

## **One dimensional modelling of conical spouted beds**

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### **Abstract**

A one-dimensional model that considers the solid cross-flow from the annulus into the spouted has been developed. The model is based on mass and momentum balance equations for the gas and solid in both the annulus and spout and empirical. The great advantage of the model is the requirement of only two empirical equations, one for obtaining the minimum spouting velocity and the other one for obtaining the pressure gradient through the bed. Based on the data obtained at the top of the spout, the model allows for calculating properties in the fountain, namely fountain height and the voidage at the core of the fountain.

Keywords: spouted beds, conical spouted beds, modelling

### **1. Introduction**

Conical spouted beds have good perspectives for operations and processes where gas-solid contact is required, as in addition to the characteristics of the conventional spouted beds (cylindrical with conical bottom) they may operate in a wider range of gas velocities, which allows for attaining a vigorous gas-solid contact (Olazar et al., 1992). Furthermore, conical spouted beds may operate with a wide particle size distribution without segregation (Olazar et al., 1993a; San José et al., 1994). This versatility makes conical spouted beds especially useful for treatment of solids that are difficult to handle due to their irregular texture or because they are sticky. Likewise, conical spouted beds allow for attaining low gas residence times. A good behaviour of conical spouted beds has been proven in combustion of bituminous coals (Tsuji et al., 1989), catalytic polymerizations, treatment of sawdust and agroforest residues (Olazar et al., 1994a) and pyrolysis of waste plastics and tyres (Aguado et al. 2005).

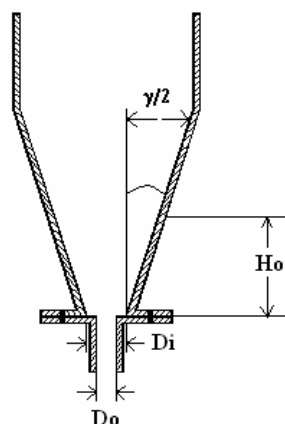
To improve the knowledge about the gas-solid contact in conical spouted beds a mathematical model has been developed. The aim of the model is to obtain velocity distributions of gas and particles in the different regions of conical spouted beds.

Furthermore, design of conical spouted bed reactors for pyrolysis of biomass, plastics and tyres requires to couple the kinetic model with the hydrodynamic one. For this purpose, a simple but reliable hydrodynamic model should be used.

The two most commonly used approaches for modelling spouted beds are discrete element method (DEM) (Kawaguchi et al., 2000) and two fluid model (TFM) (Lu et al., 2001, Krzywansky et al., 1992). In the DEM approach, gas phase is described by a locally average Navier-Stokes equation, the motion of individual particles is traced, and the two phases are coupled by interphase forces. In the TFM approach, the two phases are mathematically treated as interpenetrating continua, and the conservation equations for each phase are derived to obtain a set of equations that have similar structure, which makes the mathematical manipulation of the system relatively easier and computationally less expensive, and thus greatly benefits the practical applications of the TFM approach. The success of the TFM approach depends on the proper description of the interfacial forces and the solid stress. The interfacial forces are used to describe the momentum transfer between the two phases, which has the primary effect on the hydrodynamic behaviour (Du et al., 2006), whereas the solid stress, which represents the solid phase forces due to particle-particle interactions, has the secondary effect.

