

# Dynamic Optimization of a Campus Cooling System with Thermal Storage

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**Abstract**—Thermal energy storage gives a system enhanced operational flexibility because thermal loads can be shifted, not only spatially—from one piece of equipment to another—but also temporally, using storage, from one point in time to another. The resulting optimization problems become non-convex and difficult to solve. This paper illustrates how to take advantage of the enhanced flexibility that storage provides, while simplifying the dynamic optimization problems involved. The optimization strategy presented decouples the static and dynamic pieces of the problem using a hierarchical structure, where a static sub-problem is solved for each trial decision variable in the dynamic problem. Energy savings as high as 9.4 % are observed with cost savings as high as 17.4%.

## I. INTRODUCTION

Because thermal loads account for a significant portion of peak energy consumption, thermal energy storage has proven to be a cost-effective peak reduction technology [1,2]. Thermal energy storage gives a system the ability to shift loads temporally by providing system operators more degrees of freedom in operating the system. Optimization can then be applied to help operators use the system most effectively in terms of energy or cost minimization [3]. In this work, this methodology is applied to a district cooling system with chilled water thermal energy storage.

In warm climates, cooling demand is a significant contributor to total energy demand in buildings. Cooling loads are most frequently met by running electrically-powered air conditioners or chillers. For large-scale systems, such as campuses with many buildings, performance improvement of chilling equipment through optimization is a viable cost and energy saving approach. In order to meet cooling loads and also to have redundancy for reliability purposes, large-scale systems on a district cooling loop typically have multiple chillers. From an optimization point of view, this gives the system more degrees of freedom, as different combinations of cooling load can be placed on each chiller, while still meeting the total cooling demand for the system. Optimization leads to improved energy efficiency compared to using simple rules, such as equal ratio chiller loading, where the part load ratios (the load on a chiller divided by its capacity) for each chiller are set to be equal [4]. When chiller efficiency varies widely with load and ambient conditions (most notably wet bulb temperature), an optimization-based approach to chiller loading can have a significant impact on energy savings [5].

The addition of thermal energy storage to a district cooling system further increases system flexibility. The ability to store energy means that cooling loads do not have to be exactly met by chilling equipment at all points in time. Instead, cooling can be generated in excess of the real-time demand and stored. If stored cooling (typically in the form of chilled water or ice) is available, chilling equipment can run at loads less than the real-time demand using the stored energy to make up the difference [6]. This enhanced flexibility allows chillers to shift cooling loads to periods where ambient conditions may allow them to operate more efficiently and to optimally distribute these loads in time and over the range of available chillers. Under a time-of-use electricity pricing structure, significant cost savings will also be achieved.

While thermal energy storage can greatly enhance a system's ability to operate more efficiently, the addition of energy storage translates the optimal chiller loading problem from a static optimization problem to a dynamic optimization problem. This increases the size of the optimization problem as chiller loading must be solved at every step in the time horizon. Another complicating factor is that, for the static problem, the only load of concern is the instantaneous load, while for the dynamic problem chiller loading must be determined for some period into the future, where exact loads as well as ambient conditions are largely unknown. Therefore, solving the dynamic optimization problem typically requires incorporating a forecast of weather and of cooling load for the duration of the prediction horizon.

## II. CHILLER MODELING

Industrial sized chillers are large, complex pieces of equipment, making accurate first principles modeling a difficult task. Empirical, or black box, models can be relatively easily developed. These models use equations with a particular structure (e.g., polynomial, neural network, etc.) to describe the chiller performance. The models are generally equipped with a number of parameters, which can be fit to the given model structure through regression techniques [7]. Because there is generally no physical basis for the equations and parameters, these models can be accurate over the range of data to which they are fit because of the high number of parameters available for fitting [8–11], but they are generally inaccurate outside that range.

