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Nonlinear model predictive control strategies applied to a fed-bath sugar crystallizer

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Abstract

The present work is focused on a comparative study of two nonlinear MPC (NMPC) control schemes implemented to a fed-batch sugar crystallization process – i) NMPC that does not exploit the batch nature of the process (termed as classical NMPC) and ii) the batch NMPC that takes into account the end-point control objectives. They are also compared with the classical PI controller and a linear MPC scheme. Two main scenarios are considered: a nominal case without disturbances and a case with disturbances and variations in the initial conditions. The results demonstrate that the batch NMPC outperforms the other control structures but to the expense of high computational cost.

Keywords: nonlinear model predictive control, fed-batch sugar crystallization, shrinking nonlinear discrete time optimization

1. Introduction

During the last decade the model based predictive control (MPC) became an attractive control strategy implemented in a variety of process industries. However, it can be considered as industrial alternative only for continuous and predominantly linear processes (Qin and Badgewell, 2003). The application of MPC for batch nonlinear cases is still far from being an industrial reality and represents an interesting theoretical and practical control challenge (Balasubramhanya and Doyle, 2000). The batch or fed-batch mode is a typical production scheme for a large group of pharmaceutical, biotechnological, food and chemical processes. It is related with the formulation of a control problem in terms of economic or performance objective at the end of the process (Nagy and Braatz, 2003). For example, the crystallisation quality is evaluated by the particle size distribution (PSD) at the end of the process which is quantified by two parameters - the final average (in mass) particle size (MA) and the final coefficient of particle variation (CV). The main challenge of the batch production is the large batch to batch variation of the final PSD. This lack of process repeatability is caused mainly by improper control policy and results in product recycling and loss increase. MPC, being one of the approaches that inherently can cope with process constraints, nonlinearities, and different objectives derived from economical or environmental considerations, has the potential to overcome the problem of the lack of repeatability and drive the process to its optimal state of profit maximization and cost minimization. These problems are the

main motivation for the present work, which is divided into the following sections. In section 2 the sugar production stages are shortly presented. In section 3 the crystallization phenomenological macro model is introduced. In section 4 the nonlinear MPC strategies are formulated and finally in section 5 and 6 the results of the tests with the two control paradigms are commented and conclusions are summarised.

2. Process description

Sugar crystallisation occurs through the mechanisms of nucleation, growth and agglomeration. There are two basic types of sugar production: from cane sugar or from beet. The process considered in this work is of the first type and a typical industrial unit can be divided into the following sequential phases.

Charging. During the first phase the pan is partially filled with a juice containing dissolved sucrose (termed liquor). The initial liquid charged in the pan corresponds approximately to 40% of the total vessel height. The charge is usually performed by complete opening of the feeding valve until the level sensor indicates 40%. Therefore, no special control policy is required at this stage.

Concentration. The next phase is the concentration. The liquor is concentrated by evaporation, under vacuum, until the supersaturation reaches a predefined value. At this value seed crystals are introduced into the pan to start the production of crystals. This is the beginning of the crystallisation phase.

Crystallisation (main phase)**.** At this phase as evaporation takes place further liquor is added to the pan in order to guarantee crystal growth at a controlled supersaturation level and to increase the sugar content of the pan. Near to the end of this phase and for economical reasons, the liquor is replaced by other juice of lower purity (termed syrup).

Tightening**.** The fourth phase consists of tightening which is principally controlled by evaporation capacity. The pan is filled with a suspension of sugar crystals in heavy syrup, which is dropped into a storage mixer. At the end of the batch, the massecuite undergoes centrifugation, where final refined sugar is separated from (mother) liquor that is recycled to the process.

The different phases are comparatively independent, they have different driving forces and efforts to derive a single controller for all of them are most likely to fail. From all of the phases the crystallisation is the most challenging one because of its strong non-linear and non-stationary behaviour. Moreover, the crystallization phenomenon is typical in a great number of industrial processes such as pharmaceutical and food engineering and is responsible for the final product quality expressed in terms of PSD. Therefore, the objective of the present work is to derive an efficient operation strategy specifically for the crystallization phase.

3. Crystallization macro model

The general phenomenological model of the fed-batch crystallization process consists of mass, energy and population balances (Georgieva et al., 2003). While the mass and energy balances are common expressions in many chemical process models, the population balance is related with the crystallization phenomenon which is still an open modelling problem.