## 2. Model equations

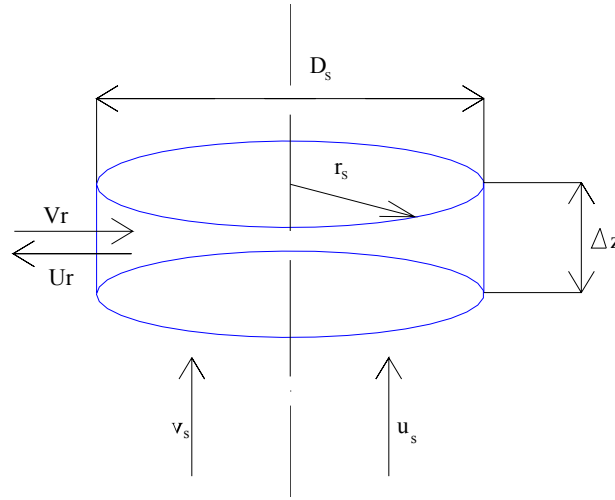
Generally, the gas–solids flow in a spouted bed can be divided into three zones: a spout zone in the center of the bed, where the gas and particles rise at high velocity and the particle concentration is low; an annulus zone between the spout and the wall, where particles move slowly downward; and a fountain zone where the particles rise to their highest positions in the bed and then rain back to the surface of the annulus. Thus, a cyclic pattern of solids movement is established. Figure 1 shows a schematic representation of particle movement in the different regions of a conical spouted bed.



**Figure 1.** Outline of a conical spouted bed reactor

2.1. Conservation equations in the spout zone

Within the spout, air is rushing upwards through particles that have been entrained from the annulus, the relative upward motion between air and particles generating the pressure gradient that both supports the spout and causes leakage of air from spout to annulus. It is therefore necessary to write down equations that allow for the mass acceleration of the air and particles and for the interaction between air and particles.



**Figure 2.** Volume element in the spout zone

Across any horizontal section in the spout, Figure 2, it is assumed that the particles have uniform velocity,  $v_s$ , the air has an absolute (interstitial) velocity,  $u_s$ , and the voidage fractions  $\varepsilon_s$  is uniform. Then the continuity of air flow:

$$\frac{d(\varepsilon_s u_s)}{dz} = -\frac{4U_r}{D_s} \quad (1)$$

and for continuity of particle flow:

$$\frac{d[(1 - \varepsilon_s) \cdot v_s]}{dz} = +\frac{4V_r}{D_s} \quad (2)$$

where  $U_r$  is the velocity representing leakage of air form the spout into the annulus and  $V_r$  is a velocity representing entrainment of particles into the spout.

From a momentum balance on the air:

$$-\rho_g \frac{d(\varepsilon_s \cdot u_s^2)}{dz} - \varepsilon_s \frac{dP}{dz} - \beta \cdot (u_s - v_s) = 0 \quad (3)$$

and a momentum balance on the particle gives:

$$\rho_s \frac{d[v_s^2(1-\varepsilon_s)]}{dz} + (1-\varepsilon_s) \frac{dP}{dz} - \beta(u_s - v_s) + \rho_s g(1-\varepsilon_s) = 0 \quad (4)$$

Here  $\beta$  is the fluid-particle interaction coefficient in drag mode. Du et al. (2006) show that the Gidaspow (1994) drag model gave the best agreement with experimental data. This model uses the Ergun (1952) equation for dense phase calculation and Wen and Leva (1956) for dilute phase calculation, i.e.:

$$\beta_{\text{Ergun}} = 150 \frac{(1-\varepsilon_s)^2 \cdot \mu}{\varepsilon_s (d_p \cdot \phi)^2} + 1.75 \frac{\rho_g (1-\varepsilon_s) \cdot |u-v|}{(d_p \phi)} \quad (5)$$

$$\beta_{\text{Wen-Yu}} = \frac{3}{4} \cdot C_D \cdot \frac{\varepsilon_s \cdot \rho_g \cdot (1-\varepsilon_s) \cdot |u-v|}{d_p \cdot \phi} \cdot \varepsilon_s^{-2.65} \quad (6)$$

where the drag coefficient  $C_D$  is calculated by the expression of Richardson and Zaki (1954), as a function of particle Reynolds number:

$$C_D = \begin{cases} \frac{24}{\text{Re}_p} \cdot [1 + 0.15 \cdot (\text{Re}_p)^{0.687}] \\ 0.44 \end{cases} \quad (7)$$

$$\text{Re}_p = \frac{|u-v| \cdot \rho \cdot d_p \cdot \varepsilon}{\mu} \quad (8)$$