A reasonable compromise between a first principles model and a purely empirical model is a semi-empirical model, which provides the structure to the model equations,

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based on the physics of the actual system, but allows for some unmeasured constants to be varied in order for the model to provide a good fit to the data. A thermodynamic chiller performance model, which has proven to be accurate, yet sufficiently simple, is that developed by Gordon and Ng [12]. This model can be used to evaluate a chiller's coefficient of performance (COP) as a function of the cooling load on the chiller, the condenser inlet temperature (which is a strong function of the ambient wet bulb temperature), and the chilled water temperature exiting the evaporator. The COP is then used to determine the power consumption by the chiller. The model is well formulated in that it provides several terms that can be used as fitting parameters, while still possessing a structure that allows the model to be extrapolated beyond the range of the data over which it is fit [13]. The relationship for COP is shown in (1).

$$\begin{aligned} \frac{1}{COP} = & -1 + \left( \frac{T_c^{in}}{T_e^{out}} \right) + \left( \frac{1}{Q} \right) \left( \frac{q_e T_c^{in}}{T_e^{out}} - q_c \right) + \\ & \left( \frac{1}{Q} \right) \left( \frac{q_e}{M_e T_e^{out}} \right) \left( \frac{q_e T_c^{in}}{T_e^{out}} - q_c \right) + \left( \frac{Q}{T_e^{out}} \right) \left( \frac{T_c^{in}}{T_e^{out}} \right) \left( \frac{1}{M_c} + \frac{1}{M_e} \right) \\ & + \frac{1}{T_e^{out}} \frac{q_c}{M_e} + \frac{q_e T_c^{in}}{T_e^{out} M_c} + \left( \frac{T_c^{in} q_e}{T_e^{out}} - q_c \right) \left( \frac{1}{M_c} + \frac{1}{M_e} \right) \end{aligned} \quad (1)$$

This thermodynamic chiller model can be fit to data for chillers using some of the constants in the model as fitting parameters and employing a least-squares model fitting algorithm. The parameters used for fitting in this work are  $q_e$  and  $q_c$  (the heat losses at the evaporator and condenser, respectively) and  $M_e$  and  $M_c$  (the product of overall heat transfer coefficient and heat transfer area for the evaporator and condenser, respectively). Although the model parameters appear nonlinearly in the model, they can be grouped together in a manner that the least squares minimization problem can be obtained by solving a linear system of equations. The model fit is demonstrated in Figure 1.

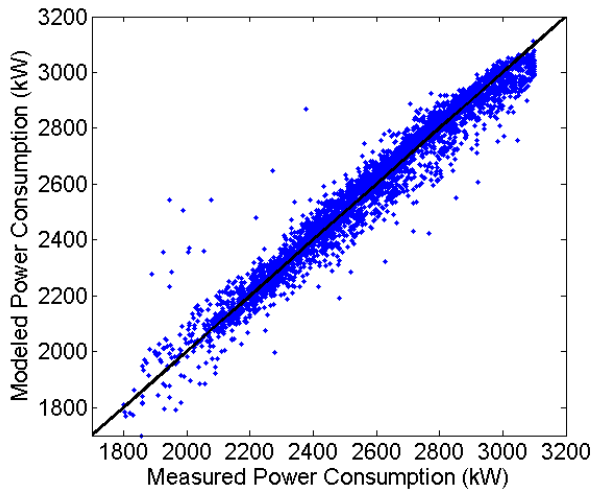


Figure 1: A parity plot demonstrating the model fit for Chiller 1.

The performance curve for one of the four chillers in the network is shown in Fig. 2. The performance curves for the

other three chillers are similar, but not identical due to differences in make, age, and capacity. These curves are generated using (1) and fitting the above-specified parameters for a four chiller system in Austin, TX. The models generated from these fits are used for subsequent analysis in this paper. The chillers are electric-powered, industrial-size centrifugal chillers using a refrigeration cycle with R-134a.