Mass balance:

The mass of all participating solid and dissolved substances are included in a set of conservation mass balance equations

$$\dot{M} = f(M(t), F(t), P1), \quad 0 \le t \le t_f, \quad M(0) = M_0 \tag{1}$$

where $M(t) \in R^q$ and $F(t) \in R^m$ are the mass and the flow rate vectors, with q and m dimensions respectively, and t_f is the final batch time. $P1$ is the vector of physical parameters as density, viscosity, purity, ect.

Energy balance:

The general energy (E) balance model is

$$\dot{E} = f(E(t), M(t), F(t), P2), \quad 0 \le t \le t_f, \quad E(0) = E_0 \tag{2}$$

where $P2$ incorporates the enthalpy terms and specific heat capacities derived as functions of physical and thermodynamic properties.

Population balance:

Mathematical representation of the crystallization rate can be achieved through basic mass transfer considerations (Galvanauskas et al., 1998) or by writing a population balance represented by its moment equations (Georgieva et al, 2003). Employing a population balance is generally preferred since it allows to take into account initial experimental distributions and, most significantly, to consider complex mechanisms such as those of size dispersion and/or particle agglomeration/aggregation. Hence

$$\dot{\eta}_i = f(\eta_i(t), \tilde{B}_0, G, \beta'), \quad 0 \le t \le t_f, \quad i = 0,1,2,....\eta_i(0) = \eta_{i0} \tag{3}$$

where η_i is the j-th moment of the mass-size particle distribution function, $\tilde{B}_0$, G and β' are the kinetic variables nucleation rate, linear growth rate and the agglomeration kernel, respectively. The PSD measures: the final average (in mass) particle size (MA) and the final coefficient of particle variation (CV) are derived from (3) as follows

$$\mathrm{MA} = \eta_1/\eta_0 \tag{4.1}$$

$$\mathrm{CV} = \left(\eta_0\eta_2/\eta_1^2 - 1\right)^{1/2} \tag{4.2}$$

For more details with respect to the detailed process model see Georgieva et al, 2003.

4. Nonlinear MPC and batch nonlinear MPC – discrete time formulation

4.1 Nonlinear MPC

Nonlinear model predictive control (NMPC) is an optimisation-based multivariable constrained control technique that uses a nonlinear dynamic model for the prediction of the process outputs (Nagy and Braatz, 2003). At each sampling time the predictions are updated on the basis of new measurements and state variable estimates. Then the open-loop optimal manipulated variable moves are calculated over a finite prediction horizon with respect to some cost function, and the manipulated variables for the subsequent prediction horizon are implemented. Then the prediction horizon is shifted by usually one sampling time into the future, and the previous steps are repeated.

Discrete process model

Considering the discrete nature of the on-line control problem the continuous time optimisation problem involved in the NMPC formulation is solved by formulating a discrete approximation to it, that can be handled by conventional nonlinear programming (NLP) solvers (Biegler, 2000). The time horizon $t = [t_0, t_f]$ is divided into equally spaced time intervals Δt, with discrete time steps $t_k = t_0 + k\Delta t$, and $k = 0,1,....,N$. The process model is discretised

$$x(k+1) = F[(x(k), u(k)] \tag{5.1}$$

$$y(k) = h[(x(k)] \tag{5.2}$$

Where x is an n-dimensional vector of state variables, u is a l-dimensional vector of manipulated input variables and y is a p-dimensional vector of controlled output variables.

Process constraints

Input constraints arise due to actuator limitations such as saturation and rate-of-change restrictions and can be expressed as

$$u_{\min} \le u \le u_{\max} \tag{6}$$

$$\Delta u_{\min} \le \Delta u \le \Delta u_{\max}\ , \tag{7}$$

where $u_{\min}$ and $u_{\max}$ are the minimum and the maximum values of the input u, $\Delta u_{\min}$ and $\Delta u_{\max}$ are the minimum and the maximum values of the rate-of-change of the same input.
Output constraints are usually associated with operational limitations such as equipment specifications and safety considerations and can be expressed as:

$$y_{\min} \le y \le y_{\max}, \tag{8}$$

where $y_{\min}$ and $y_{\max}$ are the minimum and maximum values of the output y. Constraints on the state variables also may be specified if appropriate.