To avoid the discontinuity of the two equations, Gidaspow (1994) introduced a switch function that gave a rapid transition from one regime to the other:

$$\varphi_{\text{gs}} = \frac{\arctan[150 \times 1.75 (0.2 - (1-\varepsilon))]}{\pi} + 0.5 \quad (9)$$

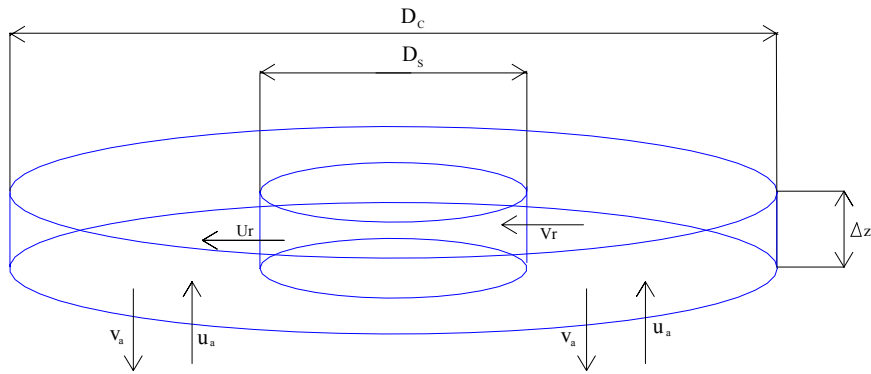
Thus, the fluid-particle interaction coefficient can be expressed as:

$$\beta = (1 - \varphi_{\text{gs}}) \beta_{\text{Ergun}} + \varphi_{\text{gs}} \beta_{\text{Wen-Yu}} \quad (10)$$

## 2.2. Conservation equations in the annular zone

Frictional stress plays an important role in the annulus of spouted beds due to the high solids volume fraction. In this case, the particles are enduring frictional contacts with multiple neighbours and the normal interaction forces and the associated tangential frictional forces due to sliding contacts have the major contribution to the particles stress

As in the spout, across any horizontal section in the annulus, Figure 3, it is assumed that the particles have uniform velocity,  $v_a$ , and the air has an absolute (interstitial) velocity,  $u_a$ .



**Figure 3.** Volume element in the annular zone

Concerning bed voidage, a constant and uniform value equal to that of the loose bed is assumed for the whole annulus. Then the continuity of air flow:

$$\frac{d[u_a(D_c^2 - D_s^2)]}{dz} = \frac{4U_r \cdot D_s}{\varepsilon_a} \quad (11)$$

and for continuity of particle flow

$$\frac{d[v_a(D_c^2 - D_s^2)]}{dz} = \frac{4D_s V_r}{(1 - \varepsilon_a)} \quad (12)$$

In a conical spouted bed, bed diameter changes with bed level according to the following expression:

$$D_c = D_i + 2 \cdot z \cdot \text{tg}(\gamma/2) \quad (13)$$

Introducing eq. 13 in eqs. 12 and 13 yields:

$$\frac{du_a}{dz} = \frac{4D_s U_r}{\varepsilon_a (D_c^2 - D_s^2)} - \frac{u_a D_c 4 \cdot \text{tg}(\gamma/2)}{(D_c^2 - D_s^2)} \quad (14)$$

$$\frac{dv_a}{dz} = \frac{4D_s V_r}{(1 - \varepsilon_a) \cdot (D_c^2 - D_s^2)} - \frac{4v_a D_c \cdot \text{tg}(\gamma/2)}{(D_c^2 - D_s^2)} \quad (15)$$

From a momentum balance on the air:

$$\rho_g \varepsilon_a \frac{d[u_a^2 (D_c^2 - D_s^2)]}{dz} + \varepsilon_a (D_c^2 - D_s^2) \frac{dP}{dz} + \beta (u_a - v_a) (D_c^2 - D_s^2) = 0 \quad (16)$$