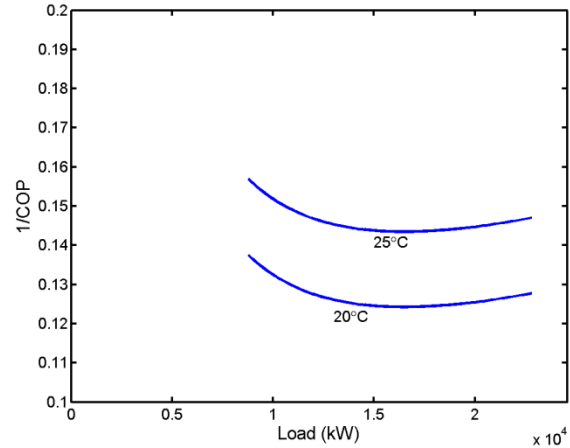


Fig. 2: Performance curve for Chiller 1 at two different wet bulb temperatures (20°C and 25°C).

As Fig. 2 indicates, chiller efficiency strongly depends on the load placed on each chiller and can vary considerably from one chiller to another. Furthermore, chillers operate much more efficiently at a lower ambient wet bulb temperature because heat is rejected by the condenser at a lower temperature.

### III. STATIC OPTIMAL CHILLER LOADING

In order to enhance the steady-state operation of a chilling system by optimal chiller loading, a static optimization problem must be solved first, with the objective of minimizing total power consumption by optimally distributing the cooling load across the available chillers. The decision for choosing an optimization method ultimately depends on the model used to represent the system and the optimization resources that are available. Close examination of (1) reveals that, upon multiplying through by  $Q$ , the cooling load, to get power consumption ( $P$ ) as a function of  $Q$ , the model is quadratic with respect to  $Q$ , as (2) indicates.

For convenience, the terms in (2) can be grouped based on their dependence on  $Q$ , where the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  are assumed to be independent of  $Q$ . This assumption is justified by the fact that  $T_e^{out}$ , the temperature of the chilled water exiting the evaporator maintained at a constant set-point and  $T_c^{in}$ , the cooling water temperature entering the condenser, is predominantly a function of the cooling tower performance and the ambient conditions, primarily the wet bulb temperature.

$$P = \alpha + \beta Q + \gamma Q^2 \quad (2)$$

#### A. Static Problem Formulation

The objective of minimizing the total power consumption for a set of  $N_c$  chillers while meeting or exceeding a total

cooling demand ( $Q_{tot}$ ) at a given instant in time yields the optimization problem in (3). Because it may be optimal to turn some chillers off, the binary decision variable  $\delta$  is added to the formulation, taking on a zero value when chiller  $i$  is off and one when it is on. This makes the problem a mixed integer nonlinear programming (MINLP) problem, which is typically solved using a branch and bound or another MINLP solution algorithm.

$$\min_{Q_i, \delta_i} \sum_{i=1}^{N_c} (\alpha_i + \beta_i Q_i + \gamma_i Q_i^2) \delta_i \quad (3a)$$

subject to

$$\sum_{i=1}^{N_c} Q_i \delta_i \geq Q_{tot} \quad (3b)$$

$$Q_{min,i} \leq Q_i \leq Q_{max,i} \quad (3c)$$

$$\delta_i \in [0,1] \quad (3d)$$

If a known subset of chillers is anticipated to be on, however, the problem is reduced to a simple quadratic programming (QP) problem, which takes on the form:

$$\min_x x^T H x + F^T x \quad (4a)$$

subject to

$$A x \leq B \quad (4b)$$

$$x_{min} \leq x \leq x_{max} \quad (4c)$$

where  $x$  is a vector containing the load placed on each chiller, with  $N_{AC}$  referring to the total number of chillers assumed to be active,  $H$  is a diagonal matrix containing the quadratic terms from each chiller model, and  $F$  is a vector containing the linear terms from the chiller model. The constraint that the total demand must be met is enforced using the  $A$  and  $B$  terms, which are a vector and a scalar, respectively. In this case  $A$  is a horizontal  $N_{AC}$ -vector of ones and  $B$  is the negative of the total cooling demand.

