

## **Nonlinear Prediction of Fluidized Bed Pressure Fluctuation**

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### **Abstract**

Nonlinear time series techniques have been applied to predict pressure fluctuation data in fluidized beds. The method of delays is used to reconstruct the state space attractor, by using time delay and embedding dimension, to carry out analysis in the reconstructed state space. Traditional linear auto-regression method and several state space state based prediction methods (SSBPMs), i.e., nearest neighbors, locally linear, and Kernel regression methods are applied to predict of pressure fluctuation signals. The quality of prediction is assessed by comparison of the predicted data for last segment of known sample time series of pressure signal with its original benchmark. In addition, chaotic invariants (dimension and entropy) of measured and predicted time series of pressure signals were compared. The results show that SSBPMs are preferred to the traditional linear method.

Keywords: nonlinear prediction, Kernel regression, locally linear, nearest neighbors, pressure fluctuation

### **1. Introduction**

Several researchers (e.g., van den Bleek and Schouten, 1993; van der Stappen et al., 1993) have found characteristics of low-dimensional nonlinear time series of pressure fluctuations in fluidized beds. The state of a fluidized bed (dissipative system) at a certain time could be determined by projecting all variables governing the system into a multidimensional space, i.e., the state space. The collection of the successive states of the system during its evolution in time is called the attractor. However, it is practically impossible to know all governing variables of a fluidized bed. Takens (1981) proved that the dynamic state of a system can be reconstructed from the time series of only one characteristic variable such as the local pressure in a fluidized bed. Time series of pressure measurements in a fluidized bed, extracted from Johnsson et al. (2000), for single and multiple bubble regimes has been considered in this work. The sampling frequency was 400 Hz for both fluctuating signals and 4096 samples were taken, corresponding to 10 sec of total sampling time. A great advantage of

pressure signals is that they include the effect of many different dynamic phenomena taking place in the bed, such as bubble formation, bubble coalescence, and bubble passage. Determining the time delay and the embedding dimension is considered as one of the most important steps in nonlinear time series modelling and prediction.

Mutual information function can be used to determine the optimum value of the time delay for the state space reconstruction, as first proposed by Fraser et al. (1986). The false nearest neighbors is a method of choosing the minimum embedding dimension of a time series, suggested by Kennel et al. (1992).

Prediction of future observations is an important problem in the analysis of time series and some existing SSBPMs, such as nearest neighbors (McNames, 1998), locally linear prediction (Kantz et al., 2002), Kernel regression method (Borovkova, 2001), and linear Autoregressive Moving Average (ARMA) method have been applied. First part of data, about 50 % for single bubble regime and 80 % for multiple bubble regimes, is considered as training section and last segment is used for prediction range. The prediction quality is determined by normalized error. In addition, to characterize the predicted attractor, maximum likelihood estimation of correlation dimension and the (Kolmogorov) entropy, Schouten et al. (1994a, b), of predicted time series is compared with actual signals.

## 2. Prediction methods

The question is to find point  $x(n+1)$  for a given time series  $x(n)$ , with  $n=1,2,\dots,N$ , where  $N$  is last point of time series.

In traditional linear models, such as the ARMA model, a future observation, point  $x(n+1)$ , is taken to be a linear combination of a certain number of previous observations and random, mostly Gaussian, disturbances.

In SSBPMs, the last known state of the system which is represented by vector (point in reconstructed attractor)  $X = [x(n), x(n-\tau), x(n-2\tau), x(n-(d-1)\tau)]$ , where  $d$  is the embedding dimension and  $\tau$  is the time delay, is determined to predict point  $x(n+1)$ . Then the time series is searched to find  $k$  similar states that have occurred in the past, where “similarity” is determined by evaluating the distance between vector  $X$  and its neighbor vector  $X'$  in the  $d$ -dimensional state space. The idea is that if the observable signal was generated by some deterministic map  $M(x(n), x(n-\tau), x(n-2\tau), x(n-(d-1)\tau)) = x(n+1)$ , that map can be recovered (reconstructed) from the data by looking at its behavior in the neighborhood of  $X$ . The map of  $M$  can be approximated by fitting a (low order) polynomial which maps  $k$  nearest neighbors (similar states) of  $X$  onto their next immediate values. Now it can be used this map to predict  $x(n+1)$ . In other words, with assumption that  $M$  is fairly smooth around  $X$ , and so if a state  $X' = [x'(n), x'(n-\tau), x'(n-2\tau), x'(n-(d-1)\tau)]$  in the neighborhood of  $X$  resulted in the observation  $x'(n+1)$  in the past, the point  $x(n+1)$  must be somewhere near  $x'(n+1)$ . In SSBM, such a model can be constructed from nearest neighbor, locally linear, and kernel regression models.