And a momentum balance on the particles gives:

$$\rho_s (1 - \varepsilon_a) \frac{d[v_a^2 (D_c^2 - D_s^2)]}{dz} + (1 - \varepsilon_a) (D_c^2 - D_s^2) \frac{dP}{dz} + \rho_s g (1 - \varepsilon_a) + \beta (u_a - v_a) (D_c^2 - D_s^2) = 0 \quad (17)$$

### 2.3. Constitutive equations

The great advantage of the model is the requirement of only two empirical constitutive equations, one for obtaining the minimum spouting velocity and the other one for obtaining the pressure gradient through the bed. Apart from that, only the geometrical factors of the conical spouted bed and the physical properties of gas and particles are required.

The minimum spouting velocity is given by (Olazar et al., 1992):

$$(\text{Re}_0)_{ms} = 0.126 Ar^{0.5} \left( \frac{D_b}{D_0} \right)^{1.68} \left[ \text{tg} \left( \frac{\gamma}{2} \right) \right]^{-0.57} \quad (18)$$

A crucial information for the model is the evolution of pressure drop with bed level, which is obtained from the well-proven expression for calculating total pressure drop in conical spouted beds (Olazar et al., 1993b):

$$-\frac{\Delta P_s}{H_o \rho_b g} = 1.20 [\text{tg}(\gamma/2)]^{-0.11} (\text{Re}_0)_{ms}^{-0.06} \left( \frac{H_o}{D_0} \right)^{0.08} \quad (19)$$

Experimental observations have proven that this expression is also valid for calculating the pressure drop up to any bed level  $z$ :

$$-\frac{\Delta P_z}{z \rho_b \cdot g} = 1.20 [\text{tg}(\gamma/2)]^{-0.11} (\text{Re}_0)_{ms}^{0.06} \left( \frac{z}{D_0} \right)^{0.08} \quad (20)$$

Then, the absolute pressure at a given bed level  $z$  is given by:

$$P_z = \Delta P_z - \Delta P_s + P_{atm} = P_{atm} + 1.20 \cdot \rho_b \cdot g \cdot [\text{tg}(\gamma/2)]^{-0.11} (\text{Re}_0)_{ms}^{-0.06} \cdot D_0^{-0.08} [H_0^{1.08} - Z^{1.08}] \quad (21)$$

### 3. Results and discussion

In the present calculation, the dimensions of the contactor used are: contactor inlet diameter,  $D_0=0.04$  m; contactor bottom diameter, 0.06 m; angle of the contactor,  $33^\circ$ ; stagnant bed height, 0.20 m. The particles are glass spheres of 3 mm diameter and 2360 kg/m<sup>3</sup> density. Air at ambient temperature has been used as spouting agent.

The spout diameter is assumed to be constant and equal to the inlet diameter. The model also allows for considering a constant diameter different to the inlet one. In fact, experimental observations have shown that it is usually larger (San José et al., 1998) in conical spouted beds.

The boundary conditions are:

At the entrance, the gas injects in the axial direction with a velocity given by eq. 18 and inlet solid velocity is zero. Pressure drop is given by eq. 19.

At the outlet, pressure drop is atmospheric.