Formulating the problem as a QP has several advantages. First, the problem can be solved quickly using an off-the-shelf QP solver. Second, because  $H$  is a diagonal matrix with only positive values, it will always be positive definite, ensuring convexity of the problem and guaranteeing that the solution to the problem will be a global minimum. The major disadvantage of this method, however, is that the problem must be solved multiple times in order to explore all possible combinations of active chillers. This can be largely bypassed, however, by methodically testing specific active sets of chillers, beginning with the most efficient chillers in the system and gradually adding the less efficient chillers. This can greatly reduce the number of combinations tried. Upon solving the problem for these combinations, the best of these solutions can then be implemented. Because the QP problems being solved are fairly trivial, computation times for the static optimal chiller loading problem are not a major issue. The overriding concern is reaching a global solution, which is guaranteed by using this method but cannot necessarily be guaranteed by the MINLP formulation

of the problem.

#### IV. MULTI-PERIOD OPTIMAL CHILLER LOADING WITH STORAGE

If the objective is to minimize total energy consumption over some time horizon by optimally placing the cooling load on certain chillers at suitable times of the day, the addition of thermal energy storage to a cooling network adds many more degrees of freedom. Using energy storage, cooling loads can be shifted, not only to the most efficient chillers, but to the times of the day when chillers operate most efficiently, typically when ambient wet bulb temperatures are lower. While the extra degrees of freedom by virtue of energy storage are very useful, they also make the problem much more difficult to solve. Typically, a dynamic optimization problem is discretized temporally into a certain number of time intervals,  $N_t$ , during which, it is assumed that inputs are held constant. For a system with  $N_C$  chillers, this creates a total of  $N_t \times N_C$  degrees of freedom. If binary variables,  $\delta$ , are used to represent the on/off states of the chillers, the total number of degrees of freedom becomes  $2N_t \times N_C$ . While solving problems of this size is certainly within the realm of some MINLP solvers, the problem of finding a global solution within a reasonable amount of time may be a limiting factor.

The high dimensionality may be mitigated, however, by re-formulating the problem. The number of degrees of freedom can be reduced by a factor of  $N_C$  if the set of chillers is considered to be a single optimal chiller, rather than  $N_C$  individual chillers. Essentially, this entails solving the static optimal chiller problem to determine the optimal total power consumption for a given load. Solutions to this problem are shown in Fig. 3 for ambient wet bulb temperatures of 20°C and 25°C, over a range of total chiller loads. As the figure shows, higher efficiencies are obtained at a lower wet bulb temperature. A general upward trend in  $1/COP$  as load increases is also observed. However, the curves have some locations with sharp peaks. These non-smooth points indicate a change in the active set of chillers that is optimal for a given load.

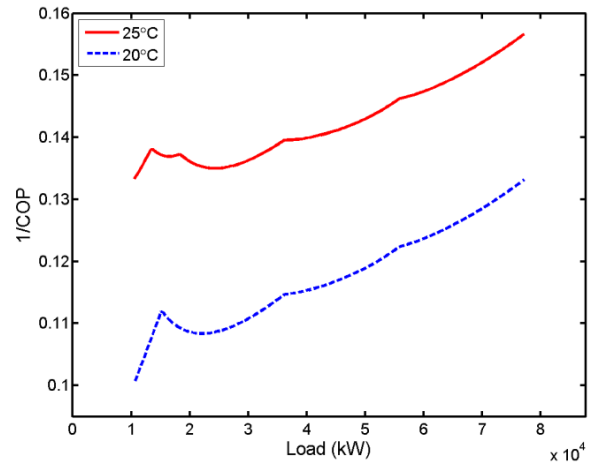


Fig. 3: A composite chiller performance curve created by solving the static optimal chiller loading problem over the range of total cooling loads at certain ambient conditions.

