

On MIMO PID Control of the quadruple-tank process via ILMIs Approaches : Minimum and Non-Minimum Case studies

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Abstract: This paper addresses the problem of the multivariable control of the quadruple-tank process via Iterative Linear Matrix Inequality (ILMI) approaches. Three methods are revised and compared to design the feedback PID gain controllers. To evaluate the performances of each approach, we consider the two case studies of the minimum and non-minimum phase linear time invariant models. We also examine the feasibility, the stabilization problem resolution and the complexity of each ILMI algorithm.

Keywords: Iterative methods; PID control, Multivariable feedback control, Minimum phase systems; Stabilization.

Nomenclature

- h_i : Level of water in tank i
- a_i : Area of the pipe flowing out from tank i
- A_i : Area of tank i
- γ_1 : Ratio of water diverting to tank 1 and tank 4
- γ_2 : Ratio of water diverting to tank 2 and tank 3
- k_1 : gain of pump 1
- k_2 : gain of pump 2
- K_c : Level sensor
- g : gravitational constant
- ϑ_1 : voltage input 1 (pump 1)
- ϑ_2 : voltage input 2 (pump 2)
- y_1 : Voltage from level measurement devices of tank 1
- y_2 : Voltage from level measurement devices of tank 2

1. INTRODUCTION

The PID controllers have been at the heart of control engineering practice over the last decades. They are widely used in industrial applications as no other controllers match the simplicity, clear functionality, applicability and ease of use. The PID controllers was introduced in 1910 and their use and popularity had grown particularly after the Ziegler–Nichols empirical tuning rules in 1942 (Ziegler and Nichols, 1942). This control approach is an online and proven method however it requires experiences and very aggressive tuning for the process. Several approaches have been reported in literature (Astrom and Hagglund, 2001) to tune PID parameters for SISO systems. In this framework, Ziegler Nichols (Astrom and Hagglund, 2004) and Cohen-Coon (Cohen and Coon, 1953) are considered as the most commonly used methods. However, most industrial processes are of multivariable

nature. MIMO PID controller is much less understood and developed than single variable case. Recently, several books and surveys reported research works about tuning MIMO PID controllers, see e.g (Luan et al., 2010), (Vilanova and Visioli, 2012), (Rames and Panda, 2012). MIMO PID controllers tuning approaches can be classified into empirical (Zhuang and Atherton, 1994), artificial intelligence (Willjuice and Baskar, 2009) and analytical approaches (Isakson and Graebe, 1999).

Analytical approaches have particularly emerged to tune the PID parameters. The most popular techniques in this category are optimal methods (Yanchevsky, 1987), robust methods (Ho, 2003), placement pole methods (Zhang et al., 2002) and iterative methods (Lequin et al., 2003). On the other hand, Linear Matrix Inequalities (LMIs) are the most efficient tools in controller design in this framework. A great deal of LMI-based design methods have been proposed by (Geromel et al., 1994; Cao et al., 1999; Wang et al., 2007) where the Iterative Linear Matrix Inequality (ILMI) methods was proposed by (Cao et al., 1998) and later used to solve several MIMO PID controller design problems (Soliman et al. , 2010 ; Bevrani et al. , 2011 ; Zheng et al., 2002 ; Lin et al., 2004; He and Wang ,2006). The basic idea is to transform a PID controller into an equivalent static output feedback (SOF) controller. This can be realized by augmenting, using some new state variables, the dimension of the PID controller system. Established results in SOF field can be then used to design a multivariable PID controller for various specifications such as stability, H_2/H_∞ performances... In this context, new additional matrix-valued variable can be introduced so that the involved stability conditions become conservative (sufficient but far from necessary). The iterative algorithm in (Zheng et al., 2002), for example, tried to find a sequence of the additional variables such that the relevant sufficient conditions are close to the necessary and sufficient ones. The similar idea is used in

the so-called substitutive LMI method in (Fujimori, 2004). The merit of our work is to present the procedures of three new ILMI approaches and to establish a comparative analysis by evaluating the performances of each one. Comparative criteria considered are the feasibility, the stabilization problem resolution and the degree of the implementation complexity.