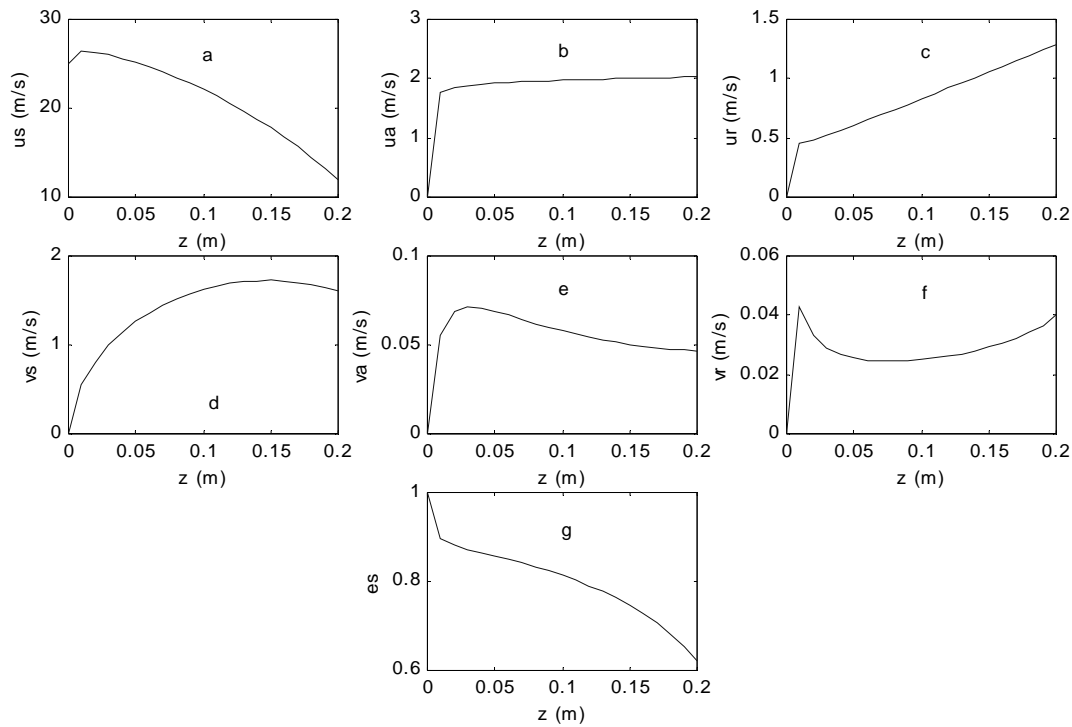
At the interphase between the spout and annulus, axial component of particle velocity is zero.

At the spout axis, velocity gradients for the two phases and their radial components are zero.

On the wall, a no slip boundary condition is assumed ( $u=v=0$ )

The model has been by using MATLAB (Version 6.5). The experimental results used for comparison are those obtained in previous papers (San José et al., 1995, 1998).

Figure 4 shows the values predicted by the model for the base case used for the simulation and the experimental ones for gas velocity and bed voidage along the spout (San José et al., 1998).

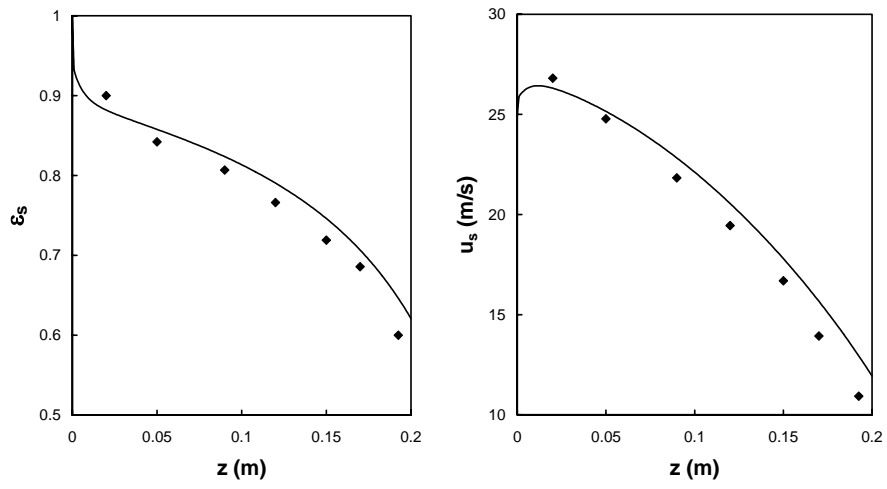


**Figure 4.** Values predicted by the model for the evolution with bed level (a) gas velocity in spout, (b) gas velocity in the annulus, (c) radial component of gas velocity at the interphase between spout and annulus, (d) particle velocity in the spout, (e) particles velocity in the annulus, (f) radial component of particle velocity at the interphase between spout and annulus, (g) bed voidage