Optimal control problem

The discrete time formulation of the optimal control problem is (Rawlings et al., 1994).

$$\min_{u(k|k),u(k+1|k),\ldots,u(k+H_u-1|k)} J = \phi\left(y(k+H_p|k)\right) + \sum_{j=0}^{H_p-1} L\left(y(k+j|k), \Delta u(k+j|k)\right) \tag{9}$$

where J is the performance function, $\Delta u(k+j|k) = u(k+j|k) - u(k+j-1|k)$, $u(k+j|k)$ is the future input $u(k+j)$ calculated at time k, $y(k+j|k)$ is the output $y(k+j)$ calculated from information available at time k, H_u is the control horizon, H_p is the prediction horizon and ϕ and L are (possibly) nonlinear functions of their arguments. The optimisation problem (9) is solved subject to constraints (6-8) which are reformulated to fit the iterative procedure:

$$u_{\min} \le u(t+j|k) \le u_{\max} \qquad 0 \le j \le H_u - 1 \tag{10}$$

$$\Delta u_{\min} \le \Delta u(k+j|k) \le \Delta u_{\max} \qquad 0 \le j \le H_u - 1 \tag{11}$$

$$y_{\min} \le y(k+j|k) \le y_{\max} \qquad 1 \le j \le H_p \tag{12}$$

In addition, the nonlinear model is also considered as a set of equality constraints,

$$x(k+j+1|k) = F\left[x(k+j|k), u(k+j|k)\right] \qquad 0 \le j \le H_p - 1 \tag{13}$$

$$y(k+j|k) = h\left[x(k+j|k)\right] \qquad 1 \le j \le H_p \tag{14}$$

(9) is a quite general form to express a wide range of objectives encountered in NMPC applications, however, a quadratic function for L is the most typical form

$$L = \left[y(k+j|k) - r(k)\right]^T Q\left[y(k+j|k) - r(k)\right] + \Delta \mathrm{u}^{\mathrm{T}}(k+j|k) S \Delta \mathrm{u}(k+j|k) \tag{15}$$

Where $r(k)$ is the steady-stage target for $y(k)$ and Q, and S are positive-definite weighting matrices. The predicted outputs are obtained from the non-linear model as follows

$$\begin{aligned} y(k+1|k) &= h\left[x(k+1|k)\right] = h\left[F\left[x(k|k), u(k|k)\right]\right] \equiv G_1\left[x(k), u(k|k)\right] \\ y(k+2|k) &= G_1\left[x(k+1|k), u(k+1|k)\right] = G_1\left[F\left[x(k|k), u(k|k)\right], u(k+1|k)\right] \\ &\equiv G_2\left[x(k), u(k|k), u(k+1|k)\right], \\ &\vdots \\ y(k+j|k) &= G_j\left[x(k), u(k|k), u(k+1|k), \ldots, u(k+j-1|k)\right], \end{aligned} \tag{16}$$

where $x(k|k) = x(k)$ is the vector of current state variables and G_j are nonlinear functions. If the control horizon (H_u) is less than the prediction horizon (H_p), the output predictions are generated by setting inputs beyond the control horizon equal to the last computed value: $u(k+j|k) = u(k+H_u-1|k)$. Only the first input vector in the sequence is implemented $u(k) = u(k|k)$. Then the prediction horizon is moved forward one time step, and the problem is resolved using new process measurements. This receding horizon formulation yields improved closed-loop performance in the presence of unmeasured disturbances and modelling errors.

4.2 Batch nonlinear MPC

The batch nonlinear MPC can be seen as a particular case of NMPC that reduces (9) to the Mayer form ($L(\cdot) = 0$). However, the main impact of the full batch NMPC formulation is that in order to reflect the end-point performance objective the optimisation problem has to be solved iteratively online, in shrinking horizon ($t_F = t_f$) approach, where t_f is the batch time. This means that at the beginning of the optimisation the prediction horizon is equal to the envisaged process duration and at each iteration it is reduced.

Closed loop process performance metrics

The closed loop process performance is quantified by the following metrics:

- *Relative Mean Square (RMS) error*

$$RMS_i = \sqrt{\frac{\sum_{k=1}^{N}\left(y_i(k) - r_i(k)\right)^2}{\sum_{k=1}^{N}\left(r_i(k)\right)^2}} \,. \tag{17}$$

- *Average Control Effort (ACE)*

$$ACE_j = \frac{\sum_{k=2}^{N}\left(u_j(k) - u_j(k-1)\right)^2}{N} \,. \tag{18}$$

y_i and u_j are the *i*-th output and the *j*-th input of the process respectively, r_i is the target for y_i and N is the total number of samples.

- *Weighted mixed performance index* (I_p)

$$I_p = \sum_i \lambda_i RMS_i + \sum_j \beta_j ACE_j \,.\ \{\lambda_i, \beta_j\} \in \Re \geq 0 \tag{19}$$

The values of λ_i, β_j are set as to compensate the magnitude orders of the different terms in (19). Note that smaller the values of RMS, ACE, I_p, better the overall closed loop performance achieved.