The paper is organized as follows. Section 2 introduces the problem statement and the motivations. Section 3 details three ILMI based approaches for MIMO PID tuning. In section 4, a quadruple-tank process model is presented as a benchmark example to illustrate the performances of MIMO PID algorithms and give an efficient comparative analysis.

2. PROBLEM STATEMENT AND MOTIVATIONS

Consider the linear time-invariant (LTI) MIMO system described by:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (1)$$

Where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, $y \in \mathbb{R}^p$ is the output vector. The matrices A, B, C are with appropriate dimensions.

The problem to be solved in this paper is to design the feedback gain matrices $F_1, F_2, F_3 \in \mathbb{R}^{m \times p}$ such that system (1) is stabilized by a PID controller of the form:

$$u = F_1 y + F_2 \int_0^t y dt + F_3 \frac{dy}{dt} \quad (2)$$

Where F_1 , F_2 and F_3 are denoted by the proportional, time integral and time derivative gain matrices respectively.

Transformation of PID controllers to SOF controllers is a good alternative to solve the complex control problem (Fujimori, 2004). Thus, the stabilization problem is reduced to calculate the closed-loop eigen values for the augmented system. Several ILMI algorithms were developed to find and lead to different approaches and methodologies. Very often, the different conditions derived are not readily implementable as numerical algorithms. Another major difficulty is due to the non-convexity of the static output feedback solution which gives an important computational task. These motivate the present work to study new ILMI approaches and to detail resolution procedures.

3. PID TUNING VIA ILMI APPROACHES

In the following section, we present the main important ILMI approaches for PID tuning.

3.1 Approach 1 (Zheng et al., 2002)

In this approach, the problem of finding the parameters of MIMO PID gains is reduced to a Static Output Feedback (SOF) stabilization problem. Consider then the augmented system:

$$\begin{aligned} \dot{z} &= \bar{A}z + \bar{B}u \\ \bar{y} &= \bar{C}z \\ u &= \bar{F}\bar{y} \end{aligned} \quad (3)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 & CA \\ 0 & I & 0 \end{bmatrix} \text{ and}$$

$$\bar{F} = [\bar{F}_1 \quad \bar{F}_2 \quad \bar{F}_3]$$

$$\bar{F} = \left[(I - F_3 CB)^{-1} F_1 \quad (I - F_3 CB)^{-1} F_2 \quad (I - F_3 CB)^{-1} F_3 \right]$$

The original PID gain matrices can be recovered as:

$$F_3 = \bar{F}_3 (I + C\bar{B}\bar{F}_3)^{-1} \quad (4)$$

$$F_2 = (I - F_3 CB) \bar{F}_2 \quad (5)$$

$$F_1 = (I - F_3 CB) \bar{F}_1 \quad (6)$$

The invertibility of matrix $I + C\bar{B}\bar{F}_3$ is guaranteed by the following proposition:

Proposition 1: Matrix $I - F_3 CB$ is invertible if and only if matrix $I + C\bar{B}\bar{F}_3$ is invertible,

Theorem:

The system (1) is stabilizable via static output feedback if and only if there exist $P > 0$ and \bar{F} satisfying the following matrix inequality:

$$\bar{A}^T P + P\bar{A} - P\bar{B}\bar{B}^T P + (\bar{B}^T P + \bar{F}\bar{C})^T (\bar{B}^T P + \bar{F}\bar{C}) < 0 \quad (7)$$

The negative sign of the term $-P\bar{B}\bar{B}^T P$ makes its solution very complicated. This approach introduced a new variable X to deal with the problem. Thus, we consider a matrix Ψ which depends on P affinely and satisfies:

$$\Psi \leq P\bar{B}\bar{B}^T P \quad (8)$$

with $\Psi = X^T \bar{B}\bar{B}^T P + P^T \bar{B}\bar{B}^T X - X^T \bar{B}\bar{B}^T X$ where $X > 0$.