### 3. Results and Discussion

As it can be seen in Figure 1 (left), first minimum of average mutual information of single and multiple bubble regime time series occur at 36 and 20 respectively. The embedding dimension of 16 and 7 for single and multiple bubble regimes respectively obtained using the false nearest neighbor method as shown in Figure 1 (right).

In general, the quality of prediction was found to be good, Figures 2-3. Figure 2 demonstrates predicted signal versus measured time series for single bubble regime for different methods of prediction. As shown, semi-periodicity is the same as measured values with equal frequency. Figure 3 shows the same trends for multiple-bubble regime which has more chaotic behaviour than single bubble regime.

In addition, Figures 2 and 3 shows there is an uncertainty based on both time and magnitude errors. Three major sources of uncertainty are reconstructed attractor (embedding dimension and time delay), measurement noise, and predicted methods.

Normalized error is calculated for predicted time series in compare with original signal for each prediction method. In addition, a maximum likelihood estimation of the correlation dimension,  $D_{ML}$ , and Kolmogorov entropy,  $K_{ML}$ , of measured and predicted time series of pressure signals are compared. The correlation dimension expresses the number of degrees of freedom of the system and spatial complexity of the attractor in state-space, whereas the entropy is measures of the predictability of the system and the sensitivity to initial conditions.

The predictions errors, correlation dimensions, and Kolmogorov entropies corresponding to each method and time series are incorporated in the Table 1 below.

Table 1: Normalized Error,  $D_{ML}$ ,  $K_{ML}$  for original and predicted time series

Single Bubble Regime	Normalized Error	$D_{ML}$	$K_{ML}$	Multiple Bubble Regime	Normalized Error	$D_{ML}$	$K_{ML}$
Original		2.67	4.5	Original		7.09	7.33
Kernel Regression	1.17	2.91	6.18	Kernel Regression	0.84	7.18	7.47
Locally Linear	1.68	2.99	6.93	Locally Linear	1.11	5.36	6.44
Nearest Neighbors	1.82	2.91	5.23	Nearest Neighbors	1.97	6.06	5.71
Linear	17.2	1.21	4.94	Linear	16.7	3.89	7.34

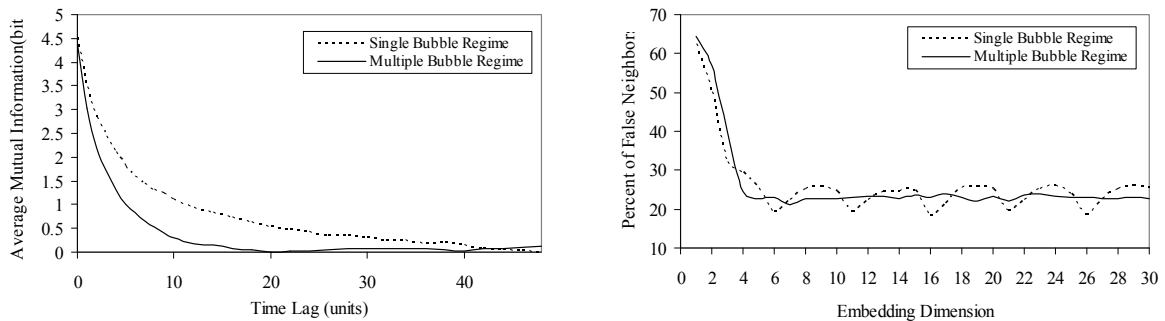


Figure 1: Average mutual information (left) and percent of false neighbors (right)