5. Nonlinear MPC control tests

Our objective is to define an efficient control strategy for the crystallization phase of an industrial sugar production process based on the NMPC concept. This phase starts when seed crystals are introduced into the pan and goes through three sub-stages where either the controlled variable or the manipulated variable is changed. Therefore, a sequence of NMPCs is designed to reflect the relevant input-output relationships along the crystallization phase. During the *first control loop* the controlled variable is the supersaturation and the manipulated variable is the liquor feed flowrate. When the amount of liquor is over the supersaturation is controlled by manipulating the steam flowrate (*second control loop*). At the end of the crystallization phase the controlled variable is switched to the volume fraction of crystals and for economical reasons the manipulated variable is the feed flowrate of a juice with lower purity termed syrup (*third control loop*).

5.1 Influence of feed flow parameters

Practical experience indicates that the characteristics (purity and brix) of the feeding have a strong effect on the crystallization path. In order to test the NMPC robustness against the most influencing variations of these parameters, an open loop test was first designed with constant (mean) values set at the process inputs, i.e. the liquor/syrup feedflow rate $F_f = 0.01\,\mathrm{m}^3/\mathrm{s}$ and the steam flowrate

$F_s = 1.5833$ m^3/s. First changes in the purity of liquor/ syrup were simulated keeping the brix of the feeding flows (liquor or syrup) at their most typical values and then vice versa. The results collected in Table 1 and 2, show clearly that the most influencing parameter is the purity of syrup. Note, that it's variations in the interval [0.85 0.95], which are rather common in practice, lead to significant changes in the final CSD characteristics (AM and CV). Since variations in the other parameters do not provoke a considerable effect, in the next simulations they were fixed at their most expected values.

Table 1 Effect of changes in the purity of liquor/ syrup

Purity of liquor	0.99	0.99	0.99	0.999
Purity of syrup	0.9	0.85	0.95	0.9
Brix of liquor	72			
Brix of syrup	77			
Maximum Supersaturation	1.14	1.22	1.14	1.14
Maximum Temperature of massecuite	76.76	76.5	76.73	76.75
Final (averaged in mass) crystal size (MA)	0.6	0.56	0.79	0.6
Final Coefficient of size variation (CV)	38.14	39.15	28.98	38.14
Final Volume	35.22	34.77	35.27	35.22
Final Mass fraction of crystals	0.6	0.58	0.61	0.6

Table 2 Effect of changes in the brix of liquor/ syrup

Purity of liquor	0.99			
Purity of syrup	0.9			
Brix of liquor	71	73	72	72
Brix of syrup	77	77	76	80
Max. Supersaturation	1.14	1.14	1.14	1.14
Max. Temperature of massecuite	76.76	76.76	76.57	77.27
Final (averaged) crystal size (MA)	0.6	0.6	0.61	0.61
Final Coefficient Variation (CV)	38.14	38.14	37.93	37.45
Final Volume	35.22	35.22	34.74	35.01
Final Mass fraction of crystals	0.60	0.60	0.59	0.64

5.2 Operation without disturbances (Scenario 1)

Test A: $H_u = 1$, $H_p = [3,5,8,10,20]$

The focus of this study is not only to analyse if the control strategy is able to fulfil the performance objectives but to evaluate what would be the computational cost and if the NMPC is feasible in a real-time application. Therefore, the first scenario of operation is the nominal case without disturbance and noise. At the first test (A) the control horizon is fixed ($H_u = 1$) and the prediction horizon varies. The results are summarised in Table 3, where the RMS error lines correspond to the output performance metrics of the three control loops and the ACE lines quantify the two control input efforts. It is easy to see that H_p variation does not affect the general process performance expressed in terms of maximum values of key variables as supersaturation, temperature in the pan and the quality of the final product (MA, CV, final mass fraction of crystals). However, the batch duration, the average and the maximum computational time per iteration are affected by the prediction horizon. It is worth to note that while the average CPU time is

proportional to H_p, the relation between the maximum CPU time and H_p is an exponentional curve. Since the sampling time in the real plant is usually 10 s., $H_p = 20$ is not an acceptable value.

Test B : $H_u = 3$, $H_p = 8$, Purity of syrup = [0.85, 0.9, 0.95]

In this test (B) the NMPC robustness to the most influencing feeding parameter is studded. Based on the previous test we choose $H_p = 8$, increase the control horizon to $H_u = 3$ and vary the purity of syrup. Naturally, the average and the maximum CPU time per iteration increase due to the higher value of H_u. The control effort of the feed flow rate is more than 3 times bigger then in test A and this is confirmed by the curly trajectory of F_f depicted in Fig.1a. Note that the general NMPC performance, defined by (17-18), is practically not affected by the purity of syrup variations.