The system (1) can be stabilized if the following inequality has solution for (P, \bar{F}) :

$$\bar{A}^T P + P\bar{A} - \Psi + (\bar{B}^T P + \bar{F}\bar{C})^T (\bar{B}^T P + \bar{F}\bar{C}) < 0 \quad (9)$$

Using Schur complement, inequality (9) is equivalent to the following inequality:

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} - \Psi & (\bar{B}^T P + \bar{F}\bar{C})^T \\ (\bar{B}^T P + \bar{F}\bar{C}) & -I \end{bmatrix} < 0 \quad (10)$$

Once X is given, matrix inequality (10) can be solved very efficiently.

To compute the feedback gain matrices F_1, F_2, F_3 , the ILMI algorithm is presented in details in (Zheng et al., 2002) using the previous procedure.

3.2 Approach 2 (Lin et al., 2004)

Based on system (1), a new state variable $\tilde{x}(t) = [x^T(t), \int_0^t x^T(\theta)d\theta, \dot{x}^T(t)]^T$ and a new output $\tilde{y}(t) = [y^T(t), \int_0^t y^T(\theta)d\theta, \dot{y}^T(t)]^T$ are introduced.

The system (1) is then transformed into the following SOF control system:

$$\begin{aligned} \bar{E}\dot{\tilde{x}}(t) &= \bar{A}\tilde{x}(t) + \bar{B}u(t) \\ \bar{y}(t) &= \bar{C}\tilde{x}(t) \\ u(t) &= \bar{F}\tilde{y}(t) \end{aligned} \tag{11}$$

where

$$\begin{aligned} \bar{E} &= \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} 0 & 0 & I_n \\ I_n & I_n & 0 \\ A & 0 & -I_n \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}, \quad \bar{F} = [F_1 \quad F_2 \quad F_3]. \end{aligned}$$

Theorem :

The system (11) is admissible if and only if there exist matrices P and \bar{F} such that

$$\bar{E}^T P = P^T \bar{E} \geq 0 \tag{12}$$

$$\bar{A}^T P + P^T A - P^T \bar{B} \bar{B}^T P + (\bar{B}^T P + \bar{F} \bar{C})^T (\bar{B}^T P + \bar{F} \bar{C}) < 0 \tag{13}$$

Remark:

Equations (12) and (13) can be combined to a single LMI. Let $E_L = [0, I_n]$ which is a maximum left annihilator of \bar{E} . Then the conditions in (12) and (13) are equivalent to the following LMI:

$$\begin{bmatrix} \bar{\Sigma}_i & (\bar{B}^T (Z_i \bar{E} + E_L^T Y_i) + \bar{F} \bar{C})^T \\ \bar{B}^T (Z_i \bar{E} + E_L^T Y_i) + \bar{F} \bar{C} & -I \end{bmatrix} < 0 \tag{14}$$

For additional matrices $Z_i > 0$ and $Y_i \in \mathbb{R}^{n \times 3n}$ where $\bar{\Sigma}_i = \bar{A}^T (Z_i \bar{E} + E_L^T Y_i) + (Z_i \bar{E} + E_L^T Y_i) \bar{A} - X_i^T \bar{B} \bar{B}^T (Z_i \bar{E} + E_L^T Y_i) - (Z_i \bar{E} + E_L^T Y_i)^T \bar{B} \bar{B}^T X_i + X_i \bar{B} \bar{B}^T X_i - 2\alpha_i \bar{E}^T (Z_i \bar{E} + E_L^T Y_i)$. Indeed, (14) implies (12) and (13) by letting $P_i = Z_i \bar{E} + E_L^T Y_i$; and conversely, (12) and (13) ensure (14) by noting that P_i can be decomposed as $Z_i \bar{E} + E_L^T Y_i$ with $Z_i = \text{diag}\{P_{i11}, I_n\} > 0$ and $Y_i = [P_{i21}, P_{i22}]$.

Note that P has the following form:

$$P = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}, 0 < P_{11} \in \mathbb{R}^{2n},$$

$P_{22} \in \mathbb{R}^n$ is invertible .

