## ON-LINE PARAMETER ESTIMATOR OF AN INDUCTION MOTOR AT STANDSTILL

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#### Abstract

This paper describes an automatic identification procedure for an induction motor. The transfer function of the motor at standstill is used to obtain a linear parametric model. An on-line parameter estimator is then derived from this model. In the implementation of the proposed estimator, a PI current controller is constructed to stabilize the current signal and to prevent the flux from saturation. An experiment with a persistently exciting input verifies the theory of the proposed estimator and demonstrates its usefulness in industry applications.

#### 1 Introduction

Recently, many research efforts have been spent in developing high performance AC drives for the induction motor (IM) in accordince with critical industrial demands. As a result of these efforts, a fast dynamic response of induction machine can be achived by the field-oriented control (FOC) [1, 8]. The FOC techniques demand a good motor parameter's knowledge to find an effective decoupling between motor torque and motor flux actuating signals.

An efficient automatic measurement procedure in industry application is developed which has to compromise constructor low-cost needs and customer satisfaction. Therefore the automatic measurement procedure must present simple, user friendly, and the accuracy on measures parameters comparable to that obtained from classical test procedures; no need to mechanical-locking the shift or load-disconnecting [6]. Consider this, a practical inverter system contain with standstill parameters identification scheme is a present trend in drive technology as it allows the automatic set-up of the control system (selfcommissioning) [2].

In the past years, the methods used for identifying the IM parameters at standstill are: fitting of time responses or of the frequency response by means of the linear squares method [5], processing of time responses by the maximum likelihood method [4, 9], considering the saturation effects in the recursive least squares (RLS) method [11], and the adjust model of model reference adaptive system (M-RAS) [3]. A fully-automated procedure for measuring the parameters of an induction motor are performed by the PWM inverter [6].

In this paper an on-line estimator to determine stator resistor, rotor resistor, stator inductance, and mutual inductance is proposed, which also requires the transfer function at standstill. This is an analytic method so that the convergence of the identification procedure is assured. Actually, the theory of the proposed estimator is a basis for adaptive control. Thus, the estimator is easily implemented on a FOC control system of AC drives. The persistently exciting input signal is suggested to verify the theory in the experiment. Furthermore, a feedback current control loop is added in the implementation of the estimator and then the persistently exciting voltage signal is generated by the feedback current control loop to constrain the current under rated value. Experimental results validate the identification procedure with good agreement between estimated and nominal electrical parameters.

# 2 Model of an induction motor at standstill

The mathematical model of an induction motor in a stator-fixed frame  $(\alpha, \beta)$  can be described by five nonlinear differential equations with four electrical variables [stator currents  $(i_{\alpha s}, i_{\beta s})$  and rotor fluxes  $(\varphi_{\alpha r}, \varphi_{\beta r})$ ], a mechanical variable [rotor speed  $(\omega_m)$ ], and two control variables [stator voltages  $(u_{\alpha s}, u_{\beta s})$ ] [8] as follows:

$$\dot{\varphi}_{\alpha r} = \frac{L_m}{\tau_r} i_{\alpha s} - \frac{1}{\tau_r} \varphi_{\alpha r} - p \omega_m \varphi_{\beta r} \tag{1}$$

$$\dot{\varphi}_{\beta r} = \frac{L_m}{\tau_r} i_{\beta s} - \frac{1}{\tau_r} \varphi_{\beta r} + p \omega_m \varphi_{\alpha r}$$
(2)

$$\dot{i}_{\alpha s} = -\gamma i_{\alpha s} + \frac{K}{\tau_r} \varphi_{\alpha r} + p K \omega_m \varphi_{\beta r} + \alpha_s u_{\alpha s} \quad (3)$$

$$\dot{i}_{\beta s} = -\gamma i_{\beta s} + \frac{K}{\tau_r} \varphi_{\beta r} - pK\omega_m \varphi_{\alpha r} + \alpha_s u_{\beta s} \quad (4)$$

$$\dot{\omega}_m = -\frac{B}{J}\omega_m + \frac{T_e}{J} - \frac{T_L}{J} \tag{5}$$

where  $R_s$  and  $R_r$  are the stator and rotor resistance,  $L_s$ ,  $L_r$ , and  $L_m$  are the stator, rotor, and mutual inductance, B and J are the friction coefficient and the the moment of inertial of the motor, p is the number of polepairs. Furthermore,  $\tau_r = L_r/R_r$  is the rotor time constant and the parameters used in (1)-(5) are defined as  $\sigma \equiv 1 - M^2/(L_s L_r)$ ,  $K \equiv L_m/(\sigma L_s L_r)$ ,  $\alpha_s \equiv 1/(\sigma L_s)$ ,  $\gamma \equiv R_s/(\sigma L_s) + R_r L_m^2/(\sigma L_s L_r^2)$ , and  $\mu \equiv p L_m/(J L_r)$ .