Table 3 Results of Test A

H_p	3	5	8	10	20
RMS_{S1} (*first control loop*)	3.72E-04	4.48E-04	5.85E-04	6.41E-04	9.97E-04
RMS_{S2} (*second control loop*)	1.71E-02	7.36E-03	7.52E-03	8.14E-03	1.14E-02
RMS_w (*third control loop*)	2.68E-02	2.51E-02	2.61E-02	2.83E-02	3.53E-02
ACE_{Ff} *(of the feed flowrate)*	1.33E-08	1.14E-08	9.89E-09	9.04E-09	9.16E-09
ACE_{Fs} *(of the steam flowrate)*	5.64E-06	6.63E-06	6.67E-06	6.65E-06	6.54E-06
Max. supersaturation	1.15	1.14	1.14	1.14	1.15
Max. temperature of massecuite [Cº]	75.22	75.52	75.6	75.54	75.44
Final (averaged) crystal size (MA) [mm]	0.595	0.594	0.588	0.586	0.578
Final coefficient of variation (CV)	33.7	33.59	33.12	33.04	32.87
Final volume [m^3]	33.01	33.0	33.0	33.0	33.01
Final mass fraction of crystals	0.46	0.46	0.46	0.46	0.46
Final time [s]	6400	6271	6078	6064	6005
Average CPU time per iteration [s]	0.36	0.597	1.01	1.29	2.78
Max. CPU time per iteration [s]	0.88	1.05	1.66	2.09	11.91

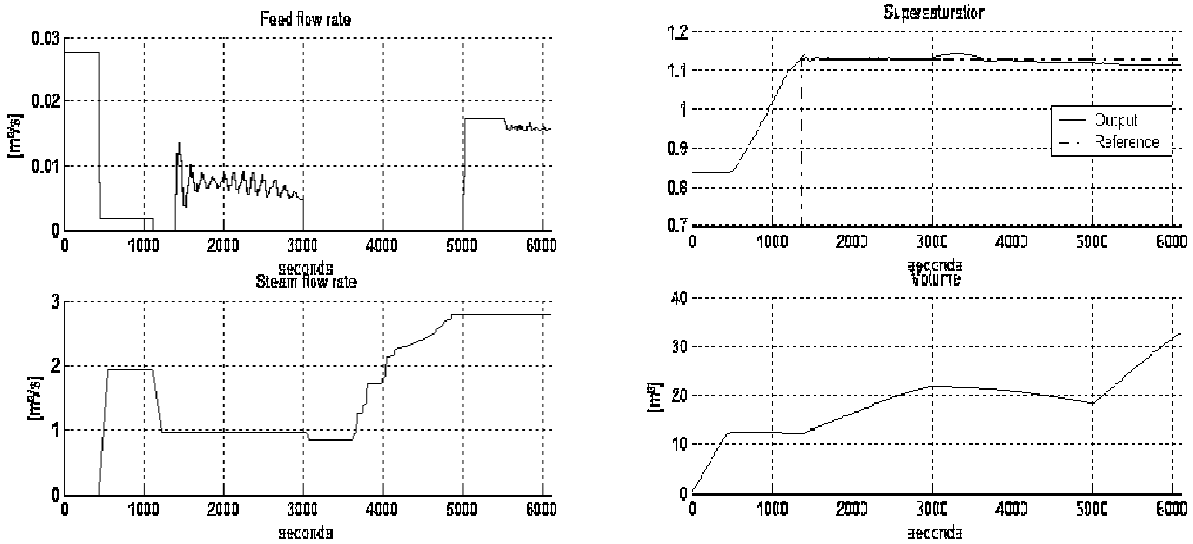
Table 4 Results of Test B

Purity of syrup	0.85	0.9	0.95
RMS_{S1} (*first control loop*)	1.23E-03	1.24E-03	1.23E-03
RMS_{S2} (*second control loop*)	6.40E-03	6.40E-03	6.39E-03
RMS_w (*third control loop*)	2.60E-02	2.64E-02	2.62E-02
ACE_{Ff}	3.63E-08	3.59E-08	3.77E-08
ACE_{Fs}	6.29E-06	6.41E-06	6.43E-06
Max. Supersaturation	1.15	1.14	1.14
Max. temperature of massecuite [Cº]	75.88	75.88	75.88
Final (averaged) crystal size (MA) [mm]	0.55	0.6	0.71
Final coefficient of variation (CV)	35.83	33.31	26.38
Final volume [m^3]	33.0	33.01	33.01
Final Mass fraction of crystals	0.46	0.46	0.46
Final time [s]	6193	6102	6106
Average CPU time per iteration [s]	2.38	2.41	2.47
Max. CPU time per iteration [s]	7.8	5.47	5.56