The system $\bar{E}\dot{\tilde{x}}(t) = \bar{A}\tilde{x}(t)$ is called admissible if it is regular, impulse-free and stable. The ILMI algorithm is detailed in (Lin et al., 2004).

3.3 Approach 3 (He and Wang, 2006)

The following assumption is made for this approach: (A, B) is stabilizable and (C, A) is detectable.

Theorem:

If

$$P(\bar{A} + \bar{B}\bar{F}\bar{C}) + (\bar{A} + \bar{B}\bar{F}\bar{C})^T P - \alpha P < 0 \tag{15}$$

holds, the closed-loop system matrix $\bar{A} + \bar{B}\bar{F}\bar{C}$ has its eigenvalues in the strict left-hand side of the line $\alpha/2$ in the complex s-plane. If a $\alpha \leq 0$ satisfying (15), the SOF stabilization problem is solved.

The key point of this approach is to divide the problem into two steps: the first one is to find an initial P; the second step is to stabilize the system and thus compute the PID gains matrices. The ILMI algorithm corresponding to this approach is detailed in (He and Wang, 2006).

4. APPLICATION

To derive a comparative analysis between the three methods based on ILMI approaches, we consider a quadruple-tank process as a benchmark for minimum-phase system.

4.1 The quadruple-tank process

The quadruple-tank process (Johansson, 2000; Gatzke et al., 2000; Rusli E. et al., 2002) is a multivariable process which consists of four interconnected water tanks and two pumps. The system is shown in figure 1. The output of each pump is split into two using a three-way valve. The inlet flow of each tank is measured by an electro-magnetic flow-meter and regulated by a pneumatic valve. The level of each tank is measured by means of a pressure sensor.

The regulation problem aims to control the water levels in the lower two tanks with two pumps. The two pumps convey water from a basin into the four tanks. The tanks at the top (tanks 3 and 4) discharge into the corresponding tank at the bottom (tanks 1 and 2, respectively). The three-way valves are emulated by a proper calculation of the set-points of the flow control loops according to the considered ratio of the three-way valve. The positions of the valves determine the location of a zero for the linearized model.

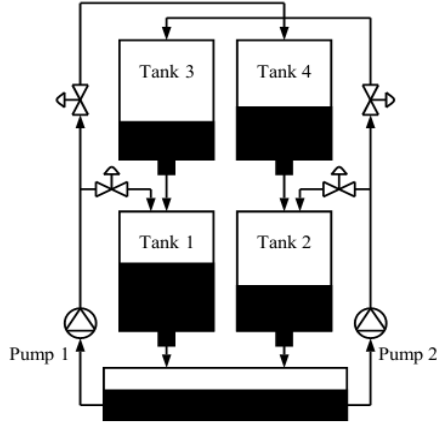


Fig. 1. Schematic diagram of the quadruple tank process

The nonlinear model of the process is described by (Gatzke et al., 2000):

$$\begin{aligned} \frac{dh_1}{dt} &= -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1 \vartheta_1}{A_1} \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2 \vartheta_2}{A_2} \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2)k_2 \vartheta_2}{A_3} \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1 \vartheta_1}{A_4} \end{aligned}$$

Let note by $x = [h_1 \ h_2 \ h_3 \ h_4]^T$ the state variable vector, $u = [\vartheta_1 \ \vartheta_2]^T$ the control vector and $y = [y_1 \ y_2]^T$ is the output vector. The linearized model around the equilibrium points $u_1^0, u_2^0, h_1^0, h_2^0, h_3^0, h_4^0, y_1^0, y_2^0$ can be expressed as in (1) (Rusli et al., 2002):

$$\begin{aligned} A &= \begin{bmatrix} -\frac{a_1}{A_1} \sqrt{\frac{g}{2h_1^0}} & 0 & \frac{a_3}{A_1} \sqrt{\frac{g}{2h_3^0}} & 0 \\ 0 & -\frac{a_2}{A_2} \sqrt{\frac{g}{2h_2^0}} & 0 & \frac{a_4}{A_2} \sqrt{\frac{g}{2h_4^0}} \\ 0 & 0 & -\frac{a_3}{A_3} \sqrt{\frac{g}{2h_3^0}} & 0 \\ 0 & 0 & 0 & -\frac{a_4}{A_4} \sqrt{\frac{g}{2h_4^0}} \end{bmatrix} \\ B &= \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \quad C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \end{aligned}$$