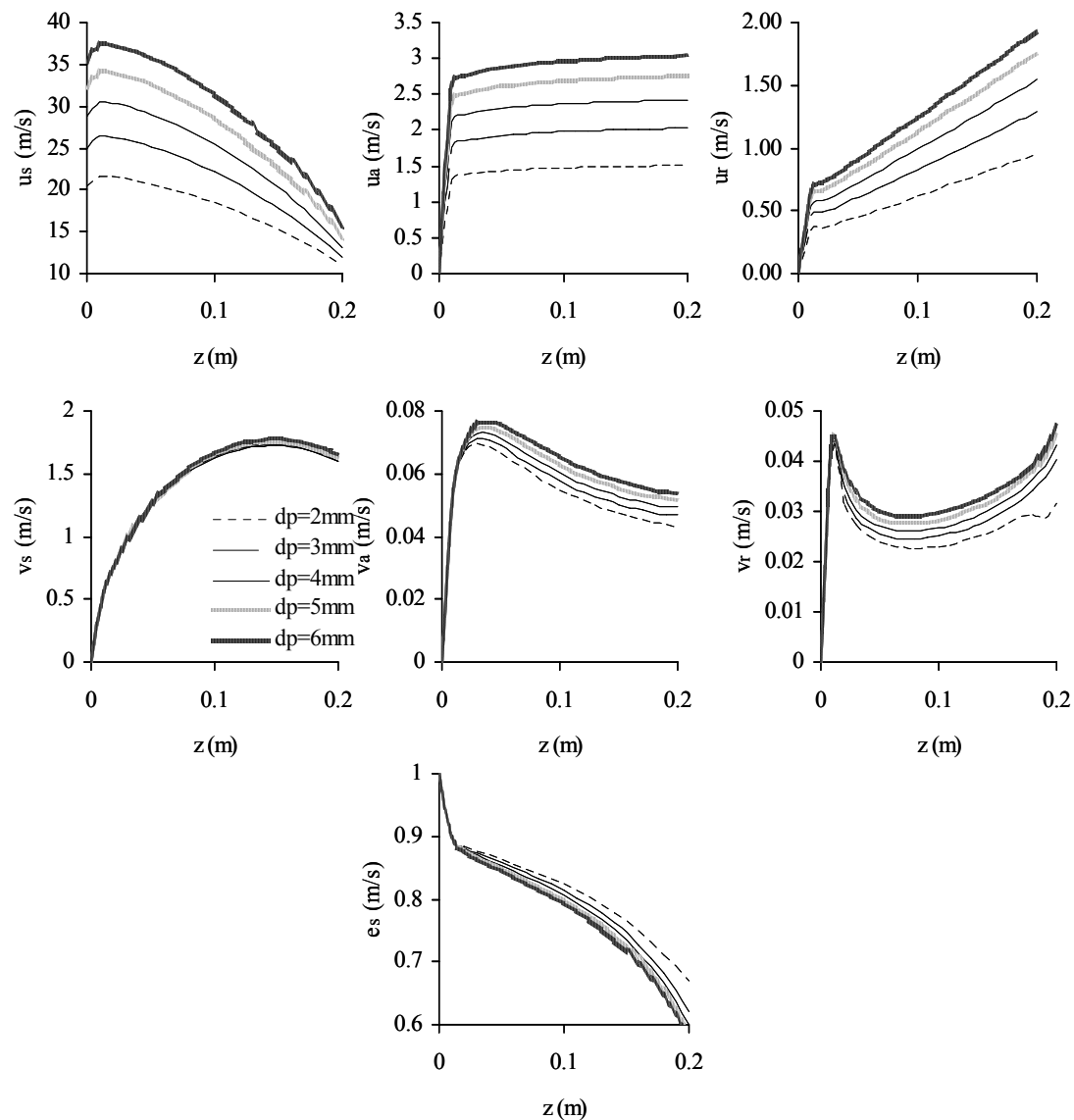
In order to validate the model, Figure 5 shows a comparison of the values obtained with the model and the experimental ones for bed voidage and gas velocity in the spout. As is observed, this simple one-dimensional model provides reasonably good predictions for these parameters. The worst prediction of the model is that corresponding to particle velocities in the spout, where it is not able to foresee the peak near the entrance.