As Fig. 3 clearly indicates, it is generally more efficient to cool during times with lower ambient wet bulb temperature. Therefore, energy storage can be used to reduce energy usage by shifting the cooling load to times when lower wet bulb temperatures are expected. There are limits to the efficiency that can be gained by doing this, however, as increasing load at the lower temperature causes the system to be less efficient. With no constraints on energy storage rate or total capacity, the optimal solution for shifting load between these two temperatures would be achieved when the marginal decrease in power consumption per unit load at the less efficient time equals the marginal increase in power per unit load at the more efficient time.

Using the solution to the static optimal chiller loading problem, all chillers can be considered as a single, optimal chiller. This significantly reduces the combinatorial complexity of the dynamic problem. Rather than solving for loading on each chiller at each time interval, only the total load is needed at every time interval. The dynamic problem uses the total loads at each time interval as decision variables, with the loads placed on individual chillers being determined by the static optimization sub-problem. This formulation is depicted graphically in Fig. 4. As the figure illustrates, the dynamic problem uses the total load ( $Q_{tot}$ ) at each time interval ( $j$ ) as its decision variables. For a given  $Q_{tot,j}$ , the static problem is then solved to determine the optimal loading on each chiller ( $Q_{i,j}$ ) required to meet  $Q_{tot,j}$ . The power consumed at each time interval under optimal loading ( $P_{tot}^*$ ) is then communicated back to the dynamic problem. By solving the static optimal chiller loading problem at each time interval, the system of chillers, therefore, behaves essentially as a single chiller, operating at its most efficient point for a given load and given ambient conditions. While the system still has  $N_C \times N_t$  degrees of freedom of which it can take advantage, the optimization problem is reduced to one with only  $N_C$  degrees of freedom with global optimality guaranteed at each time interval. One inherent disadvantage to this problem formulation is that it becomes more difficult to prevent chillers from switching on and off regularly. However, if a penalty on the change in total load from one time interval to the next is added, this will prevent excessive chiller switching, provided there are no dramatic swings in ambient conditions over the same interval.

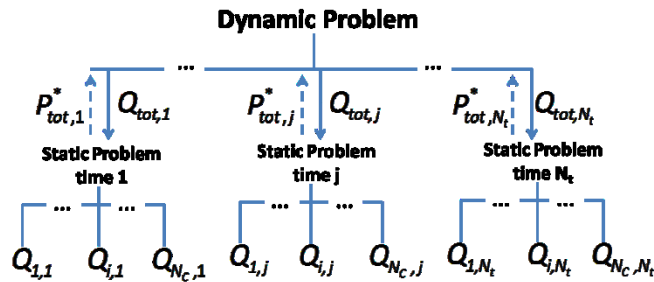


Fig. 4: The hierarchical structure used to solve the dynamic optimization problem is shown. The dynamic problem uses the total load at each time step ( $Q_{tot,j}$ ) as decision variables. These values are then fed to the static optimization sub-problem, where the optimal loading on each chiller ( $Q_{i,j}$ ) is determined. The optimal power that results from the static sub-problem at each time step ( $P_{tot,j}^*$ ) is then communicated back to the dynamic problem.

## V. CONCLUSIONS

### A. Dynamic Problem Formulation

A discrete-time version of the dynamic optimal chiller loading problem is described in (6). Here, a simple model (6b) is adopted for the energy stored ( $E$ ), where the energy stored at time  $j+1$  is equal to the difference in cooling delivered ( $Q_{tot}$ ) minus the total cooling demand ( $Q_{demand}$ ) at time  $j$  multiplied by the time interval  $\Delta t$ . The function  $\Phi$  in (6a) refers to the composite chiller function at time  $j$ , as obtained from the solution of the static optimal chiller loading problem. The system is subject to inequality constraints on the amount of energy stored (6e) and the rate at which energy can be extracted or delivered to storage (6f). Here, a negative value of  $\Delta E$  means that energy is being extracted from the storage system.