Now, consider the IM at standstill, i.e., the IM is controlled to produce zero torque, so that the motor is at standstill with  $\omega_m = 0$ . This can be achieved by magnetizing the IM in the  $\beta$ -axis. Under such a circumstance,  $u_{\alpha s}$ ,  $i_{\alpha s}$ , and  $\varphi_{\alpha s}$  are all zero. Thus, it follows from (1)-(4) that the model of an IM at standstill consists of only the state space equations along the  $\beta$ -axis:

$$\dot{\varphi}_{\beta r} = -\frac{1}{\tau_r}\varphi_{\beta r} + \frac{L_m}{\tau_r}i_{\beta s} \tag{6}$$

$$\dot{i}_{\beta s} = -\gamma i_{\beta s} + \frac{K}{\tau_r} \varphi_{\beta r} + \alpha_s u_{\beta s} \tag{7}$$

Taking Laplace transforms fo both sides of (6) and (7) and then substituting the result of (6) into that of (7), we obtain the transfer function of the present system as

$$\frac{i_{\beta s}}{u_{\beta s}} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{8}$$

where

$$\begin{cases}
 a_1 = (R_s L_r + R_r L_s)/(\sigma L_s L_r) \\
 a_0 = R_s R_r/(\sigma L_r L_s) \\
 b_1 = 1/(\sigma L_s) \\
 b_0 = R_r/(\sigma L_r L_s)
\end{cases}$$
(9)

These four parameters will be identified by an on-line parameter estimator described in the next sectin.

#### 3 On-line parameter estimator

To derive an on-line parameter estimator, we should transform (8) to a linear parametric model. Since the system is second-order, it needs a second-order filter for the transformation. It is, however, desirable to make the order of the resulting linear parametric model as low as apossible. This motivates us to use the filter of  $\Lambda(s) = (s + h_1)(s + h_0)$ . Let  $z \equiv s^2 i_{\beta s}/\Lambda(s)$ . It then follows from (8) that

$$z = \begin{bmatrix} -a_1 & -a_0 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} si_{\beta s}/\Lambda(s) \\ i_{\beta s}/\Lambda(s) \\ su_{\beta s}/\Lambda(s) \\ u_{\beta s}/\Lambda(s) \end{bmatrix}$$
(10)

According to the definition of z, it is apparent that

$$i_{\beta s} = z + \frac{(h_1 + h_0)s + h_1 h_0}{\Lambda(s)} i_{\beta s} = \frac{(h_1 + h_0 - a_1)s + (h_1 h_0 - a_0)}{\Lambda(s)} i_{\beta s} + \frac{b_1 s + b_0}{\Lambda(s)} u_{\beta s}$$
(11)

which leads to the linear parametric model as follows.

$$i_{\beta s} = \boldsymbol{\theta}^{*T} \mathbf{w} \tag{12}$$

where

$$\boldsymbol{\theta}^{*} = \begin{bmatrix} c_{1} \\ c_{2} \\ c_{3} \\ c_{4} \end{bmatrix} = \begin{bmatrix} \frac{b_{0} - b_{1}h_{1}}{h_{0} - h_{1}} \\ \frac{b_{1}h_{0} - b_{0}}{h_{0} - h_{1}} \\ \frac{-a_{0} + a_{1}h_{1} - h_{1}^{2}}{h_{0} - h_{1}} \\ \frac{a_{0} - a_{1}h_{0} + h_{0}^{2}}{h_{0} - h_{1}} \end{bmatrix}, \quad (13)$$
$$\mathbf{w} = \begin{bmatrix} w_{1} \\ w_{2} \\ w_{3} \\ w_{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{s + h_{1}}u_{\beta s} \\ \frac{1}{s + h_{0}}u_{\beta s} \\ \frac{1}{s + h_{1}}i_{\beta s} \\ \frac{1}{s + h_{0}}i_{\beta s} \end{bmatrix}$$

Note  $\boldsymbol{\theta}^*$  is the vector of the parameters to be esimated and  $\mathbf{w}$  is the vector of measured signals. The measured singals in (14) is only first-order, instead of second-order. This is because we took a factored second-order filter  $\Lambda(s)$ in advance.