Test C: Comparison of PI, GPC, NMPC, $H_u = 1$, $H_p = 8$, Purity of syrup = 0.9

Test C is focused on comparison between the proposed in this work NMPC, the industrially most celebrated Proportional-Integrative (PI) controller and the most popular MPC, namely the

Generalized Predictive Controller (GPC) (Qin and Badgewell, 2003). The results, summarised in Table 5, demonstrate slightly better performance of the NMPC approach compared to the PI with respect to I_p, RMS_{S1}, MA and shorter batch duration. Both NMPC and PI control outperform the GPC, which is expected because the latter approach is based on linear model predictions.



a) Manipulated process inputs b) Controlled process outputs

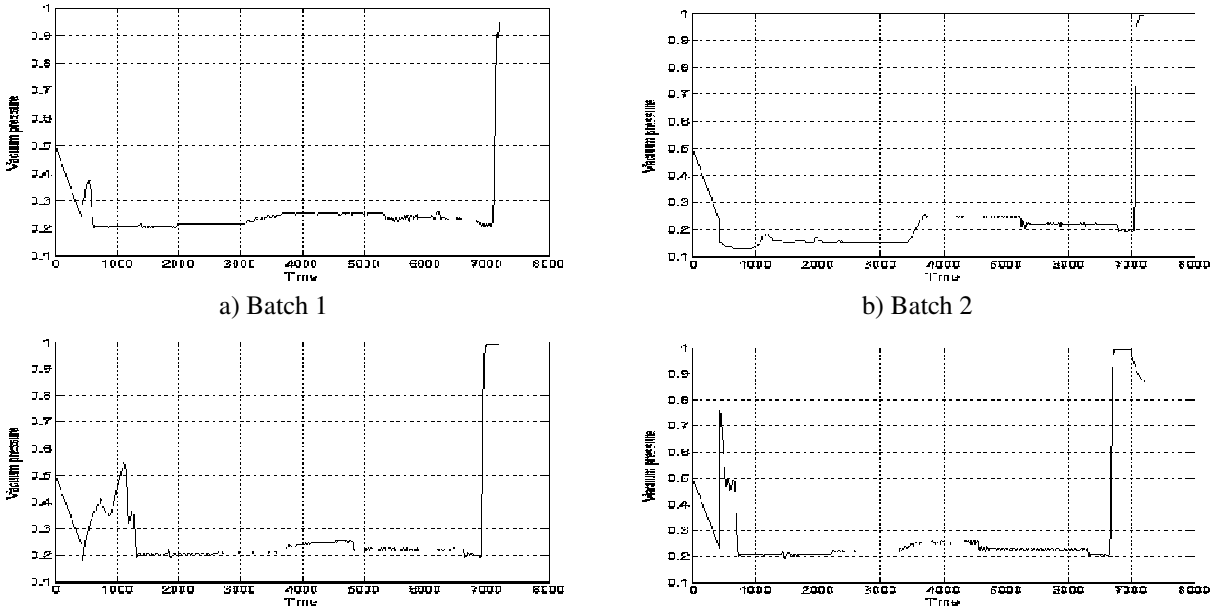
Fig1. Scenario 1, Test B: Input-output process behaviour

Table 5 Results of Test C

Control strategy	PI	GPC	NMPC
RMS_{S1}	1.92E-03	1.38E-03	5.85E-04
RMS_{S2}	5.70E-03	8.00E-03	7.52E-03
RMS_w	2.69E-02	2.58E-02	2.61E-02
ACE_{Ff}	6.81E-09	1.19E-08	9.89E-09
ACE_{Fs}	5.31E-06	5.16E-06	6.67E-06
Max. supersaturation	1.14	1.14	1.14
Max. temperature of massecuite [Cº]	75.84	75.69	75.6
Final (averaged) crystal size (MA) [mm]	0.597	0.596	0.588
Final coefficient of variation (CV)	33.3	33.55	33.12
Final volume [m^3]	33.0	33.0	33.0
Final mass fraction of crystals	0.46	0.46	0.46
Final time	6085	6263	6078
Average CPU time per iteration [s]	2.04E-04	7.60E-04	1.01
Max. CPU time per iteration [s]	0.016	0.235	1.64
$I_p = \lambda_1 RMS_{S1} + \lambda_2 RMS_{S2} + \lambda_3 RMS_w + \lambda_4 ACE_{Ff} + \lambda_5 ACE_{Fs}$			
$\lambda_1 = \lambda_2 = 10^3, \lambda_3 = 10^2, \lambda_4 = 10^8, \lambda_5 = 10^6$	22.44	28.98	21.28