The parameters of the quadruple-tank process are presented in (Johansson, 2000). The eigenvalues of the

open-loop system are - 0.0159, - 0.0111, - 0.0419 and - 0.0333. The system has two multivariable transmission zeros, which are determined by the zeros of its determinant:

$$\det G(s) = \frac{T_1 T_2 k_1 k_2 \gamma_1 \gamma_2}{\prod_{i=1}^4 (1+sT_i)} \left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \right] \quad (16)$$

Thus, the zeros can be computed analytically:

$$z_1(\eta) = \frac{-(T_3 + T_4) + \sqrt{(T_3 - T_4)^2 + 4T_3 T_4 \eta}}{2T_3 T_4} \quad (17)$$

$$z_2(\eta) = \frac{-(T_3 + T_4) - \sqrt{(T_3 - T_4)^2 + 4T_3 T_4 \eta}}{2T_3 T_4} \quad (18)$$

where

$$\eta = \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \in [0, \infty[$$

The adjustable multivariable zero z_1 given by (17) can be set to a left or the right-half plane. The valves position adjustment determines if the system is minimum-phase or non-minimum phase. The results can be written in terms of the flow ratios γ_1 and γ_2 as shown in table 1.

Table 1: Location of zeros on the linearized system as a function of the flow ratios γ_1 and γ_2

	z_1	z_2	System behavior
$1 < \gamma_1 + \gamma_2 \leq 2$	negative	negative	Minimum phase
$\gamma_1 + \gamma_2 = 1$	zero	negative	Boundary
$0 < \gamma_1 + \gamma_2 \leq 1$	positive	negative	Non minimum phase

4.2 MIMO PID via ILMI

In this section, we will apply the three ILMI approaches to design the feedback gain matrices of the MIMO PID controllers of the quadruple-tank process. Sedumi and Yalmip Toolbox (Lofberg, 2004) are used to solve the numerical problem. Simulation results for the two cases of minimum and non-minimum phase case studies are presented in table 2 and table 3, respectively.

Table 2: PID controllers for minimum phase process

Approach	Feedback matrices	Poles
1	$F_1 = \begin{bmatrix} -0.6073 & -0.0204 \\ -0.2294 & -0.4300 \end{bmatrix}$ $F_2 = \begin{bmatrix} -0.0033 & -0.0004 \\ -0.0018 & -0.0016 \end{bmatrix}$ $F_3 = \begin{bmatrix} -0.1227 & 1.1891 \\ -1.2754 & -0.1068 \end{bmatrix}$	- 10.0175 - 6.1201 - 0.0166 - 0.0598 -0.0615±0.0149i
2	$F_1 = \begin{bmatrix} -0.2284 & -0.1725 \\ -0.3656 & -0.1502 \end{bmatrix}$ $F_2 = \begin{bmatrix} -0.0013 & -0.0013 \\ -0.0025 & -0.0008 \end{bmatrix}$ $F_3 = \begin{bmatrix} -0.1493 & 2.1896 \\ -2.9234 & -0.2977 \end{bmatrix}$	0 0 -0.9389 -0.9909±0.0496i -1.0125 -0.0229±0.0170i -0.0147±0.0023i -0.0008 -0.0049
3	$F_1 = \begin{bmatrix} 0.1874 & 6.3471 \\ 6.4080 & -2.1746 \end{bmatrix}$ $F_2 = \begin{bmatrix} 1.3596 & 2.6713 \\ 2.5197 & -2.3161 \end{bmatrix}$ $F_3 = \begin{bmatrix} 30.5335 & 12.9082 \\ 9.5872 & 18.7263 \end{bmatrix}$	-0.2394± 0.5675i -0.1667± 0.3093i -0.0585 -0.0172

For the approach 1 and 3, LMIs (10) and (15) solved via the ILMI algorithms are found feasible. For the approach 2, the LMIs (12) and (13) using the appropriate ILMI algorithm are also found feasible since the system (11) is regular, impulse-free and has all its roots in the left-hand side of the complex plane.