Let the estimate of  $i_{\beta s}$  be  $\hat{i}_{\beta s}$ :

$$\hat{i}_{\beta s} = \hat{\boldsymbol{\theta}}^T \mathbf{w} \tag{15}$$

where  $\hat{\boldsymbol{\theta}}$  is the estimate of  $\boldsymbol{\theta}^*$ . Moreover, define a normalized estimation error as

$$\varepsilon = \frac{i_{\beta s} - \hat{i}_{\beta s}}{m^2} \tag{16}$$

where  $m^2 = 1 + n_s^2$  with  $n_s^2 = \alpha \mathbf{w}^T \mathbf{w}$  and  $\alpha > 0$ . The purpose of *m* is to make  $\mathbf{w}/m$  bounded. We further define a quadratic cost function as

$$J(\hat{\boldsymbol{\theta}}) = \frac{\varepsilon^2 m^2}{2} = \frac{(i_{\beta s} - \hat{\boldsymbol{\theta}}^T \mathbf{w})^2}{2m^2}$$
(17)

The gradient method to minimize  $J(\hat{\theta})$  is the trajectory of

$$\hat{\boldsymbol{\theta}} = -\Gamma \nabla J(\hat{\boldsymbol{\theta}}) = \Gamma \varepsilon \mathbf{w}$$
 (18)

where  $\Gamma$  is a diagonal matrix with positive diagonal entries. Equation (18) is then used as the gradient on-line parameter estimator for (15).

It is shown in [7] that if **w** is bounded and PE (persistently exciting), then  $\hat{\boldsymbol{\theta}}$  converges exponentially to  $\boldsymbol{\theta}^*$ . Since the

filter  $\lambda$  is a stable trasfer function, **w** is bounded if  $u_{\beta s}$ and  $i_{\beta s}$  are bounded.  $u_{\beta s}$  is the input signal and is given by the user, while  $i_{\beta s}$  is the output of a physical system whose response to a finite input is still finite. Thus, we can simply assume that the boundedness of **w** is always satsified.

The PE property of  $\mathbf{w}$  can be related to the sufficient richness of  $u_{\beta s}$  [10]. A simple result is that  $\mathbf{w} \in \Re^4$  is PE if and only if  $u_{\beta s}$  is sufficiently rich of oder 4. According to the definition, the signal  $u_{\beta s}$  is sufficiently rich of oder n = 4 if it consists of at least n/2 = 2 distinct frequencies. It is then not difficult to construct the input signal so that  $\mathbf{w}$  is PE.

The above proposed on-line estimator is used to obtain a convergent value of  $\hat{\theta}$ . The next step is to calculate the coefficients of (8) from the value of the estimate  $\hat{\theta}$  by

$$\begin{cases}
\hat{a}_{1} = h_{1} + h_{0} - \hat{c}_{3} - \hat{c}_{4} \\
\hat{a}_{0} = h_{1}h_{0} - h_{1}\hat{c}_{4} - h_{0}\hat{c}_{3} \\
\hat{b}_{1} = \hat{c}_{1} + \hat{c}_{2} \\
\hat{b}_{0} = h_{0}\hat{c}_{1} + h_{1}\hat{c}_{2}
\end{cases}$$
(19)

which follows from (14), where  $\hat{c}_i$  are the entries of  $\hat{\boldsymbol{\theta}}^*$ . The inverse relation of (9) allows us to obtain the parameter estimates as

$$\begin{cases}
\hat{R}_{s} = \hat{a}_{0}/\hat{b}_{0} \\
\hat{R}_{r} = \hat{a}_{1}/\hat{b}_{1} - \hat{R}_{s} \\
\hat{L}_{s} = \hat{L}_{r} = \hat{R}_{r}\hat{b}_{1}/\hat{b}_{0} \\
\hat{L}_{m} = \sqrt{\hat{L}_{s}^{2} - \hat{L}_{s}/\hat{b}_{1}}
\end{cases}$$
(20)

Alternatively, we can combine (19) and (20) to directly calculate out the parameter estimates as follows.

$$\hat{R}_s = (h_1 h_0 - h_0 \hat{c}_3 - h_1 \hat{c}_4) / (h_0 \hat{c}_1 + h_1 \hat{c}_2) \quad (21)$$

$$R_r = (h_1 - h_0 - \hat{c}_3 - \hat{c}_4) / (\hat{c}_1 + \hat{c}_2) - R_s \quad (22)$$

$$\hat{L}_s = \hat{L}_r = \hat{R}_r (\hat{c}_1 + \hat{c}_2) / (h_0 \hat{c}_1 + h_1 \hat{c}_2)$$
 (23)

$$\hat{M} = \sqrt{\hat{L}_s^2 - \hat{L}_s / (\hat{c}_1 + \hat{c}_2)}$$
(24)