5.3 Operation with vacuum pressure disturbances (Scenario 2)

In the second scenario the NMPC is tested in the presence of typical process disturbances, introduced by four vacuum pressure profiles taken from real industrial data bases (see Fig. 2). The simulations are made with the following parameters: $H_u = 1$, $H_p = 8$, Purity of syrup = 0.9. The NMPC controlled system is able to compensate the fluctuations in the vacuum pressure and the quality metrics summarized in Table 6 are in fact undistinguishable of the unperturbed case (the second column).



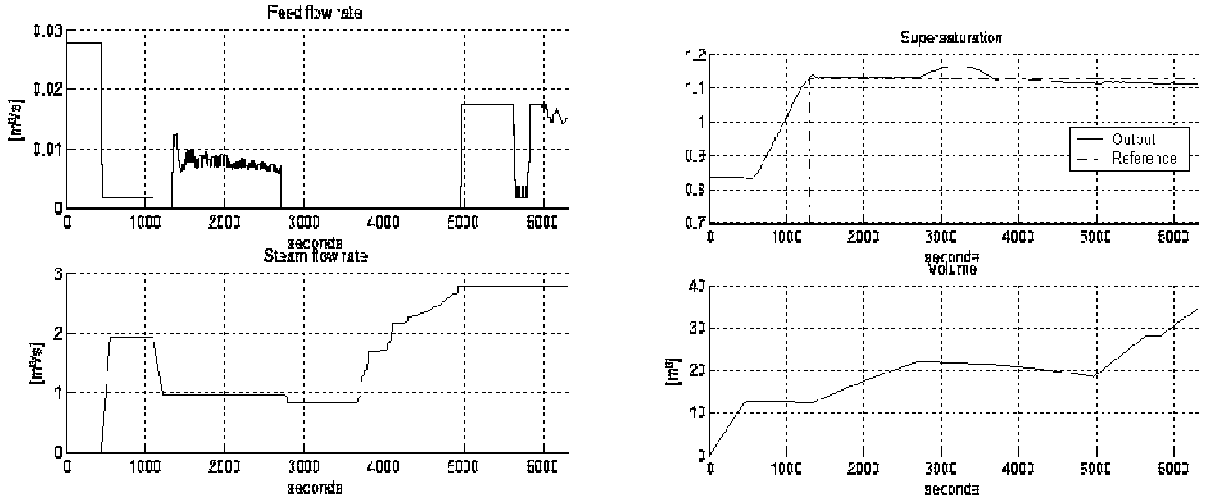
a) Batch 1 b) Batch 2

c) Batch 3 d) Batch 4

Fig. 2 Vacuum pressure real profiles

Table 6 Results of Scenario 2

Case	No perturb.	Perturb. Batch 1	Perturb. Batch 2	Perturb. Batch 3	Perturb. Bath 4
RMS_{S1}	1.23E-03	1.82E-03	5.58E-03	5.65E-03	2.48E-03
RMS_{S2}	6.39E-03	1.60E-02	6.61E-02	2.99E-02	1.96E-02
RMS_{w}	2.62E-02	2.58E-02	2.59E-02	2.75E-02	2.69E-02
ACE_{Ff}	2.98E-08	6.82E-08	9.41E-08	5.39E-08	6.67E-08
ACE_{Fs}	6.43E-06	6.56E-06	6.05E-06	6.84E-06	6.74E-06
Max. Supersaturation	1.1419	1.1647	1.2629	1.1913	1.1728
Max. temperature of massecuite [Cº]	75.8805	77.478	74.838	76.257	76.588
Final (averaged) crystal size (MA) [mm]	0.59	0.6	0.58	0.59	0.59
Final coefficient of variation (CV)	33.29	33.22	34.35	33.8	33.73
Final volume [m^3]	33.0	33.01	33.01	33.0	33.01
Final mass fraction of crystals	0.46	0.46	0.46	0.46	0.46
Final time	6100	6185	6212	5959	6091