Considering the stabilization problem, approach 1 and 3 solved the problem since the eigenvalues of the closed-loop system are in the left-hand side of the complex plane. However, the approach 2 can't solve the stabilization problem since we can observe that two zeros for the eigenvalues of the closed-loop system are given.

In terms of complexity of the ILMI implementation, approach 1 and 2 are complex as they need the introduction of additional variables, leading to higher dimension of the LMIs. Approach 3 seems to be simpler as it avoids the introduction of the additional variables. However it requires the convergence of two independent algorithms. The first one must find an initial matrix P and a second algorithm, using the initial matrix P, must compute the gain matrices F_1, F_2, F_3 .

Table 3: PID controllers for non-minimum phase process

Approach	Feedback matrices	Poles
1	$F_1 = \begin{bmatrix} -0.6047 & -0.0241 \\ -0.2320 & -0.4205 \end{bmatrix}$ $F_2 = \begin{bmatrix} -0.0033 & -0.0004 \\ -0.0018 & -0.0016 \end{bmatrix}$ $F_3 = \begin{bmatrix} -0.1287 & 1.1966 \\ -1.2915 & -0.1105 \end{bmatrix}$	-1.4903 -0.3065 -0.0228 ± 0.0083i -0.0130 -0.0173
2	$F_1 = \begin{bmatrix} -0.2284 & -0.1725 \\ -0.3656 & -0.1502 \end{bmatrix}$ $F_2 = \begin{bmatrix} -0.0013 & -0.0013 \\ -0.0025 & -0.0008 \end{bmatrix}$ $F_3 = \begin{bmatrix} -0.1493 & 2.1896 \\ -2.9234 & -0.2977 \end{bmatrix}$	0 0 -0.9398 -0.9909±0.0496 -1.0125 -0.0229±0.0170i -0.0147±0.0023i -0.0008 -0.0049
3	$F_1 = \begin{bmatrix} 2.6115 & 0.9588 \\ 1.9941 & 1.7539 \end{bmatrix}$ $F_2 = \begin{bmatrix} 0.3353 & 0.2650 \\ 0.3331 & 0.2992 \end{bmatrix}$ $F_3 = \begin{bmatrix} 41.0167 & 1.6633 \\ 0.3142 & 58.9080 \end{bmatrix}$	-1.5317±2.3132i -0.1042 -0.0009±0.0124i -0.0663

Tuning parameters, for each approach, exposed in table 4, can be also considered as an important comparative criterion since a suitable choice of each tuning parameter is required to lead to good performances.

Table 4: Convergence rate, synthesis and tuning parameters of each approach

Approach	α_i		Synthesis parameters	Tuning parameters
	MP	NMP		
1	-0.0333	-0.025	P_i, F_i, α_i	Q_0, ϵ
2	$7.62 * 10^{-6}$	$7.62 * 10^{-7}$	Z_i, L_i, F_i, α_i	X_1, ϵ
3	$1.1895 * 10^{-7}$	$2.88 * 10^{-4}$	$P_i, F_i, \alpha_i, L_i, V_1, V_2$	$\epsilon_1, \epsilon_2, \delta$

After intensive simulation results, we are able to conclude that the approach 1 is very sensitive to the variation of the tuning parameters where approach 2 and 3 are less sensitive.

5. CONCLUSION

In this paper, three new ILMI approaches have been exposed for designing feedback gain matrices for MIMO PID controllers. A benchmark of a minimum/non

minimum phase process has been used to illustrate the comparative analysis between the three approaches. This note succeeds not only to study existing ILMI approaches for MIMO PID design but mainly to propose an efficient comparative analysis which open some horizons to give a number of extensions for existing results by dealing with their advantages and disadvantages.

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