$$\min \sum_{j=1}^{N_t} \Phi(Q_{tot,j}, T_{c,j}^{in}, T_{e,j}^{out}) \Delta t \quad (6a)$$

subject to

$$E_{j+1} = E_j + \Delta E_j \quad (6b)$$

$$E_0 = E(0) \quad (6c)$$

$$\Delta E_j = (Q_{tot,j} - Q_{demand,j}) \Delta t \quad (6d)$$

$$0 \leq E_j \leq E_{max} \quad (6e)$$

$$\Delta E_{min} \leq \Delta E_j \leq \Delta E_{max} \quad (6f)$$

The system is subject to an initial condition on the amount of stored energy initially in the storage system (6c). With the objective to minimize total energy consumption over some time horizon, the optimal solution to this problem will generally be to finish with an empty storage tank. A time horizon of 24 hours is chosen in this analysis, with the tank reaching its initial condition at the end of the time horizon. Longer time horizons (two to three) may also be considered as it may be advantageous to store energy over a period of multiple days, depending on the forecasted conditions.

## VI. RESULTS

Dynamic optimal chiller loading using thermal energy storage is more effective when there is a larger swing in wet bulb temperature over the course of a day (giving the system a greater improvement in efficiency by shifting the cooling load to these times) and when there is a large swing in total cooling demand. Assuming a storage system that is initially uncharged ( $E_0=0$ ), several days were simulated using actual wet bulb temperatures and cooling demands for a large campus in Austin, TX. One of these days is shown in Fig. 5, where the total demand ranges from 39,600 to 65,300 kW of cooling, giving the system ample opportunity to shift the cooling load. The ambient wet bulb temperature ranges from 17.7°C to 23.3°C, which correlates fairly well with the cooling demand. The total storage capacity for this system is 136,800 kWh, meaning that the system has a little over 2

hours of full-load storage capacity in this case. However, because the storage can only be charged and discharged at a maximum rate of 21,100 MW, the storage lasts much longer as part-load storage.

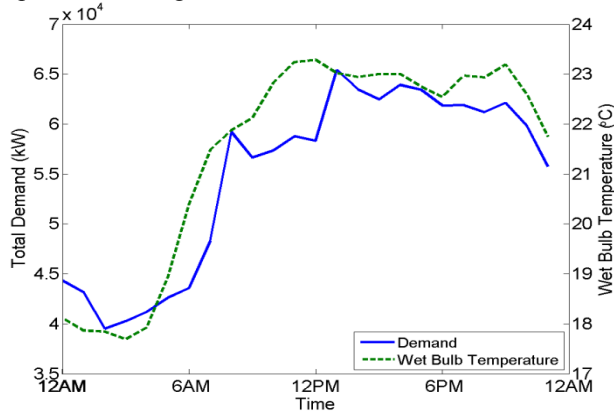


Fig. 5: Total cooling demand ( $Q_{demand}$ ) and wet bulb temperature vs. time for one day on a campus cooling network in Austin, TX.

Three different chiller operating strategies are considered. An equal ratio loading operating strategy places the same proportional load (relative to the chiller maximum capacity) on each chiller.

When static optimization is performed, the results show that, during peak times, Chiller 1 is used at full capacity. Peak loads require Chiller 3 (the least efficient chiller) to run for a total of 15 hours. Still, static optimal operation saves approximately 9% energy over an equal ratio loading strategy under these conditions. The chiller operation for static optimization is shown in Fig. 6.

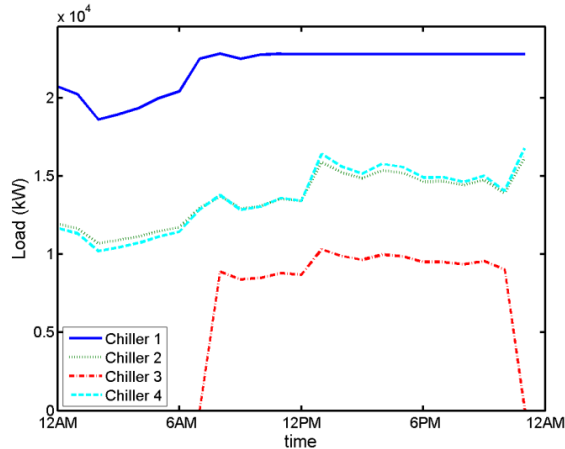


Fig. 6: Solution to the static optimal chiller loading problem for each hour of the day under the conditions given in Fig. 5. Static optimal chiller loading still requires using the least efficient chiller (Chiller 3) for long durations.