a) Manipulated process inputs b) Controlled process outputs

Fig. 3 Scenario 2: Input-output process behaviour

6. Batch nonlinear MPC control tests

The batch NMPC optimization problem is formulated as

$$\min_{u(k|k),u(k+1|k),\ldots,u(k+t_f|k)} J = (MA_{end} - MA_{ref})^2, \quad (20)$$

with the following constrains: the supersaturation ≤ 1.3 (for the first and the second control loop), the final mass fraction of crystals ≥ 0.46 and the final volume ≥ 30 m^3 (for the third control loop) . MA_{end} is the final (averaged) crystal size and MA_{ref} is the respective final desired value. Note that the prediction horizon is equal to the assumed batch duration (t_f) and at each iteration is shrunk. A number of preliminary simulations lead to the following conclusions:

i) The sampling time of 10s., used in the classical NMPC schemes of the previous section, is inadequate for the higher average computational time required at each iteration of the batch NMPC. Therefore, it is changed to 30s. However, sporadically the CPU time becomes much higher than 30s. and in such cases the control input is kept at its last calculated value.

ii) In the batch NMPC mode output predictions up the batch end are required. Therefore, we need to assume manipulated inputs till the end of the process. For inputs beyond the control horizon, constant (mean) values are assigned. During the first control loop the feed flowrate of syrup $F_{f\_syrup} = 0.01 m^3/s$ and the steam flowrate $F_s = 1.5833$ m^3/s. During the second control loop, $F_{f\_syrup} = 0.01 m^3/s$.

The main conclusion of this test is that the batch NMPC achieves quite nice the final objective ($MA_{ref} = 0.55$ mm) for purity of syrup in the range (0.85,0.9), but to the expense of high CPU time that at some iterations can become rather high (see the last line in Table 7) . In Fig. 4 is depicted the history of the CPU time at each iteration along the complete process duration. Obviously, the initial phase is the most demanding and it is at that period that the optimization procedure needs more computational power.

Table 7 Results of batch NMPC

Purity of syrup	0.85	0.9	0.95
Max Supersaturation	1.22	1.20	1.19
Max. temperature of massecuite [Cº]	75.73	75.37	74.88
Final (averaged) crystal size (MA) [mm]	0.56	0.57	0.66
Final coefficient of variation (CV)	35.11	34.04	27.08
Final volume [m^3]	30.0	29.99	30.0
Final Mass fraction of crystals	0.57	0.53	0.50
Average CPU time per iteration [s]	7.2	7.8	6.6
Max. CPU time per iteration [s]	54.3	241.1	21.8

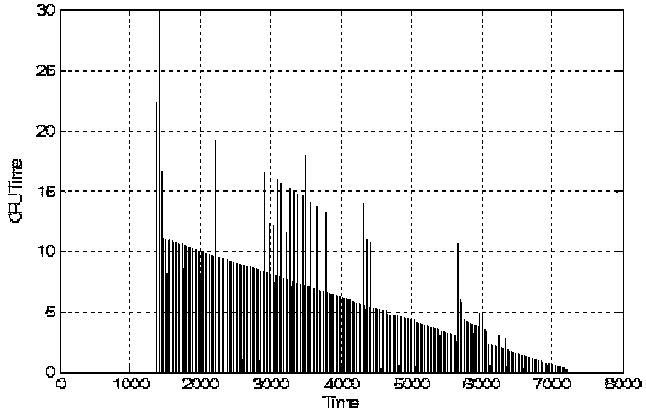


Fig. 4 CPU time per iteration [s] along the process duration

7. Conclusions

Two nonlinear model predictive control (NMPC) strategies are applied to an industrial sugar production process: i) NMPC that does not exploit the batch nature of the process (termed as classical NMPC) and ii) the Batch NMPC (BNMPC) that takes into account the end-point control objectives. They are also compared with the classical PI controller and one linear MPC scheme, namely GPC. A number of tests are performed covering most relevant practical issues as the choice of control and prediction horizons, operation without and with disturbances (in the vacuum pressure) and variations in the initial conditions (purity of syrup of the feeding). The main conclusions are that while the process runs under nominal conditions, NMPC, PI, GPC and DNMPC demonstrate practically insignificant differences and any of these controllers can be tuned to respond satisfactorily. However, in the presence of disturbances or changes in the initial conditions the DNMPC demonstrate better performance with respect to the final product quality (MA). This is due to the explicit consideration of the end point objectives in the performance function (20). However, the price to be paid is a high CPU that forced us to simplify the batch performance function. Work, which is now going on, is to reduce the computational time and to consider not only final time specifications but also tracking objectives over the batch duration in the framework of a more complex cost function.

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