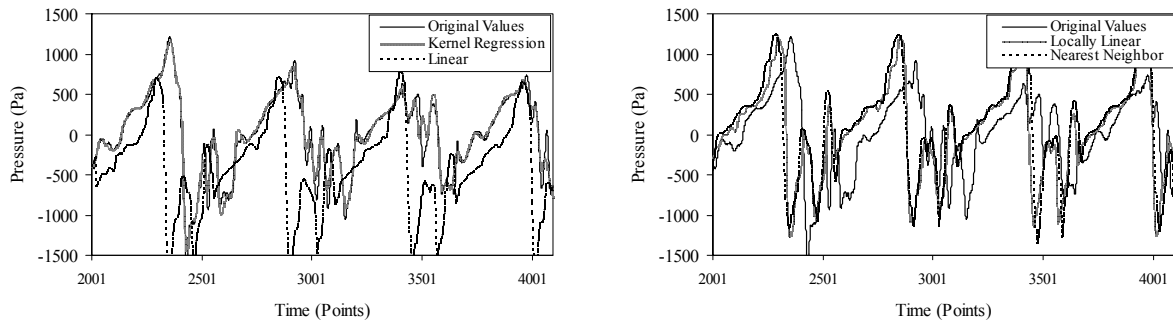


Figure 2: Original and predicted pressure fluctuations for single bubble regime with Kernel regression, linear (left), nearest neighbors, and locally linear (right) methods

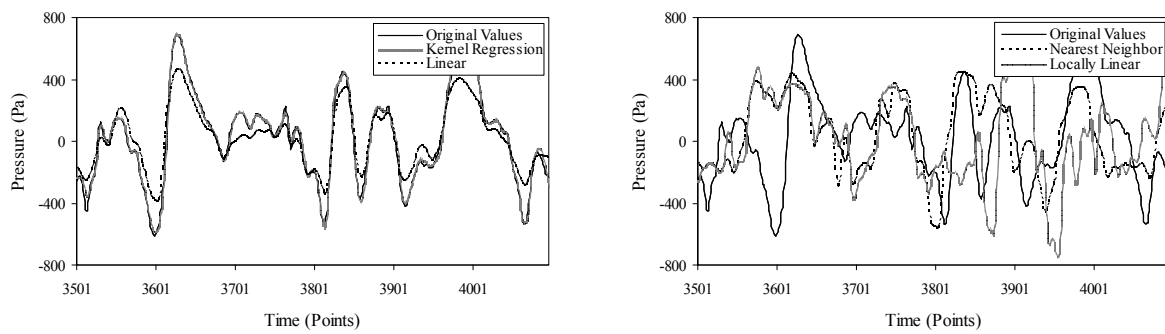


Figure 3: Original and predicted pressure fluctuations for multiple bubble regime with Kernel regression, linear (left), nearest neighbors, and locally linear (right) methods

#### 4. Conclusion

The predictions errors corresponding to each method and values of dimension and entropy show that the kernel method has a considerable advantage over all other methods for the fluidized bed time series. This is mostly because the fluidized bed time series is a good example of a chaotic time series, and the local nonlinear character of the Kernel method could best capture its local dynamics. The locally linear predictor and the nearest neighbors showed comparable performance, while the linear method performed much worse. This confirms the suggestion that for chaotic time series SSBPMs are preferred to the traditional linear method.

#### Notation

$d$	embedding dimension
$D_{ML}$	maximum likelihood estimation of Kolmogorov entropy
$k$	number of nearest neighbors around $X$
$K_{ML}$	maximum likelihood estimation of correlation dimension
$M$	deterministic map
$N$	Last point of time series
$x(n)$	time series with $n=1, 2, \dots, N$
$X$	reconstructed vector (point in reconstructed attractor)

$X'$   $X$  neighbor vector in reconstructed attractor  
 $\tau$  time delay

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