Dynamic optimal chiller loading gives the system many more degrees of freedom. This allows the system to not only shift load to the more efficient chillers, but also to shift the load to the most efficient times of the day. Fig. 7 and Fig. 8 show the results of dynamic optimal chiller loading under the same conditions. As these results indicate, much of the load is shifted to the earlier parts of the day, under cooler conditions. Chiller 3 is still used; however, it is only required for a total of 7 hours as a larger percentage of the load is placed on the more efficient chillers in the system. Chiller 1 (the most efficient chiller at full load), for example,

runs at full capacity the entire time, while Chillers 2 and 4 are kept near their optimal efficiency point. An added benefit of storage is that it allows the chillers to run at a more constant rate for the duration of the day, with only small fluctuations in the load on each chiller.

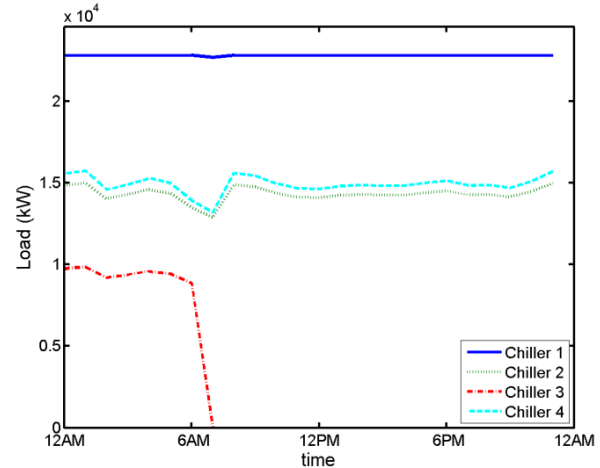


Fig. 7: Solution to the dynamic optimal chiller loading problem with storage for the conditions in Fig. 5.

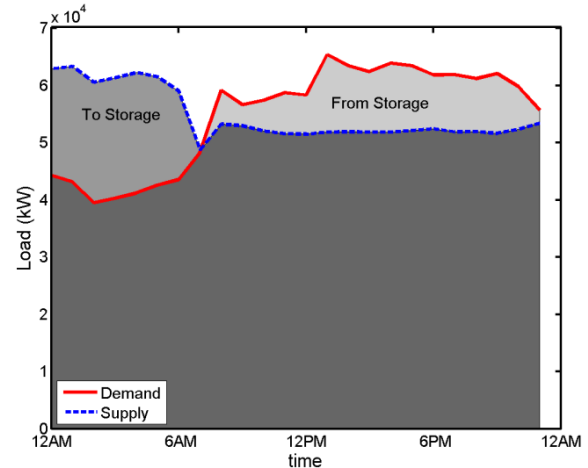


Fig. 8: A plot showing the total supply and demand vs. time of day in the dynamic optimal chiller loading case. As the plot shows, much of the cooling supply is delivered early in the day and stored. It is extracted later when wet bulb temperatures are higher and it becomes less efficient to run the chillers.

Table 1, which summarizes the results of three different days used in this case study, shows that the total energy consumption is improved by a total of up to 9.4% in going from an equal ratio chiller loading strategy to a dynamic optimal chiller loading strategy with thermal energy storage. In Case 3, a day when there is little fluctuation (a range of only 0.9°C) in ambient wet bulb temperature, the savings are 6.8%. Therefore, the benefit of using thermal energy storage solely for shifting cooling loads to more efficient periods of the day depends on how much ambient conditions change during the day.

Because COP is also a strong function of the load on each chiller, shifting load even with relatively constant ambient conditions can be beneficial. Because the load in Case 3 is relatively high, however, the system is more constrained and requires all chillers to be active. This leaves less opportunity for optimization as it reduces the system's flexibility.