Figure 6 shows the values predicted for different particle diameter using this model. As is observed, particle size greatly affects gas velocity but not particle velocity. Bed voidage along the spout decreases with particle size, which is consistent with the lower fountain heights experimentally observed for coarse particles.





**Figure 5.** Experimental values (points) and those calculated with the model (solid lines) for bed voidage (left graph) and gas velocity (right graph) along the spout.



**Figure 6.** Model predictions for different particles sizes

Based on the data obtained at the top of the spout and annulus, the model allows for calculating properties in the fountain, namely fountain height and the voidage at the core of the fountain. Thus, ignoring drag force in the fountain, which is reasonable for such a large particles, and assuming that particle velocity is decreasing only due to gravity, a fountain height of 0.13 mm is obtained for the base case (the experimental one is 0.15 cm). Cycle times may also be calculated using the radially averaged velocities in the spout and annulus for the particles. The theoretical value obtained for the base case is 3.8 s, which is slightly higher than the experimental one, 3.6 s.

#### **4. Conclusions**

A one-dimensional model based on mass and momentum equations for gas and solid phases has been proposed for conical spouted beds. The input parameters for the model are the minimum spouting velocity and the pressure drop evolution with bed level.

The model faithfully predicts radially averaged air velocities at different levels in the bed, in both spout and annulus. Predictions are also reasonable for radially averaged particle velocities in the annulus and solid cross-flow from the annulus into the spout. Nevertheless, this one-dimensional model is not able to predict that particle velocity in the spout peaks near the entrance of the contactor, although predictions of this magnitude near the surface are reasonable.

The information of the model allows to assess average cycle times and fountain heights. A great advantage of this one-dimensional model is its simplicity, which is especially interesting when hydrodynamic models are coupled with kinetic ones in the reactor design.

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## Nomenclature

$Ar$  = Archimedes number,  $g d_p^3 \rho (\rho_s - \rho) / \mu^2$

$C_D$  = drag coefficient,  $(24/Re)(1+0.15 Re^{0.687})$

$D_c, D_i, D_o, D_s$  = diameter of the column (or upper diameter needed for the cone), of the bed bottom, of the bed inlet and of the spout, respectively, m

$d_p$  = particle diameter, m

$g$  = acceleration of gravity,  $m s^{-2}$

$H_o$  = height of the stagnant bed, m

$\Delta P_s, \Delta P_z, P$  = spouting pressure drop, pressure drop from the inlet to a  $z$  level and pressure at any level, Pa

$(Re_o)_{ms}, Re_p$  = Reynolds number of minimum spouting referred to  $D_o$  and particle Reynolds number.

$U_r$  = radial gas velocity at the interphase between spout and annulus,  $m s^{-1}$

$u_a, u_s$  = interstitial gas velocity in the annulus and spout  $m s^{-1}$

$V_r$  = radial particle velocity at the interphase between spout and annulus,  $m s^{-1}$

$v_a, v_s$  = interstitial particle velocity in the annulus and spout  $m s^{-1}$

$z$  = bed level, m

## Greek Letters

$\beta$  = fluid-particle interaction coefficient

$\gamma$  = cone angle, degrees

$\epsilon_a, \epsilon_s$  = bed voidage in the annulus and spout bed of minimum spouting and of static bed

$\varphi_{gs}$  = parameter defined by eq 9

$\mu$  = viscosity,  $kg m^{-1} s^{-1}$

$\phi$  = particle sphericity

$\rho_b$ ,  $\rho_g$ ,  $\rho_s$  = bulk density, density of the gas and apparent density of the solid, respectively,  $\text{kg m}^{-3}$

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