#### 4 Experiment

The experimental system is a PC-based control system. A servo control card on the ISA bus of the PC provides eight A/D converters, four D/A converters, and an encoder counter. The ramp comparison modulation circuit is used to generate the PWM for driving the IGBT module inverter. The sampling time for the adaptive identification is 0.3ms. The induction motor in the experimental system is a 4-pole, 5HP, and 220V with the rated current 14 A, rated speed 1700 rpm, and rated torque 20.95 Nm.

In our rigorious concept , the overall impedence of IM at standstill is smaller for the sake of the senondary-side

is taken as short-circuit while IM motor equavilent circuit model is applied, which inducing the current is more sensitive to the input voltage variation. Little voltage incresement will enlarge the output current of IM which will be simpler falling into the condition of flux saturation. If the saturation flux is excited by over current, then the IM parameter identification accuracy will be influenced. To alleviate this problem, we implemented the proposed parameter estimator on a current-controlled PWM inverter, which is shown in Fig. 1. In the experiment, the input of voltage  $v_{\alpha s}$  is always set to be zero, whereas  $v_{\beta s}$  is generated by a PI controller as follows:

$$u_{\beta s} = (k_p + k_i/s)(i^*_{\beta s} - i_{\beta s}) \tag{25}$$

where  $k_p$  and  $k_i$  are the regulator gains. The feedback signal  $i_{\beta s}$  is the measured current by a Hall sensor. A dead-time compensator [6] is also considered in this experimental system in order to reduce the effects of the inverter dead-time. The input current command is



Figure 1: Block diagram of the identifying implement of an induction motor at standstill.

 $i_{\beta s}^* = 2\sin(t) + 4\sin(4t) + 8\sin(8t)$ , which consists fo three distinct frequencies and can make the signals **w** PE. The parameters of the filter in the parameter estimator are  $h_0 = 30$  and  $h_1 = 80$ , while the PI controller has the gains of  $k_p = 1.6$  and  $k_i = 116$ .

The experimental results are reported in Fig. 2 to Fig. 4. The history of the current command and that of the measured one are shown Fig. 2. They match very well. It can be seen from Fig. 3 that the estimates  $\hat{c}_i$  converge to some values. The stead-state values aare recorded as  $\hat{c}_1 = 0.23$ ,  $\hat{c}_2 = 0.22$ ,  $\hat{c}_3 = 0.34$ , and  $\hat{c}_4 = 0.32$ . We then use (24) to obtain the parameter estimates  $\hat{R}_s = 0.5$ ,  $\hat{R}_r = 0.65$ ,  $\hat{L}_s = \hat{L}_r = 0.064$ ,  $\hat{L}_m = 0.05$ . Besides, the estimate histories of the electrical parameters are also depicted in Fig. 4. As a conclusion, we found that the resulting estimates of the electrical parameters are very close to the nominal values given by the manufacturer.

#### 5 Conclusions

An approach based on the adaptive gradient algorithm has been developed for identifying the IM parameters in PWM inverter-fed drivers at standstill. A linear regression model of the induction motor at standstill is obtained taking in zero torque which is generated in the only  $\beta$ -axis magnetized voltage applied condition. Identification scheme of adaptive gradient algorithm is implement based on the the parametric form of linear model of the induction motor. Persistent exciting propriety of the input voltage waveform is ensure the the parameters' convergence property. So rich input signal is required in the adaptive identifying procedure. For inhibit the over flux saturation to influrence the identification results, the  $\beta$ -axis current contrl loop controller of IM is developed to overcome this problem. The identification procedure system is implemented in a PC-based controller to drive a 5HP induction motor. Experimental results show the parameters identifying with good agreement between estimated and nominal electrical parameters.



Figure 2: Identification procedure input signal: (a) command current, (b) measured current.



Figure 3: Identification procedure of the estimated parameters , (a)  $\hat{c}_1$ , (b)  $\hat{c}_2$ , (c)  $\hat{c}_3$ , and (d)  $\hat{c}_4$ .

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Figure 4: Identification procedure of the IM parameters: (a)  $\hat{R}_s$ , (b)  $\hat{R}_r$ , (c)  $\hat{L}_s$ , and (d)  $\hat{L}_m$ .

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