**Table 1: A summary of the results of 3 different cases with an objective to minimize total energy consumption. Three solutions are compared: equal ratio chiller loading (where each chiller is assigned the same ratio of load with respect to its maximum capacity), static optimal chiller loading, and dynamic optimal chiller loading using storage.**

	Case 1	Case 2	Case 3
<b>T<sub>WB</sub> Range (°C)</b>	6.4	5.6	0.9
<b>Min Cooling Load (kW)</b>	37,900	39,600	62,500
<b>Max Cooling Load (kW)</b>	55,400	65,300	76,400
<b>Total Energy Consumption (MWh)</b>			
<b>Equal Ratio Chiller Loading</b>	165.6	197.5	279.7
<b>Static Optimal Chiller Loading</b>	152.4	183.0	261.8
<b>Dynamic Optimal Chiller Loading</b>	150.0	179.7	260.8
<b>Total Savings</b>	<b>9.4%</b>	<b>9.0%</b>	<b>6.8%</b>

While thermal storage can shift cooling loads to periods where chillers can operate more efficiently, the major benefit of having thermal energy storage is its ability to shift electrical loads, not only cooling loads, temporally. Therefore, the true benefit of thermal energy storage must be quantified in terms of the savings achieved for the larger electrical system. The real-time value of electricity, for example, is often reflected in a time-of-use pricing structure, where electricity costs more during peak times of the day. If a simple time-of-use pricing structure is applied to this district cooling system, the savings change significantly. The price structure used in this case study is \$0.1/kWh during off peak times and \$0.2/kWh during peak times (12:00 PM to 8:00 PM). When this pricing structure is applied with an objective to minimize total cost, rather than total energy, the savings from optimization and thermal energy storage increases to as much as 17.4%, as Table 2 indicates.

**Table 2: Daily cost for the three cases with an objective to minimize total cost subject to time of use electricity pricing.**

	Case 1	Case 2	Case 3
<b>Equal Ratio Chiller Loading</b>	\$23,600	\$28,600	\$39,500
<b>Static Optimal Chiller Loading</b>	\$21,800	\$26,500	\$37,000
<b>Dynamic Optimal Chiller Loading</b>	\$19,500	\$23,900	\$34,500
<b>Total Savings</b>	<b>17.4%</b>	<b>16.4%</b>	<b>12.7%</b>

## VII. CONCLUSIONS

Optimization can be one of the most cost-effective methods to improve a utility network. For a cooling network with multiple chillers, several degrees of freedom exist, allowing an optimization scheme to dictate which chillers should be used and their corresponding cooling loads. The addition of thermal energy storage to a cooling network can also have a profound impact. While it does require some capital investment, a thermal energy storage tank is significantly less expensive than an industrial scale chiller, yet it can shift load to off-peak hours. Thermal energy

storage also provides more degrees of freedom to a system, which can be exploited through optimization. Thermal storage allows for cooling loads to be shifted temporally, so that the system can take advantage of ambient conditions that are more amenable to efficient chiller operation. However, as this paper has shown, the true value of thermal storage comes by its ability to shift electrical loads, allowing the system to take advantage of less expensive off-peak rates. Thermal energy storage, therefore, can essentially be used as electrical storage, given that a significant portion of the electrical load in most climates is for HVAC purposes.

It has been shown that for a quadratic chiller model the power minimization problem of static optimal chiller loading can be easily solved by a series of QPs, assuming different sets of chillers to be active. The convexity of such problems guarantees a global solution for each active set assumed. This QP problem can be solved very quickly. The dynamic optimal chiller loading problem is much more difficult to solve as it is inherently non-convex and has many degrees of freedom. However, it has been shown that the solution to the static optimal chiller loading problem can be used by the dynamic problem, significantly reducing the number of degrees of freedom and allowing for much faster solution times in addition to a better probability of converging to a global minimum for total energy consumption.

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