 & -1 & 0.25 \\ 1 & 0.25 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} f$$
$$y = [1\ 1\ 0]\, x + [1\ 1.5]\, u + [0\ 1]\, f\ . \tag{32}$$

The system is excited over 45 s by a chirp signal varying from 0 to 10 rad s^{-1} for the first input channel and from 10 to 0 rad s^{-1} for the second channel. The response data is sampled at $T_s = 50\,\text{ms}$. Set $\nu = 9$, $T = 5\,\text{s}$ and $\varphi(t) = (T-t)^m t^m$ with $m = 10$ in order to fulfill Equation (25) for the needed derivatives. Filter the sampled input and output data using the discrete approximation presented in Appendix A with order of $T/T_s = 100$. Build the matrices $Y_N \in \mathbb{R}^{10\times 800}$, $U_N \in \mathbb{R}^{20\times 800}$ and aggregate them to $Z_N \in \mathbb{R}^{30\times 800}$. Note that due to the receding nature of the filtering, the integral interval is only completely covered after $T = 5\,\text{s}$, therefore the first 100 values of the filtered responses are neglected in the data collection, resulting in $N = 800$. Do an SVD of Z_N, a QL decomposition of the resulting partition $U_{12}^\top = U_Q U_L \in \mathbb{R}^{7\times 10}$ and select α as the first row of $U_Q^\top$, namely

$$\alpha = [0,\ 0,\ 0,\ 0,\ 0.7032,\ -0.6356,\ -0.3188]\ .$$

Using α, the design of a (reduced) observer-based residual generator as in Zhang (2005) leads to a similar result for the observer-based FD system. The example system and the designed observer-based FD system have been simulated for 80 s with a unit step at 0 s for the first input channel and $\sin(t)$ for the second input channel. Step-wise faults of amplitude 1 at 50 s for each component have been considered separately. Figure 4 shows the responses of the residual for both simulations, confirming the results compared to Zhang (2005). It can be seen, that each fault can be detected based on the residual behavior.

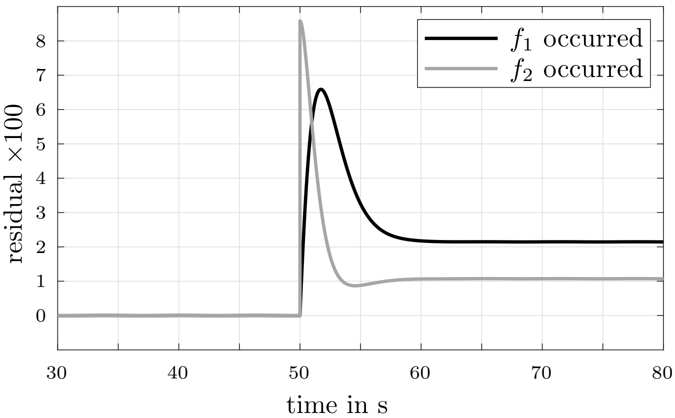

Fig. 4. Simulation result: Residual response for each fault

5. CONCLUSION AND FUTURE WORK

In this paper, a data-driven or model-free approach is presented to design a fault detection system of continuous-time LTI systems based on input and output data in the time domain. The main idea of the approach is to directly identify the subspaces and their related matrices relevant for parity-space-based residual generation based on a filtered output equation using modulation functions and their properties. Thus, the design effort has been reduced as the relevant subspaces are directly identified without intermediate step depending on the filtering. Overall, the explicit model identification of the process for a model-based approach in a conventional two-step procedure can be avoided saving design efforts especially for large-scale systems. Future work will consider the choice of modulation functions for specific processes and the influence of noise and external disturbances.

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Appendix A. DISCRETE-TIME IMPLEMENTATION OF MODULATION FUNCTION FILTERING

For discrete-time implementation with sample time T_s of a receding horizon integral as in (27), consider the simplest approximation of the integral by the endpoint rule

$$\int_0^T (-1)^i \varphi^{(i)}(\tau) y(\tau) \mathrm{d}\tau = \sum_{k=1}^{T/T_s} (-1)^i \varphi^{(i)}(kt_k) y(kt_k) T_s \tag{A.1}$$

where T is a multiple of T_s and T/T_s is the order of approximation. The following linear discrete system $(A_k, b_k, c_k^\top)$ with input y realizes such an approximation of the receding horizon integral

$$A_k = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \; b_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$c_k^\top = (-1)^{(i)} \begin{bmatrix} \varphi^{(i)}(t_k) \\ \vdots \\ \varphi^{(i)}(T - t_k) \\ \varphi^{(i)}(T) \end{bmatrix}. \tag{A.2}$$

However, there are many other options of implementing the modulation-function-based filtering starting with different methods for numerical integration.